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# Computational Complexity in Timed Argumentation Frameworks

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### Abstract

Timed Argumentation Frameworks (TAFs) allow taking into account the availability of arguments and attacks in abstract argumentation. We propose a new reasoning approach for TAFs, where a standard Dung-style AF can be associated with each timepoint. We show that, although this framework is more expressive than Dung's framework, our approach does not lead to an increase in computational complexity for most reasoning problems and classical extension-based semantics.

## 1 Introduction

Abstract argumentation (Dung 1995) is a major topic in Knowledge Representation and Reasoning, allowing the elegant modeling of conflicting pieces of information and providing methods for reasoning in presence of this conflicting information. An Argumentation Framework (AF) is a directed graph where nodes represent the arguments and the edges are the attacks (*i.e.* oriented conflicts) between arguments. Classical reasoning methods, called *extension-based semantics* rely on so-called extensions, which are sets of jointly acceptable arguments (Dung 1995; Baroni, Caminada, and Giacomin 2018).

For modeling real-world scenarios, it may be necessary to take into account the actual availability of arguments through time. For instance, the natural language argument "I cannot go on vacation (in a particular city) because hotels are expensive" is not true all along the year. Timed AFs (TAFs) were proposed by Cobo, Martínez, and Simari (2010), where classical semantics are adapted to take into account the availability of arguments in the notion of defense. Timing was studied in different directions in argumentation (Mann and Hunter 2008; Cobo, Martínez, and Simari 2011b; Cobo, Martínez, and Simari 2011a; Pardo and Godo 2011; Barringer, Gabbay, and Woods 2012; Budán et al. 2012b; Budán et al. 2012a; Budán et al. 2015; Budán et al. 2015; Budán et al. 2017; Baumann and Heinrich 2020; Bistarelli et al. 2023a; Bistarelli et al. 2023b; Bistarelli et al. 2024). Here, we propose an alternative approach for reasoning of TAFs, which does not require adapting the semantics. We map a TAF to a set of AFs corresponding to different timepoints, and apply classical extension-based semantics to these AFs. This induces, for each classical decision problem, two variants: is a property (*e.g.* the credulous acceptability of a given argument) true with respect to at least one (possible view) or to each (necessary view) of these standard AFs. This kind of approach is reminiscent of reasoning with Incomplete AFs (Baumeister et al. 2018; Fazzinga, Flesca, and Furfaro 2020; Baumeister et al. 2021; Mailly 2022). Yet, in contrast to incomplete AFs where complexity of reasoning typically increases (Baumeister et al. 2018; Baumeister et al. 2021), we show that complexity of reasoning in TAFs coincides with classical AF reasoning for most tasks.

## 2 Background

We recall basics of abstract argumentation (Dung 1995).

**Definition 1.** *An argumentation framework (AF) is a pair $F = \langle A, R\rangle$, where $A$ is a finite non-empty set of arguments and $R \subseteq A \times A$ denotes an attack relation. If $(a, b) \in R$ then argument $a$ attacks argument $b$.*

In an AF $F = \langle A, R\rangle$ a set of arguments $S \subseteq A$ defends an $a \in A$ if $\forall(b, a) \in R$ we find a $(c, b) \in R$ with $c \in S$. Semantics can then be defined, as follows, for $\mathcal{F}_F(S) = \{a \in A \mid S \text{ defends } a\}$.

**Definition 2.** *Let $F = \langle A, R\rangle$ be an AF and $S \subseteq A$.*

- *$S$ is conflict-free in $F$ iff $\nexists a, b \in S$ s.t. $(a, b) \in R$.*
- *$S$ is an admissible extension in $F$ iff $S$ is conflict-free in $F$ and $S \subseteq \mathcal{F}_F(S)$.*
- *$S$ is a complete extension in $F$ iff $S$ is an admissible extension in $F$ and $S = \mathcal{F}_F(S)$.*
- *$S$ is the grounded extension in $F$ iff $S$ is conflict-free and $S$ is the least fixed point of $\mathcal{F}_F$.*
- *$S$ is a stable extension in $F$ iff $S$ is conflict-free in $F$ and $\forall a \in A \setminus S$, there is $b \in S$ with $(b, a) \in R$.*
- *$S$ is a preferred extension in $F$ iff $S$ is a $\subseteq$-maximal admissible extension in $F$.*

Reasoning for a given semantics, AF, and an argument includes credulous acceptance of the argument, i.e., is the argument in at least one extension of the chosen semantics, and skeptical acceptance, i.e., is the argument in all extensions of the chosen semantics. For admissible, complete, stable, and preferred semantics credulous acceptance is NP-complete and skeptical acceptance is trivial for admissible, in P for complete, coNP-complete for stable, and $\Pi_2^P$-complete for preferred semantics. Verifying whether a set of arguments is admissible, complete, or stable is in P, and coNP-complete for preferred semantics (Dvořák and Dunne 2018).

Cobo, Martínez, and Simari (2010) augment AFs with a time component, indicating availability of arguments.

**Definition 3.** *A Timed abstract Argumentation Framework (TAF) is a tuple $T = \langle A, R, Av_A, Av_R \rangle$ where*

- *$A$ is a set of arguments,*
- *$R \subseteq A \times A$ is an attack relation,*
- *$Av_A : A \to \mathfrak{P}(\mathbb{Q})$ is the availability function over arguments, and*
- *$Av_R : R \to \mathfrak{P}(\mathbb{Q})$ is the availability function over attacks.*

**Example 1(a)** (Vacation)**.** *Consider* e.g. *this example of decision making: where (and when) to go on vacation? There are three options: going to the grandparents' farm in the countryside ($g$), or visiting one of the nice cities of Toulouse ($t$) or Vienna ($w$). We assume that the person would prefer to go to Toulouse or Vienna over the grandparents' farm, but because of financial reasons, (s)he can not do it if the hotels in Vienna are expensive ($h$) or if the flights to Toulouse are expensive ($f$). The prices of hotels and flights evolve through time, so the arguments $h$ and $f$ are not always relevant, i.e., when prices are cheap then $h$ and/or $f$ are not available for reasoning. Figure 1 represents this scenario, with time intervals given close to the nodes $f$ and $h$. For a matter of simplicity, we assume that arguments and attacks without explicit time intervals are always available, and that time is restricted to months in $[Jan, Dec]$ with $Jan$ corresponding to January 2026 and $Dec$ to December 2026.*

## 3 Time-point Reasoning

In this paper, we assume that the availability of arguments and attacks is represented by a finite set of intervals. Formally, $Av_A(a)$ and $Av_R(a, b)$ both map to sets of rational intervals denoted by $I_x^y$. For the sake of a clearer presentation, we assume that each interval is closed, i.e., of the form $[x, y]$, indicating that the argument or attack is present between $x$ and $y$, including $x$ and $y$. We also allow for $x$ or $y$ to be $\infty$ or $-\infty$ (indicating no bounds).

In a TAF we identify for $t \in \mathbb{Q}$ an associated AF.

**Definition 4.** *Given a TAF $T = \langle A, R, Av_A, Av_R \rangle$, a time-point AF is $F_t = \langle A_t, R_t \rangle$ such that*

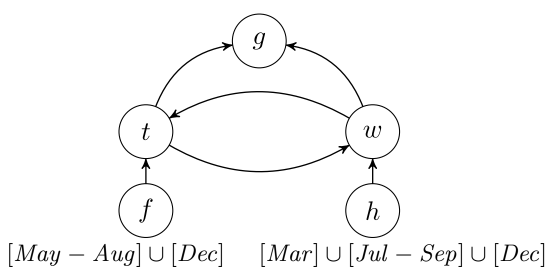



Figure 1: A Timed AF for deciding where to go on vacation

- *$A_t = \{a \in A \mid t \in Av_A(a)\}$,*
- *$R_t = \{(a, b) \in R \mid a, b \in A_t, t \in Av_R(a, b)\}$.*

Naturally, asking for argument acceptance at a specific time-point leads to reasoning coinciding to the classical case.

**Definition 5.** *Given a TAF $T = \langle A, R, Av_A, Av_R \rangle$, $a \in A$, $\sigma$ an extension-based semantics, and $t \in \mathbb{Q}$ a time-point, $a$ is $t$-credulously (resp. $t$-skeptically) acceptable if $a$ is credulously (resp. skeptically) acceptable in $F_t$.*

When considering a given time-point, the corresponding time-point AF can be directly obtained (in polynomial time): selecting available arguments and available attacks such that both arguments involved in the attack are available.

**Example 1(b)** (Vacation, cont.)**.** *Considering our running example, and the time-point $Mar$, we first compute the set of arguments $A_{Mar} = \{t, w, g, h\}$, and then $R_{Mar} = R \setminus \{(f, t)\}$. For the time-point $May$, on the contrary, we have $A_{May} = \{t, w, g, f\}$ and the corresponding attacks. In $Jul$ we have all the arguments and attacks, while in $Oct$ we have $A_{Oct} = \{t, w, g\}$ and $R_{Oct} = \{(t, w), (w, t), (t, g), (w, g)\}$.*

**Proposition 1.** *The complexity of checking $t$-credulous and $t$-skeptical acceptance coincides with classical credulous and skeptical acceptance, under admissible, complete, stable, and preferred semantics.*

To investigate more general notions of reasoning in TAFs, we make use of possible and necessary acceptance, borrowed from incomplete AFs (Baumeister et al. 2021).

**Definition 6.** *Given a TAF $T = \langle A, R, Av_A, Av_R \rangle$, $a \in A$, $I$ a closed interval, and $\sigma$ a semantics,*

- *$a$ is possibly credulously (resp. skeptically) accepted w.r.t. $I$ and $\sigma$ if $a$ is credulously (resp. skeptically) accepted w.r.t. $\sigma$ in some $F_t$ with $t \in I$, and*
- *$a$ is necessarily credulously (resp. skeptically) accepted w.r.t. $I$ and $\sigma$ if $a$ is credulously (resp. skeptically) accepted w.r.t. $\sigma$ in each $F_t$ with $t \in I$.*

**Example 1(c)** (Vacation, cont.)**.** *Continuing the previous example, we observe that argument $g$ is possibly skeptically accepted: in situations where both $f$ and $h$ are available (in $[Jul, Aug] \cup [Dec, Dec]$): when all arguments are present, there is a single extension $\{f, g, h\}$ under* e.g. *stable semantics. For similar reasons, when considering the whole interval $[Jan, Dec]$, $t$ (resp. $w$) is possibly skeptically accepted when only $h$ (resp. $f$) is available. For this reason, none of the arguments is necessarily accepted under the stable semantics.*

Towards complexity results, we next define some useful shorthands.

**Definition 7.** *For a TAF $T = \langle A, R, Av_A, Av_R \rangle$, let*

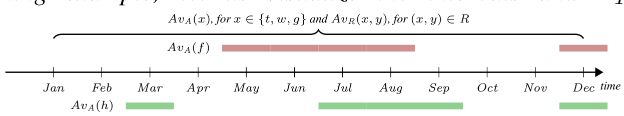
- *$I_T = \{I_x^y \mid I_x^y \in Av_A(a), a \in A$ or $I_x^y \in Av_R(r), r \in R\}$ be the set of intervals of arguments and attacks,*
- *$N_T = \{z \mid I_x^y \in I_T, x = z$ or $y = z, z \neq -\infty, z \neq \infty\}$ be the set of all start and end points of intervals,*
- *$S_T = \{x \mid I_x^y \in I_T\}$ the starting time-points and $E_T = \{y \mid I_x^y \in I_T\}$ the endpoints in $T$.*

In the next definition we define "invariant intervals", *i.e.*, intervals that represent *no change* is occurring during the time elapsing in this interval.

**Definition 8.** *Let $T = \langle A, R, Av_A, Av_R \rangle$ be a TAF. For $(t_1, \ldots, t_n)$ the sorted time-points $t_i \in N_T$ of $T$, define the invariant intervals of $T$ by the set $P_T$ containing exactly the following intervals:*

- $]-\infty, t_1[$ *and* $]t_n, \infty[$,
- $[t_i, t_i]$ *if* $t_i \in S_T \cap E_T$,
- $[t_i, t_{i+1}]$ *if* $t_i \in S_T \setminus E_T$ *and* $t_{i+1} \in E_T \setminus S_T$,
- $]t_i, t_{i+1}]$ *if* $t_i \in E_T$ *and* $t_{i+1} \in E_T \setminus S_T$,
- $[t_i, t_{i+1}[$ *if* $t_i \in S_T \setminus E_T$ *and* $t_{i+1} \in S_T$,
- $]t_i, t_{i+1}[$ *if* $t_i \in E_T$ *and* $t_{i+1} \in S_T$.

**Example 1(d)** (Vacation, cont.)**.** *Continuing the running example, let us visualize the intervals and $P_T$.*




$$P_T = \big\{]-\infty, Mar[, [Mar, Mar], ]Mar, May[, [May, Jul[,$$
$$[Jul, Aug], ]Aug, Sep], ]Sep, Dec[, [Dec, Dec], ]Dec, +\infty[\big\}$$

By restricting our attention to time-point AFs based on $P_T$ we do not miss any critical time-point AF, as witnessed by the next observation. For convenience, if $i \in P_T$ is some interval, let $F_i$ be some time-point AF $F_t$ with $t$ in the interval $i$. For instance, for $]-\infty, t_1[$ and $]t_n, \infty[$ we can consider any $t < t_1$ or $t > t_n$, respectively. And for all the other intervals, we can consider the mean of the bounds.[1]

**Lemma 1.** *Let $T = \langle A, R, Av_A, Av_{atts} \rangle$ be a TAF represented as intervals. It holds that*

$$\{F_t \mid t \in \mathbb{Q}\} = \{F_i \mid i \in P_T\}.$$

Regarding computation, note that we can obtain $P_T$ in polynomial time w.r.t. a given TAF $T$.

**Proposition 2.** *Let $T = \langle A, R, Av_A, Av_{atts} \rangle$ be a TAF represented as intervals. The set of intervals $P_T$ can be constructed in $\mathcal{O}(n \times \log(n))$ steps where $n = |\bigcup_{a \in A} Av_A(a)| + |\bigcup_{(a,b) \in R} Av_R((a, b))|$. Moreover, $|P_T|$ is linear w.r.t. $T$.*

Now we will establish the complexity of reasoning with TAFs. Notice that a TAF where all arguments and attacks are always available is equivalent to an AF, so AF reasoning induces lower bounds (Dvořák and Dunne 2018).

**Proposition 3.** *Given a TAF represented as intervals with rational numbers, the problem of deciding whether a given argument is*

- *possibly credulously accepted is NP-complete under admissible, complete, preferred, and stable semantics,*
- *necessarily credulously accepted is NP-complete under admissible, complete, preferred, and stable semantics,*
- *possibly skeptically accepted is trivial under admissibility, in P under complete semantics, coNP-complete under stable semantics and $\Pi_2^P$-complete under preferred semantics,*
- *necessarily skeptically accepted is trivial under admissibility, in P for complete semantics, coNP-complete under stable semantics and $\Pi_2^P$-complete under preferred semantics.*

*Proof.* For all problems, for a given TAF $T$, construct $P_T$, which can be computed in polynomial time, including restricting $P_T$ to a given interval.

For possibly credulously accepted arguments, consider the following non-deterministic algorithm. Guess an interval $i \in P_T$ and a subset of arguments available in $F_i$. Check whether this set is admissible (for admissible, complete, and preferred semantics) or stable in $F_i$ and whether the set contains the queried argument.

For necessarily credulously accepted arguments, guess for each $i \in P_T$ a subset of arguments available in the corresponding $F_i$. Check whether the subset is admissible (stable) in $F_i$ and whether the queried argument is part of the guessed subset.

For possibly skeptically accepted arguments, the algorithm depends on the semantics. For complete semantics, construct the grounded extension for $F_i$ for each $i \in P_T$ and check whether the queried argument is part of some grounded extensions. For stable semantics, consider the complementary problem. Guess for each $i \in P_T$ a subset of arguments in $F_i$. Check whether the queried argument is excluded in each guessed subset and whether each subset is stable in the corresponding $F_i$. For preferred semantics, also consider the complementary problem and guess for each $F_i$ a subset and check whether the subset excludes the queried argument and the subset is preferred in $F_i$.

For necessarily skeptically accepted arguments, again consider the semantics individually. For complete semantics, construct again the grounded extension for each $F_i$, $i \in P_T$ and check whether the queried argument is part of all these grounded extensions. For stable and preferred semantics, consider the complementary problems. Guess one $i \in P_T$ and a subset of arguments in $F_i$. Check whether this set is stable or preferred and whether the queried argument is excluded.

Hardness for all cases can be inferred from credulous and skeptical reasoning in AFs. □

Similarly, extension verification can be adapted.

**Definition 9.** *Given a TAF $T = \langle A, R, Av_A, Av_R \rangle$, $S \subseteq A$ and $\sigma$ an extension-based semantics,*

- *$S$ is a possible extension w.r.t. $I$ and $\sigma$ if $S$ is a $\sigma$-extension in $F_t$ for some $F_t$ with $t \in I$.*
- *$S$ is a necessary extension w.r.t. $I$ and $\sigma$ if $S$ is a $\sigma$-extension in $F_t$ for each $F_t$ with $t \in I$.*

**Proposition 4.** *Given a TAF represented as intervals with rational numbers, the problem of deciding whether a given set of arguments is*

[1]The proof of the next lemma and other missing proofs are available at https://hal.science/hal-05609909.

- *a possible extension is polynomial under admissible, complete, and stable semantics, and coNP-complete under preferred semantics,*
- *a necessary extension is polynomial under admissible, complete, and stable semantics, and coNP-complete under preferred semantics,*

Then, we are interested in the opposite question: when is an argument (or a set of arguments) acceptable? We start with the notion of acceptability period of an extension.

**Definition 10.** *Given a TAF* $T = \langle A, R, Av_A, Av_R \rangle$*,* $S \subseteq A$ *and* $\sigma$ *an extension-based semantics, the* acceptability period *of* $S$ *under* $\sigma$ *is the set of intervals* $\mathcal{I} = \{I_1, \dots, I_k\}$ *such that* $S$ *is a* $\sigma$*-extension for all time-points in* $I_1 \cup \dots \cup I_k$*, and no other time-points.*

**Example 1(e)** (Vacation, cont.)**.** *We continue the running example. The set* $S = \{t, h\}$ *is a stable extension when* $h$ *is available, and* $f$ *is not, so its acceptability period w.r.t. stable semantics is* $[Mar, Mar] \cup [Sep, Sep]$*.*

**Proposition 5.** *Given a TAF represented as intervals with rational numbers,* $S$ *a set of arguments, and* $\mathcal{I}$ *a set of intervals, the problem of deciding whether* $\mathcal{I}$ *is the acceptability period of* $S$ *is polynomial under the admissible, stable and complete semantics.*

We handle the preferred semantics separately, first focusing on a slightly simpler version of the problem. Indeed, in Proposition 5, we want to know if the set $\mathcal{I}$ is exactly the acceptability period of $S$. Now we want to determine (under the preferred semantics) if $\mathcal{I}$ is *part of* the acceptability period. Formally, given $\mathcal{I}$ a set of intervals and $\mathcal{I}_S$ the acceptability period of $S$ under preferred semantics, we write $\mathcal{I} \sqsubseteq \mathcal{I}_S$ if $\bigcup_{I \in \mathcal{I}} I \subseteq \bigcup_{I' \in \mathcal{I}_S} I'$.

**Proposition 6.** *Given a TAF represented as intervals with rational numbers,* $S$ *a set of arguments, and* $\mathcal{I}$ *a set of intervals, the problem of deciding whether* $\mathcal{I} \sqsubseteq \mathcal{I}_S$ *is coNP-complete under the preferred semantics.*

Regarding the verification of the acceptability period under the preferred semantics, we provide the following bounds.

**Proposition 7.** *Given a TAF represented as intervals with rational numbers,* $S$ *a set of arguments, and* $\mathcal{I}$ *a set of intervals, the problem of deciding whether* $\mathcal{I}$ *is the acceptability period of* $S$ *is coNP-hard and in DP under the preferred semantics.*

Although this remains to be proven, we conjecture this problem to be DP-complete.

Finally, we remark that, *e.g.*, for admissible semantics and credulous acceptance, one can compute all intervals (in $P_T$) where the argument is accepted via a linear number of NP oracle (SAT) calls, also in a parallel fashion. Thus, this problem is in $\text{FP}^{\text{NP}}_{||}$. A tight bound is currently open. However, hardness holds for $\text{FP}^{\text{NP}[log\ n]}$, *i.e.*, the functional problem allowing for a logarithmic number of NP oracle calls, via reducing from forms of Maximum Satisfiability. While the relation between $\text{FP}^{\text{NP}}_{||}$ and $\text{FP}^{\text{NP}[log\ n]}$ does not appear fully clarified in complexity theory, one can consider it plausible that these two classes are distinct (Jenner and Torán 1995). We further remark that the underlying oracles are for decision problems. In case of search problems, like is the case here, it was previously discussed that complexity classes permitting oracles to return whole witnesses might be a fruitful research direction (de Haan 2019), which we leave for future work.

**Proposition 8.** *Computing the set of intervals in* $P_T$ *where an argument is credulously accepted under admissible, complete, stable, or preferred semantics is* $\text{FP}^{\text{NP}[log\ n]}$*-hard, under metric reductions, and in* $\text{FP}^{\text{NP}}_{||}$*.*

*Proof sketch.* For hardness, we reduce from the $\text{FP}^{\text{NP}[log\ n]}$-complete problem of computing the maximum number of clauses in a Boolean formula in CNF that are simultaneously satisfiable (Krentel 1988). Construct for each $1 \leq k \leq C$, with $C$ the number of clauses, a formula $\phi_k$ being satisfiable if at least $k$ many clauses are satisfiable in the given formula (e.g., via standard cardinality constraints). Construct for each of the $\phi_k$ a time-point whose time-point AF directly corresponds to the AF that has a specific argument credulously accepted iff $\phi_k$ is satisfiable (using standard reductions) (Dvořák and Dunne 2018). □

## 4 Discussion

In previous works on Timed AFs (Cobo, Martínez, and Simari 2010; Cobo, Martínez, and Simari 2011b; Cobo, Martínez, and Simari 2011a) (and their subsequent generalization with support relations (Budán et al. 2017), or adaptations to structured argumentation (Budán et al. 2012a; Budán et al. 2015)) one relies on the notion of t-profiles, which are pairs of an argument associated with a set of time-points where this argument is available. Then, extension-based semantics are adapted using sets of t-profiles instead of sets of arguments. So, compared to our approach where time is associated to time-point AFs in order to compute sets of acceptable arguments, in these works time remains attached to arguments in a different notion of extension. Determining whether there are some formal connections between the result of reasoning with the t-profile-based approach or with our approach based on time-point AFs is an interesting future work.

There is a relation between our approach based on time-points and standard reasoning approaches with Incomplete Argumentation Frameworks (IAFs) (Baumeister et al. 2018; Fazzinga, Flesca, and Furfaro 2020; Baumeister et al. 2021; Mailly 2022), where arguments and attacks can be identified as uncertain, which induces a set of completions, *i.e.* standard AFs corresponding to the possible resolutions of the uncertainty. In dynamic contexts, uncertainty regarding the existence of arguments and attacks can be seen as a form of uncertainty about their availability (for instance, whether they will or will not be used in the next steps of a debate), and completions thus have a similar role to time-point AFs. However, since temporal information is not explicitly encoded in IAFs, it is not possible reason about when an argument is acceptable, only about the possibility of its acceptability. Recently, IAFs have been extended in various directions (*e.g.* adding constraints regarding the valid completions (Fazzinga, Flesca, and Furfaro 2021a; Fazzinga, Flesca, and Furfaro 2021b; Mailly 2024), adding supports relations (Fazzinga, Flesca, and Furfaro 2023; Lagasquie-Schiex, Mailly, and Yuste-Ginel 2024) or higher-order attacks (Doutre et al. 2025), or using quantitative reasoning (Fazzinga et al. 2024)). All these generalizations could be considered for future work in the context of Timed Argumentation Frameworks.

Our complexity results pave the way for algorithmic solutions of TAFs. Our results are that the complexity coincides, in many cases, with classical AF reasoning, which signals that SAT(isfiability) or Answer Set Programming (ASP (Brewka, Eiter, and Truszczynski 2011)) based approaches present themselves for implementations of TAFs. Additionally, since a linear number of, *e.g.*, SAT calls can determine the intervals an argument is credulously acceptable (e.g., under complete semantics), it is an interesting avenue whether associated concepts, such as Maximum SAT solving can be fruitfully applied in this scenario.

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## AI Declaration

The authors have not employed any Generative AI tools.

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