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Collective Satisfaction Semantics for Opinion Based Argumentation

Juliete Rossie[1], **Jérôme Delobelle**[2], **Sébastien Konieczny**[1], **Clément Lens**[1], **Srdjan Vesic**[1]
[1]CRIL, CNRS, Univ. Artois, Lens, France
[2]Université Paris Cité, LIPADE, F-75006 Paris, France
{rossie,konieczny,lens,vesic}@cril.fr, jerome.delobelle@u-paris.fr

Abstract

Voting on arguments in a debate is a natural approach for reaching a consensual decision. Despite this, there are few formal methods of abstract argumentation dealing with the use of votes in the process of selecting accepted arguments. We introduce the Opinion Based Argumentation (OBA) framework, where individuals can vote (or abstain) for or against arguments in a Dung argumentation framework. Our research aims to determine the most appropriate collective decisions within this framework. We propose a new semantics for this framework, called Collective Satisfaction Semantics (CSS), to evaluate the acceptability of arguments and study their properties. Additionally, we compare these semantics against alternative methods adapted from related literature to provide insights into their relative effectiveness.

1 Introduction

Argumentation stands as a powerful tool for e-democracy, facilitating the exchange of arguments and attacks among citizens when grappling with pivotal issues. However, for a true democratic outcome, the arguments (and attacks) alone are not enough to make a collective decision. It is imperative to gauge the level of agreement among the citizens and consider their collective sentiment. Several online platforms[1] exist that allow the collective creation of an argumentation graph, with some offering participants the ability to vote for or against the arguments. But these platforms serve as a visualization of the debate's current state, lacking a reasoning tool to analyze the arguments and evaluate the collective outcome of the debate. The simplest way for formally representing arguments and their interaction is Dung's (1995) abstract argumentation framework (AF) which can be seen as a directed graph where the arguments are the nodes and the attacks are the edges. In this work, this AF is assumed to be complete, i.e., that all the related arguments and attacks to the current debate are considered in the graph[2]. In particular, we suppose that invalid arguments and attacks have been removed from the argumentation graph. The completeness of the AF implies that it thoroughly addresses all pertinent issues. Classical semantics for reasoning with an AF are based on the notion of extensions, representing sets of arguments that can be jointly accepted and encode every possible decision within the AF (see (Baroni, Caminada, and Giacomin 2011) for an overview).

In this work, we introduce Opinion Based Argumentation Framework (OBAF) which extends Dung's AFs by allowing every individual to vote for or against each argument, but she can also abstain. We consider these extensions as encoding the constraints on the outcome of the voting process over the arguments. Then, the question is, given an OBAF, what are the optimal outcome(s)? To address this problem, we introduce a new family of semantics, called Collective Satisfaction Semantics, that we compare axiomatically with existing work dealing with a similar problem. These existing works include the work on judgment aggregation for argumentation by Caminada and Pigozzi (2011), where the AF is also complete and the agents can provide a labelling (Caminada 2006) on the set of arguments in the AF. The problem is to define a satisfactory collective labelling. Although the input in that work is different from ours, we show how to transform votes into labellings in order to compare approaches. Another method is provided by Bernreiter et al. (2024) exploring Approval-Based Social AFs (ABSAF). The approach begins with a predefined AF, a fixed set of agents, and approval ballots. Two operators are used to assign a score to each extension based on these ballots, subsequently selecting the "best" extension(s) according to a predefined semantics. This approach shares some similarities with our approach, but we highlight in Section 2.3 several important distinctions. Notably, ABSAF lacks neutral votes in its model, leading to information loss and potentially impacting the final outcome.

The different ways of representing votes on arguments in existing approaches led us to define a unified and complete framework called Opinion Based Argumentation (OBA) in addition to a unified semantics, Collective Opinion Semantics (COS). In Section 4, we show that the labelling aggregation approach (Caminada and Pigozzi 2011) and the approval ballots approach (Bernreiter et al. 2024) can be defined as COS. In the same section, we also define two new approaches: i) the first one is based on a combination of vote aggregation inspired by (Leite and Martins 2011) and the elimination of attacks used in preference-based argumentation frameworks (Amgoud and Cayrol 2002);

[1]DebateGraph, Kialo, idebate, DebateArt, Arguman, etc.

[2]In practice, it should be possible to construct the graph conjointly with the voting on the arguments, but we consider those as two separate steps in this work.

and ii) the second one is a family of methods, called Collective Satisfaction Semantics (CSS), based on the aggregation of the votes focusing on three different metrics (satisfaction, dissatisfaction, and utility) with utilitarian and egalitarian aggregation functions. In Section 5, we introduce a list of ten properties for COS used to perform an axiomatic analysis in Section 6, and show in particular that only CSS satisfies all essential properties we put forward.

2 Background Notions

In this section, we establish key definitions and concepts from existing literature to provide a foundation for our discussion and analysis.

2.1 Abstract Argumentation

An abstract argumentation framework (AF) (Dung 1995) serves as a fundamental structure, containing a set of arguments and a binary relation representing their attacks.

Definition 1 (AF). *An AF is a pair $\mathcal{F} = \langle \mathcal{A}r, att \rangle$ where $\mathcal{A}r$ is a finite set of arguments and att is a binary relation on $\mathcal{A}r$, i.e. $att \subseteq \mathcal{A}r \times \mathcal{A}r$, called the attack relation. A set of arguments $S \subseteq \mathcal{A}r$ attacks an argument $y \in \mathcal{A}r$ if there exists $x \in S$ such that $(x, y) \in att$.*

Dung also defines several extension-based semantics to select sets of arguments, called extensions, which can be collectively accepted based on chosen semantics' criteria for a given argumentation framework.

Definition 2 (Dung's Extensions). *Let $\mathcal{F} = \langle \mathcal{A}r, att \rangle$ be an AF and $\mathcal{E} \subseteq \mathcal{A}r$ be a set of arguments. An argument x is said acceptable w.r.t. $\mathcal{E}$ if for every argument y such that $(y, x) \in att$, there exists some argument $z \in \mathcal{E}$ such that $(z, y) \in att$. A set of arguments $\mathcal{E}$ is said admissible if each $x \in \mathcal{E}$ is acceptable w.r.t. $\mathcal{E}$ and is conflict-free, i.e., the attack relation does not hold for any pair of arguments belonging to $\mathcal{E}$. Then we say that $\mathcal{E}$ is:*

- *a complete extension, if it is an admissible set and every argument which is acceptable w.r.t. $\mathcal{E}$, belongs to $\mathcal{E}$;*
- *a preferred extension if it is any maximally (w.r.t. set inclusion) admissible set of $\mathcal{F}$;*
- *a stable extension if it is conflict-free and attacks every argument not belonging to $\mathcal{E}$.*

We represent the set of extensions for a semantics $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{stb}\}$ as $\mathcal{E}_\sigma(\mathcal{F})$.

An alternative representation of admissibility and Dung's semantics involves a labelling-based approach (Caminada 2006). In this approach, each argument is assigned one of three labels: `in` (accepted), `out` (rejected), or `undec` (undecided). The notion of reinstatement labelling ensures that the mapping takes the attack relation into account.

Definition 3 (Labellings). *Let $\mathcal{F} = \langle \mathcal{A}r, att \rangle$ be an AF. $\mathcal{L}$ is a labelling of $\mathcal{F}$ iff $\mathcal{L}$ is a mapping from $\mathcal{A}r$ to $\{\texttt{in}, \texttt{out}, \texttt{undec}\}$. We denote $\texttt{in}(\mathcal{L})$ as $\{x \in \mathcal{A}r \mid \mathcal{L}(x) = \texttt{in}\}$, $\texttt{out}(\mathcal{L})$ as $\{x \in \mathcal{A}r \mid \mathcal{L}(x) = \texttt{out}\}$ and $\texttt{undec}(\mathcal{L})$ as $\{x \in \mathcal{A}r \mid \mathcal{L}(x) = \texttt{undec}\}$. A labelling $\mathcal{L}$ is a reinstatement labelling of $\mathcal{F}$ iff $\forall x \in \mathcal{A}r$*

1. *$\mathcal{L}(x) = \texttt{in}$ iff $\forall y \in \mathcal{A}r$, $(y, x) \in att$, $\mathcal{L}(y) = \texttt{out}$;*
2. *$\mathcal{L}(x) = \texttt{out}$ iff $\exists y \in \mathcal{A}r$, $(y, x) \in att$ s.t. $\mathcal{L}(y) = \texttt{in}$;*
3. *$\mathcal{L}(x) = \texttt{undec}$ iff $\nexists y \in \mathcal{A}r$, $(y, x) \in att$ s.t. $\mathcal{L}(y) = \texttt{in}$ and $\exists z \in \mathcal{A}r$, $(z, x) \in att$ s.t. $\mathcal{L}(z) = \texttt{undec}$.*

A formal correspondence exists between extensions and labelling semantics (see (Caminada 2006; Baroni, Caminada, and Giacomin 2011) for an overview). For example, a complete labelling is exactly a reinstatement labelling whereas an admissible labelling is a labelling that satisfies only conditions 1 and 2 of Definition 3.

Example 1. *Let us consider the AF $\mathcal{F}$ represented in Figure 1 (left). The set of extensions of $\mathcal{F}$ with the extension-based semantics $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{stb}\}$ is:*
$\mathcal{E}_{pr}(\mathcal{F}) = \mathcal{E}_{stb}(\mathcal{F}) = \{\{a, b\}, \{b, c\}, \{c, d\}, \{a, e\}, \{c, e\}\}$
$\mathcal{E}_{co}(\mathcal{F}) = \mathcal{E}_{pr}(\mathcal{F}) \cup \{\emptyset, \{b\}, \{c\}, \{e\}\}$
$\mathcal{L} = \{(a, \texttt{out}), (b, \texttt{out}), (c, \texttt{in}), (d, \texttt{undec}), (e, \texttt{in})\}$ *is a labelling of $\mathcal{F}$. In the remainder of this paper, we will also use the simplified notation $\mathcal{L} = (\{c, e\}, \{a, b\}, \{d\})$ where the first set represents arguments with the label `in`, the second set contains arguments with the label `out` and the third set represents arguments with the label `undec`.*

Caminada and Pigozzi (2011) introduced down-admissible and up-complete labellings. These concepts retrieve the closest admissible (resp. complete) labelling to a given labelling, as formally defined by Gabbay and Rodrigues (2014).

Definition 4. *(Down-admissible) Let $\mathcal{L}$ be a labelling of the AF $\mathcal{F} = \langle \mathcal{A}r, att \rangle$. The down-admissible labelling of $\mathcal{L}$ is the biggest element $\mathcal{L}'$ of the set of all admissible labellings s.t. $\texttt{in}(\mathcal{L}') \subseteq \texttt{in}(\mathcal{L})$ and $\texttt{out}(\mathcal{L}') \subseteq \texttt{out}(\mathcal{L})$.*

Definition 5. *(Up-complete) Let $\mathcal{L}$ be an admissible labelling of the AF $\mathcal{F} = \langle \mathcal{A}r, att \rangle$. The up-complete labelling of $\mathcal{L}$ is the smallest element $\mathcal{L}'$ of the set of all complete labellings s.t. $\texttt{in}(\mathcal{L}') \supseteq \texttt{in}(\mathcal{L})$ and $\texttt{out}(\mathcal{L}') \supseteq \texttt{out}(\mathcal{L})$.*

Following the previous definitions, the largest and smallest labelling are defined in terms of the inclusion of arguments in the "in" and "out" sets. According to (Caminada and Pigozzi 2011), a labelling $\mathcal{L}_1$ is smaller or equal to a labelling $\mathcal{L}_2$ iff $\texttt{in}(\mathcal{L}_1)$ is included or equal to $\texttt{in}(\mathcal{L}_2)$ and $\texttt{out}(\mathcal{L}_1)$ is included or equal to $\texttt{out}(\mathcal{L}_2)$. This comparison allows them to establish a partial order on a set of labellings and select the biggest or smallest from this set. The uniqueness and well-definition of these elements are established (Caminada and Pigozzi 2011, Theorems 5 and 11).

2.2 Labellings Aggregation Operators

Caminada and Pigozzi (2011) focus on the challenge of combining individual views into a coherent group decision. The general idea is to aggregate a set of labellings (each labelling can be seen as an agent's point of view regarding the acceptability of arguments in an AF) to obtain a collective labelling. The result of this aggregation can be obtained using a labelling aggregation operator which is a function $LA_\mathcal{F} : 2^{\mathcal{L}abellings} - \{\emptyset\} \rightarrow \mathcal{L}abellings$ where $\mathcal{L}abellings$ is the set of all labellings of an AF $\mathcal{F}$. They define three labelling aggregation operators: skeptical, credulous, and super credulous.

Skeptical Aggregation Operator The skeptical aggregation operator requires the unanimous agreement among all labellings for an argument to be initially accepted or rejected. All other arguments are left `undec`. A second phase is carried out to determine the down-admissible labelling of the result, as it may not always be admissible.

Definition 6 (Skeptical Operator). *The skeptical initial aggregation operator is a function* $sio_{\mathcal{F}} : 2^{\mathcal{L}abellings} - \{\emptyset\} \to \mathcal{L}abellings$ *such that* $sio_{\mathcal{F}}(\{\mathcal{L}_1, ..., \mathcal{L}_m\}) =$
$\{(a, \texttt{in}) \mid \forall i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{in}\} \cup$
$\{(a, \texttt{out}) \mid \forall i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{out}\} \cup$
$\{(a, \texttt{undec}) \mid \exists i \in [1, ..., m] : \mathcal{L}_i(a) \neq \texttt{in} \land \exists i \in [1, ..., m] : \mathcal{L}_i(a) \neq \texttt{out}\}$.
The skeptical aggregation operator $so_{\mathcal{F}}(\{\mathcal{L}_1, ..., \mathcal{L}_m\})$ *is the down-admissible labelling of* $sio_{\mathcal{F}}(\{\mathcal{L}_1, ..., \mathcal{L}_m\})$.

Example 2. *Let us consider the argumentation framework* $\mathcal{F}$ *represented in Figure 1 and the set of labellings* $\mathcal{L}abs = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$ *such that* $\mathcal{L}_1 = (\{c, e\}, \{a, b\}, \{d\})$, $\mathcal{L}_2 = (\{e\}, \{a\}, \{b, c, d\})$, *and* $\mathcal{L}_3 = (\{b, c\}, \{a, e\}, \{d\})$.
We have $sio_{\mathcal{F}}(\mathcal{L}abs) = (\emptyset, \{a\}, \{b, c, d, e\})$. *The result is not an admissible labelling of* $\mathcal{F}$*, so we need to determine its down-admissible labelling, which gives* $so_{\mathcal{F}}(\mathcal{L}abs) = (\emptyset, \emptyset, \{a, b, c, d, e\})$ *where all arguments are* `undec`.

Credulous Aggregation Operator The credulous aggregation operator serves as a more lenient counterpart of the skeptical aggregation operator. The idea is to initially accept (resp. reject) an argument if it is accepted (resp. rejected) in at least one labelling and there is no labelling going against this decision. All other arguments are left `undec`. Similarly to the skeptical operator, in the second phase, we consider the down-admissible labelling of the initial result.

Definition 7 (Credulous Operator). *The credulous initial aggregation operator is a function* $cio_{\mathcal{F}} : 2^{\mathcal{L}abelling} - \{\emptyset\} \to \mathcal{L}abelling$ *such that* $cio_{\mathcal{F}}(\{\mathcal{L}_1, ..., \mathcal{L}_m\}) =$
$\{(a, \texttt{in}) | \exists i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{in} \land \nexists i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{out}\} \cup$
$\{(a, \texttt{out}) | \exists i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{out} \land \nexists i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{in}\} \cup$
$\{(a, \texttt{undec}) | \forall i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{undec} \lor (\exists i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{in} \land \exists i \in [1, ..., m] : \mathcal{L}_i(a) = \texttt{out})\}$.
The credulous aggregation operator $co_{\mathcal{F}}(\{\mathcal{L}_1, \dots, \mathcal{L}_m\})$ *is the down-admissible labelling of* $cio_{\mathcal{F}}(\{\mathcal{L}_1, ..., \mathcal{L}_m\})$.

Example 2 (cont.). *Let us consider the set of labellings* $\mathcal{L}abs = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$ *such that* $\mathcal{L}_1 = (\{c, e\}, \{a, b\}, \{d\})$, $\mathcal{L}_2 = (\{e\}, \{a\}, \{b, c, d\})$, *and* $\mathcal{L}_3 = (\{b, c\}, \{a, e\}, \{d\})$.
We have $cio_{\mathcal{F}}(\mathcal{L}abs) = (\{c\}, \{a\}, \{b, d, e\})$. *The result of cio is an admissible labelling of* $\mathcal{F}$*, so* $co_{\mathcal{F}}(\mathcal{L}abs) = cio_{\mathcal{F}}(\mathcal{L}abs)$.

Super Credulous Aggregation Operator The super credulous aggregation operator expands the credulous aggregation operator using the up-complete labelling. Thus, arguments considered as `undec` by the credulous operator *co* can be accepted (resp. rejected) if all their direct attackers are rejected (resp. at least one of these attackers is accepted).

Definition 8 (Super Creduous Operator). *The super credulous aggregation operator* $sco_{\mathcal{F}}(\{\mathcal{L}_1, ..., \mathcal{L}_m\})$ *is the up-complete labellings of* $co_{\mathcal{F}}(\{\mathcal{L}_1, ..., \mathcal{L}_m\})$.

Example 2 (cont.). *Let us consider the set of labellings* $\mathcal{L}abs = \{\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3\}$ *such that* $\mathcal{L}_1 = (\{c, e\}, \{b, a\}, \{d\})$, $\mathcal{L}_2 = (\{e\}, \{a\}, \{c, b, d\})$, *and* $\mathcal{L}_3 = (\{b, c\}, \{a, e\}, \{d\})$.
We have $co_{\mathcal{F}}(\mathcal{L}abs) = (\{c\}, \{a\}, \{b, d, e\})$ *which is a complete labelling of* $\mathcal{F}$*, so* $sco_{\mathcal{F}}(\mathcal{L}abs) = co_{\mathcal{F}}(\mathcal{L}abs)$.

2.3 Approval-Based Social AFs (ABSAF)

Bernreiter et al. (2024) focus on the challenge of combining approval ballots and an AF. To this end, they introduce Approval-Based Social Argumentation Frameworks (ABSAFs) which depict discussions where agents endorse arguments they find persuasive.

Definition 9. *(ABSAF) An ABSAF* $\mathcal{A}b = (\mathcal{F}, N, \bar{A})$ *consists of an AF* $\mathcal{F} = \langle \mathcal{A}r, att \rangle$*, a finite set of voters N and a vector* $\bar{A} = (A(i))_{i \in N}$ *of approval ballots where* $\forall i \in N$, $A(i) \subseteq \mathcal{A}r$ *with* $A(i) \neq \emptyset$ *is the set of arguments approved by* i.

From Definition 9, this framework defines votes as non-empty sets of arguments. The outcome Ω of an ABSAF $\mathcal{A}b = (\mathcal{F}, N, \bar{A})$ is a set of extensions w.r.t. an extension-based semantics σ, i.e., $\Omega \subseteq \mathcal{E}_\sigma(\mathcal{F})$. This implies that voters can only express their approval, leaving other arguments ambiguous as voters can either disagree or remain neutral towards them. This design likely stems from the definition of the operator that will follow, which solely considers the positive votes.

Representation operator The primary focus of this operator is identifying an outcome that represents a broad spectrum of voters. This operator defines the degree to which an extension represents a voter.

Definition 10. *(Representation operator) Let* $\mathcal{A}b = (\mathcal{F}, N, \bar{A})$ *be an ABSAF and* σ *be an extension-based semantics. The score of an extension* $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$ *w.r.t. a voter* $i \in N$ *is* $rep_i(\mathcal{E}) = \frac{|\mathcal{E} \cap A(i)|}{|A(i)|}$.
The score of an outcome $\Omega \subseteq \mathcal{E}_\sigma(\mathcal{F})$ *takes the maximum score among its extensions:* $rep_i(\Omega) = \max_{\mathcal{E} \in \Omega} rep_i(\mathcal{E})$.

Core representation operator As the set of arguments approved by a voter i (i.e., $A(i)$) does not necessarily correspond to an existing extension, Bernreiter et al. propose a variant of the representation operator to ensure that at least one extension obtains the maximum score of 1 even if it does not correspond perfectly to any $A(i)$.

Definition 11. *(Core representation operator) Let* $\mathcal{A}b = (\mathcal{F}, N, \bar{A})$ *be an ABSAF and* σ *be an extension-based semantics. For* $i \in N$ *let* $\mu(i) = max_{\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})} |\mathcal{E} \cap A(i)|$. *If* $\mu(i) = 0$ *the core-score of an extension* $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$ *is* $rep_i^c(\mathcal{E}) = 1$*, otherwise it is* $rep_i^c(\mathcal{E}) = \frac{|\mathcal{E} \cap A(i)|}{\mu(i)}$.

To denote these two operators interchangeably we use $op \in \{rep, rep^c\}$. If $op = rep$ then $op_i(\mathcal{E}) = rep_i(\mathcal{E})$.

Approval ballot aggregation In order to optimally represent the voters in an ABSAF with a fixed number of extensions, Bernreiter et al. use a family of rules based on Ordered Weighted Averaging (OWA) vectors.

Definition 12. *(OWA) Let* $\mathcal{A}b = (\mathcal{F}, N, \bar{A})$ *be an ABSAF with* $|N| = n$*,* σ *be an extension-based semantics* *and $k \in \{1,\ldots,n\}$. Given an outcome $\Omega \subseteq \mathcal{E}_\sigma(\mathcal{F})$, let $\vec{s}(\Omega)$ be the vector sorted in non-decreasing order $\vec{s}(\Omega) = (op_1(\Omega),\ldots,op_n(\Omega))$ with $op \in \{rep, rep^c\}$. For a non-increasing vector of non-negative weights $\vec{w} = (w_1,\ldots,w_n)$, where $w_1 > 0$, the corresponding OWA rule is defined as follows:*

$$OWA_\sigma^{\vec{w},op}(\mathcal{A}b) \in \underset{\Omega\subseteq\mathcal{E}_\sigma(\mathcal{F}):|\Omega|\leq k}{\operatorname{argmax}} \vec{w}\cdot\vec{s}(\Omega)$$

Tthe egalitarian, utilitarian, and harmonic rules are denoted respectively by e, u, and h. By adjusting the vector $\vec{w}$ in the previous definition, the egalitarian rule is given by $\vec{w}_e = (1,0,\ldots,0)$, the utilitarian rule is given by $\vec{w}_u = (1,\ldots,1)$, and the harmonic rule is given by $\vec{w}_h = (1,1/2,\ldots,1/n)$.

Another method consists in selecting the set of extensions Ω which maximize the number of voters whose set of arguments approved by these agents obtains a score of 1.

Definition 13. *(MaxCov) Let $\mathcal{A}b = (\mathcal{F}, N, \bar{A})$ be an ABSAF with $|N| = n$, σ be an extension-based semantics and $k \in \{1,\ldots,n\}$. Given an outcome $\Omega \subseteq \sigma(\mathcal{F})$ and $op \in \{rep, rep^c\}$. The max cover rule is defined as follows:*

$$MaxCov_\sigma^{op}(\mathcal{A}b) \in \underset{\Omega\subseteq\mathcal{E}_\sigma(\mathcal{F}):|\Omega|\leq k}{\operatorname{argmax}} |\{i\in N : op_i(\Omega) = 1\}|$$

The MaxCov rule is denoted as mc.

Example 3. *Consider an ABSAF $\mathcal{A}b = (\mathcal{F}, N, \bar{A})$ where $\mathcal{F}$ is the AF represented in Figure 1, with voters $N = \{1,2,3\}$ and $\bar{A} = (\{c,e\},\{e\},\{b,c\})$. Let us recall that $\mathcal{E}_{pr}(\mathcal{F}) = \{\{a,b\},\{b,c\},\{c,d\},\{a,e\},\{c,e\}\}$. For example, for $\{b,c\}$, we obtain $\vec{s}(\{b,c\}) = (0,\frac{1}{2},1)$ because $rep_1(\{b,c\}) = \frac{1}{2}$, $rep_2(\{b,c\}) = 0$, and $rep_3(\{b,c\}) = 1$. Thus, using the utilitarian rule, we obtain $\vec{w}_u\cdot\vec{s}(\{b,c\}) = (1,1,1)\cdot(0,\frac{1}{2},1) = \frac{3}{2}$. Applying the same reasoning to the other extensions, we obtain $OWA_{pr}^{\vec{w}_u,rep}(\mathcal{A}b) = \{\{c,e\}\}$.*

Before describing our contributions in the following sections, we would like to clarify several points in relation to the approaches defined in (Bernreiter et al. 2024). First, Definitions 12 and 13 specify that k denotes the number of required extensions in the final set. In Definition 12, for $k > 1$, any set of extensions containing the highest-scoring extension(s) will obtain that score (because $rep(\Omega)$ uses the function *max*). Similarly, Definition 13 specifies that any set of arguments containing the 1-scoring extension(s) will obtain the highest score. Consequently, we adopt $k = 1$ for the remainder of this paper to evaluate the score of each extension individually. Extensions with the highest score will all be included in the outcome if there is a tie. A second important remark is that, in (Bernreiter et al. 2024), the OWA and MaxCov rules are only defined for the preferred semantics. However, we generalize their definitions to other extension-based semantics, enabling a more comprehensive study of the properties associated with these methods. Finally, as detailed in Section 4.2, the status of arguments outside the approval ballot is unclear (rejection or abstention). Adapting approval ballots to our voting framework required a modification to address this issue.

3 Opinion Based Argumentation

After establishing the foundational concepts in the previous section, we now shift our focus to our first contribution, Opinion Based Argumentation (OBA). In introducing OBA, our focus lies in representing the collective opinions of voters regarding a set of arguments. OBA frameworks (OBAF) serve as a structure containing an AF alongside a tuple of votes, enabling the representation of diverse perspectives within a given context. This framework facilitates the assessment and evaluation of collective opinions expressed by voters through their stances on arguments. In OBAFs, voters express their preferences for each argument by assigning a value: 1 indicates acceptance, 0 indicates abstention, and -1 indicates rejection.

Definition 14 (Votes)**.** *Let $\mathcal{F} = \langle \mathcal{A}r, att\rangle$ be an AF. Votes on $\mathcal{A}r$, denoted as $\mathcal{V}_{\mathcal{A}r} = \langle v_1,\ldots,v_n\rangle$, represents the system's votes. Each vote $v_i \in \mathcal{V}_{\mathcal{A}r}$ is a function $v_i : \mathcal{A}r \to \{-1,0,1\}$ which assigns value for each argument in $\mathcal{F}$, indicating the voters' stance. Given $x \in \mathcal{A}r$, we note $v^+(x) = \{v_i \in \mathcal{V}_{\mathcal{A}r} \mid v_i(x) = 1\}$, $v^o(x) = \{v_i \in \mathcal{V}_{\mathcal{A}r} \mid v_i(x) = 0\}$ and $v^-(x) = \{v_i \in \mathcal{V}_{\mathcal{A}r} \mid v_i(x) = -1\}$ the set of votes assigning 1, 0 or -1 respectively to x.*

In practice, various kinds of votes could be used for voting, such as a larger scale than the 3-level scale (-1,0,1). We deliberately adhere to the (-1, 0, 1) scale (Jacoby and Matell 1971), since in our framework we assume that the votes of the participants represent their beliefs (So, by voting, they answer the question ”do you believe that this argument is true or not”). Beliefs inherently entail binary states of acceptance or rejection, parallel to logic, where a formula is either entailed or not (binary evolution). This results in three possible states: the formula is implied, its negation is implied, or neither is implied. Thus, the three-level scale is particularly suitable for accurately capturing belief states. Unlike preference frameworks, where degrees of preference can exist, belief systems are typically dichotomous. We believe adding the neutral vote (abstaining from voting with 0) is crucial. Allowing voters to abstain from expressing a belief prevents undue influence on the voting outcome, thereby fostering a more democratic decision-making process. Nonetheless, it is worth noting that our OBA framework can be seamlessly adapted into a preference framework by extending the voting scale to include additional values representing varying degrees of preference. We leave this idea for future work.

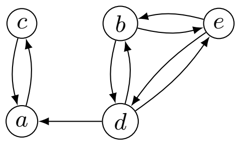
Let us now formally introduce OBAFs.

Definition 15 (OBAF)**.** *An Opinion Based Argumentation Framework (OBAF) is a pair $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ where $\mathcal{F} = \langle \mathcal{A}r, att\rangle$ is an AF and $\mathcal{V}_{\mathcal{A}r}$ are the votes on $\mathcal{A}r$.*

Example 4. *Figure 1 represents an OBAF $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ where $\mathcal{F}$ is the AF (left) and $\mathcal{V}_{\mathcal{A}r} = \{v_1, v_2, \ldots, v_6\}$ (right).*

We will only require one rationality constraint for the votes, that is that a voter can not vote (i.e., assign the positive value 1) to two arguments involved in an attack. As we suppose that the AF is known by all voters, and that they adhere to the basic property of conflict-freeness of argumentation.

Definition 16 (Votes consistency)**.** *Let $\mathcal{O} = \langle\langle \mathcal{A}r, att\rangle, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF. $\mathcal{V}_{\mathcal{A}r}$ is consistent iff*



$\mathcal{V}_{\mathcal{A}r}$	a	b	c	d	e
$v1$	1	−1	0	−1	1
$v2$	1	0	0	−1	1
$v3$	1	−1	0	−1	1
$v4$	−1	0	1	0	1
$v5$	1	0	0	−1	−1
$v6$	1	1	0	−1	−1

Figure 1: An Opinion Based Argumentation framework $\mathcal{O}$

$\nexists v \in \mathcal{V}_{\mathcal{A}r}$ *s.t.* $v(x) = v(y) = 1$ *and* $(x, y) \in att$.

4 Collective Opinion Semantics

In order to evaluate the arguments of an OBAF, we introduce a new family of semantics called Collective Opinion Semantics (COS) which can be seen as a general representation of a semantics that allows opinion aggregation and will be used in our comparative study. In this section, we start by defining the COS, followed, in Sections 4.1 and 4.2, by the adaptation of existing works to align with COS. Then we propose two new semantics in Sections 4.3 and 4.4.

Definition 17 ($\texttt{COS}$)**.** *Let $\mathcal{O} = \langle\langle\mathcal{A}r, att\rangle, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF. A Collective Opinion Semantics is a function* $\texttt{COS} : \mathcal{O} \to 2^{2^{\mathcal{A}r}}$.

4.1 Caminada and Pigozzi's Approach

Although the Caminada and Pigozzi's approach defined in Section 2.2 is not explicitly a COS, it is possible to adapt this approach to match the definition of COS. The idea is to transform each vote in $\mathcal{V}_{\mathcal{A}r}$ into a labelling, then apply one of the operators defined in Section 2.2 to obtain the collective labelling, which will then be transformed back into an extension.

Definition 18 (Vote2Lab)**.** *Let $\mathcal{V}_{\mathcal{A}r}$ be the votes on $\mathcal{A}r$ and $v \in \mathcal{V}_{\mathcal{A}r}$. The function $Vote2Lab(v) : \{-1, 0, 1\}^{\mathcal{A}r} \to \mathcal{L}abellings$ transforms a vote v into its corresponding label as follows: $Vote2Lab(v) = \{(x, \texttt{in}) \mid v(x) = 1\} \cup \{(x, \texttt{out}) \mid v(x) = -1\} \cup \{(x, \texttt{undec}) \mid v(x) = 0\}$.*

Example 5. *In Figure 1, $Vote2Lab(v_1) = (\{a, e\}, \{b, d\}, \{c\})$ and $Vote2Lab(v_6) = (\{a, b\}, \{d, e\}, \{c\})$.*

To switch from a labelling $\mathcal{L}$ to an extension in a given AF $\mathcal{F}$, we use the function $Lab2Ext_{\mathcal{F}}(\mathcal{L}) = \{a \in \mathcal{A}r \mid (a, \texttt{in}) \in \mathcal{L}\}$ (Caminada 2006).

Definition 19 ($\texttt{COS}^{LA}$)**.** *Let $\mathcal{O} = \langle\mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF with $\mathcal{V}_{\mathcal{A}r} = \langle v_1, \dots, v_n\rangle$. The COS based on labellings aggregation operator $LA \in \{so, co, sco\}$ is:*

$$\begin{aligned}\texttt{COS}^{LA}(\mathcal{O}) = &\{Lab2Ext_{\mathcal{F}}(\ell) \mid \ell \in LA_{\mathcal{F}}(\\ &\{Vote2Lab(v_1), \dots, Vote2Lab(v_n)\})\}\end{aligned}$$

Example 5 (cont.)**.** *We obtain $\texttt{COS}^{so}(\mathcal{O}) = \{\emptyset\}$, $\texttt{COS}^{co}(\mathcal{O}) = \{\{c\}\}$, $\texttt{COS}^{sco}(\mathcal{O}) = \{\{c\}\}$.*

4.2 Bernreiter et al.'s Approach

In an ABSAF, voters can only indicate their approval, leaving the status of other arguments uncertain, as voters may either disagree with them or remain neutral. To address this ambiguity and enable a fair comparison with our method, we introduce a definition to convert our votes into approval ballots. While there are several ways to achieve this, the simplest method involves considering only the positive votes in the approval ballot.

Definition 20 (Vote2Bal)**.** *Let $\mathcal{V}_{\mathcal{A}r}$ be the votes on $\mathcal{A}r$ and $v \in \mathcal{V}_{\mathcal{A}r}$. The function $Vote2Bal(v) : \{-1, 0, 1\}^{\mathcal{A}r} \to 2^{\mathcal{A}r}$ transforms a vote v into its corresponding approval ballot as follows: $Vote2Bal(v) = \{x \mid v(x) = 1\}$.*

This transformation facilitates the conversion of an OBAF into an ABSAF, allowing for the adaptation of Bernreiter et al. (2024)'s operator within our $\texttt{COS}$ framework.

Definition 21 ($\texttt{COS}_\sigma^{AB,op}$)**.** *Let $\mathcal{O} = \langle\mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF with $\mathcal{V}_{\mathcal{A}r} = \langle v_1, \dots, v_n\rangle$ and σ be an extension-based semantics. An ABSAF is defined as $\mathcal{A}b = (\mathcal{F}, \{1, \dots, n\}, \langle Vote2Bal(v) | v \in \mathcal{V}_{\mathcal{A}r}\rangle)$. The COS based on the representation operators $op \in \{rep, rep_c\}$ is:*

$$\texttt{COS}_\sigma^{AB,op}(\mathcal{O}) = \begin{cases} OWA_\sigma^{\vec{w},op}(\mathcal{A}b) & \textit{for } AB \in \{u, e, h\} \\ MaxCov_\sigma^{op}(\mathcal{A}b) & \textit{for } AB \in \{mc\} \end{cases}$$

Example 6. *We obtain $\texttt{COS}_{pr}^{u,op}(\mathcal{O}) = \{\{c, e\}\}$ for $op \in \{rep, rep_c\}$.*

4.3 Semantics based on Attack Removal

In this subsection, we define a new approach based on a combination of vote aggregation inspired by (Leite and Martins 2011) and the elimination of attacks used in preference-based argumentation frameworks (Amgoud and Cayrol 2002). The idea is to give strong priority to the "most accepted" arguments with respect to the votes. To this end, the method associates with each argument a value computed from its votes and then removes attacks on the AF where the attacking argument has a higher value than the attacked argument. This could be interpreted as a drastic change to the AF based on existing votes. However, this can present a practical benefit, especially in the context of online debates, where our attack removal method becomes particularly valuable. For instance, if there are arguments that are widely perceived as incorrect or disruptive ("troll arguments"), and most users have voted against them, removing the attacks associated with these arguments mitigates their impact on the final result without making an even more drastic change to the AF (i.e., removing the argument itself). This solution can lead to a substantial improvement in the quality and coherence of the debate.
The evaluation of this score for an argument is carried out by a general opinion aggregation function based on the number of votes at 1, 0, and -1 for this argument.

Definition 22 (Opinion aggregation function)**.** *Let $\mathcal{O} = \langle\langle\mathcal{A}r, att\rangle, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF. An opinion aggregation function $\tau : \mathbb{N} \times \mathbb{N} \times \mathbb{N} \to \mathbb{R}$ produces a score of an argument based on its votes. With abuse of notation, given $x \in \mathcal{A}r$, we note $\tau(x) = \tau(|v^+(x)|, |v^o(x)|, |v^-(x)|)$.*

There are many possible instances of opinion aggregation functions, we use the function proposed by Leite and Martins (2011) because of its simplicity. This function can be easily adjusted to adapt to other scenarios.

Definition 23 (τ_ϵ)**.** *Let $\mathcal{O} = \langle\langle \mathcal{A}r, att\rangle, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF and $x \in \mathcal{A}r$. τ_ϵ is an opinion aggregation function such that $\epsilon \geq 0$*

$$\tau_\epsilon(x) = \begin{cases} 0 & |v^+(x)| = |v^-(x)| = 0 \\ \frac{|v^+(x)|}{|v^+(x)|+|v^-(x)|+\epsilon} & \textit{otherwise} \end{cases}$$

Our approach to eliminating attacks is based on preference-based argumentation frameworks proposed by Amgoud and Cayrol (2002). They redefine the attack relation as follows: an argument x defeats an argument y only if there exists an attack (x, y) and y is not preferred to x according to the preference relation. In our case, instead of a preference relation, we employ the ranking of arguments based on the scores in τ.

Definition 24 ($\mathcal{O}_\tau$)**.** *Let $\mathcal{O} = \langle\langle \mathcal{A}r, att\rangle, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF and τ be an opinion aggregation function.*
An $\mathcal{O}_\tau$ is a triplet $\langle\langle \mathcal{A}r, att^*\rangle, \mathcal{V}_{\mathcal{A}r}, \succeq^\tau_\mathcal{O}\rangle$ where:
- *$att^* = \{(x, y) \mid (x, y) \in att$ and $x \succeq^\tau_\mathcal{O} y\}$;*
- *$\succeq^\tau_\mathcal{O}$ is the total preorder on $\mathcal{A}r$ such that $x \succeq^\tau_\mathcal{O} y$ iff $\tau(x) \geq \tau(y)$.*

We note $\mathcal{F}_\tau = \langle \mathcal{A}r, att^*\rangle$ as the AF associated to $\mathcal{O}_\tau$.

It is now possible to define a COS that builds $\mathcal{O}_\tau$ from an OBAF and returns the sets of arguments which are the result of an extension-based semantics applied to $\mathcal{F}_\tau$.

Definition 25 ($\texttt{COS}^{\texttt{AR}}$)**.** *Let $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF. Let σ be an extension-based semantics and τ be an opinion aggregation function. The COS based on attack removal is $\texttt{COS}^{\texttt{AR}}_{\sigma,\tau}(\mathcal{O}) = \mathcal{E}_\sigma(\mathcal{F}_\tau)$.*

Example 7. *Let us first apply the opinion aggregation function τ_ϵ on each argument with $\epsilon = 0.1$. We obtain $\tau_\epsilon(a) = 0.82$, $\tau_\epsilon(b) = 0.32$, $\tau_\epsilon(c) = 0.91$, $\tau_\epsilon(d) = 0$ and $\tau_\epsilon(e) = 0.66$. This gives us the following total preorder: $c \succeq^{\tau_\epsilon}_\mathcal{O} a \succeq^{\tau_\epsilon}_\mathcal{O} e \succeq^{\tau_\epsilon}_\mathcal{O} b \succeq^{\tau_\epsilon}_\mathcal{O} d$. It is now possible to define $\mathcal{O}_\tau = \langle\langle \mathcal{A}r, att^*\rangle, \mathcal{V}_{\mathcal{A}r}, \succeq^{\tau_\epsilon}_\mathcal{O}\rangle$ with $att^* = \{(c, a), (b, d), (e, b), (e, d)\}$. Therefore, we obtain $\texttt{COS}^{\texttt{AR}}_{pr,\tau_\epsilon}(\mathcal{O}) = \{\{c, e\}\}$.*

4.4 Collective Satisfaction Semantics (CSS)

In this section, we introduce our second contribution, which defines measures of satisfaction, dissatisfaction, and utility to aggregate voters' opinions. The idea is to use these measures to select the extension(s) within the OBAF's associated AF that closely align with the expressed voter opinions.

In order to facilitate the comparison between votes and extensions, an extension can be seen as a vector of values where the arguments belonging to this extension are assigned 1 and the remaining arguments -1.

Definition 26 (Vec)**.** *Let $\mathcal{F} = \langle \mathcal{A}r, att\rangle$ be an AF. Let σ be an extension-based semantics. For a given extension $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$, the function $Vec_\mathcal{E} : \mathcal{A}r \to \{-1, 1\}$ is defined s.t. $\forall x \in \mathcal{A}r$:*

$$Vec_\mathcal{E}(x) = \begin{cases} 1 & \textit{if } x \in \mathcal{E} \\ -1 & \textit{if } x \notin \mathcal{E} \end{cases}$$

With abuse of notation, $Vec(\mathcal{E})$ represents the (vector of) votes corresponding to the extension $\mathcal{E}$.

Let us now formally define the three measures used to compare a vote and an extension. The satisfaction measure counts the number of arguments for which the function Vec returns the same value as the vote (i.e. arguments being in the extension and getting 1 in the vote as well as arguments not being in the extension and getting -1 in the vote). The dissatisfaction measure, on the other hand, counts the number of arguments that did not have the same values. Finally, the utility measure is the sum of the previous two measures.

Definition 27 ((Dis)satisfaction,utility)**.** *Let $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF with $\mathcal{F} = \langle \mathcal{A}r, att\rangle$. Let σ be an extension-based semantics. For a given extension $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$, the satisfaction $\mathcal{S}_v$, dissatisfaction $\mathcal{D}_v$ and utility $\mathcal{U}_v$ of $\mathcal{E}$ w.r.t. a vote $v \in \mathcal{V}_{\mathcal{A}r}$ are defined as follows:*
- $\mathcal{S}_v(\mathcal{E}) = |\{x \in \mathcal{A}r \mid v(x) = Vec_\mathcal{E}(x)\}|$
- $\mathcal{D}_v(\mathcal{E}) = -|\{x \in \mathcal{A}r \mid v(x) = -Vec_\mathcal{E}(x)\}|$
- $\mathcal{U}_v(\mathcal{E}) = \mathcal{S}_v(\mathcal{E}) + \mathcal{D}_v(\mathcal{E})$

While dissatisfaction and satisfaction may appear complementary, in fact, they measure distinct aspects of a vote's correspondence to an extension. For example, complementary operator to dissatisfaction would account for both abstentions ($v(x) = 0$) and mismatches between the vector and the extension ($v(x) = -Vec_\mathcal{E}(x)$)

We now aim to determine the distance between an extension and the set of votes $\mathcal{V}_{\mathcal{A}r}$. This distance measure indicates how closely the extension aligns with the profile of votes. Given that our aggregations are sum, min, and leximin[3], maximizing this distance is desirable.

Definition 28 (Distance)**.** *Let $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF with $\mathcal{F} = \langle \mathcal{A}r, att\rangle$. Let σ be an extension-based semantics. For a given extension $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$, the distance of $\mathcal{E}$ w.r.t. $\mathcal{V}_{\mathcal{A}r}$ is defined as follows:*

$$d^\otimes_{\mathcal{V}_{\mathcal{A}r}}(\mathcal{E}) = \otimes_{v \in \mathcal{V}_{\mathcal{A}r}} \mathcal{M}_v(\mathcal{E})$$

with $\otimes \in \{\Sigma, min, leximin\}$ and $\mathcal{M} \in \{\mathcal{D}, \mathcal{S}, \mathcal{U}\}$.

Thus, the extensions that maximize the distance from the set of votes will be considered as the result of the CSS.

Definition 29 ($\texttt{CSS}$)**.** *Let $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r}\rangle$ be an OBAF with $\mathcal{F} = \langle \mathcal{A}r, att\rangle$. Let σ be an extension-based semantics, $\otimes \in \{\Sigma, min, leximin\}$ and $\mathcal{M} \in \{\mathcal{D}, \mathcal{S}, \mathcal{U}\}$. The collective satisfaction semantics is*

$$\texttt{CSS}^{\mathcal{M},\otimes}_\sigma(\mathcal{O}) = \underset{\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})}{\operatorname{argmax}}(d^\otimes_{\mathcal{V}_{\mathcal{A}r}}(\mathcal{E}))$$

Example 8. *Here are the calculations to obtain $\texttt{CSS}^{\mathcal{U},\Sigma}_{pr}(\mathcal{O})$. As a reminder, we have $\mathcal{E}_{pr}(\mathcal{F}) = \{\{a, b\}, \{b, c\}, \{c, d\}, \{a, e\}, \{c, e\}\}$. First, we need to convert each of these extensions into a vector of votes. For instance, $Vec(\{a, b\}) = \langle 1, 1, -1, -1, -1\rangle$. Second, we compute the utility measure between each couple (extension, vote). Table 1 shows an example with $\{a, b\}$. Following the same reasoning, for $\mathcal{U}_v$,*

[3]When applied to a vector of n real numbers, the leximin function gives the list of those numbers sorted in a increasing way. Such lists are compared with respect to the lexicographic ordering induced by the standard ordering on real numbers.

	v_1	v_2	v_3	v_4	v_5	v_6	$d^{\Sigma}_{\mathcal{V}_{\mathcal{A}r}}(\{a,b\})$
$\mathcal{S}_v(\{a,b\})$	2	2	2	0	3	4	13
$\mathcal{D}_v(\{a,b\})$	-2	-1	-2	-3	0	0	-8
$\mathcal{U}_v(\{a,b\})$	0	1	0	-3	3	4	5

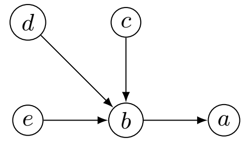
Table 1: Values for the satisfaction, dissatisfaction and utility measure between the preferred extension $\{a, b\}$ and all votes from $\mathcal{V}_{\mathcal{A}r}$ in the OBAF represented in Figure 1.

we have $d^{\Sigma}_{\mathcal{V}_{\mathcal{A}r}}(\{b, c\}) = -1$, $d^{\Sigma}_{\mathcal{V}_{\mathcal{A}r}}(\{c, d\}) = -9$, $d^{\Sigma}_{\mathcal{V}_{\mathcal{A}r}}(\{a, e\}) = 11$, *and* $d^{\Sigma}_{\mathcal{V}_{\mathcal{A}r}}(\{c, e\}) = 5$. *Finally, by applying the formula given in Definition 29, we obtain* $\mathtt{CSS}^{\mathcal{U},\Sigma}_{pr}(\mathcal{O}) = \{\{a, e\}\}$. *Using the other two aggregation functions, we obtain* $\mathtt{CSS}^{\mathcal{U},min}_{pr}(\mathcal{O}) = \{\{a, e\}, \{c, e\}\}$ *and* $\mathtt{CSS}^{\mathcal{U},leximin}_{pr}(\mathcal{O}) = \{\{a, e\}\}$.

4.5 Observations

Here, we discuss the results obtained when applying the semantics we defined in the previous section on different examples. Let us start by examining the related work with Figure 1. First, we observe that $\mathtt{COS}^{so}$ yields an empty set as a result. That is because there is no unanimity in the votes (negative or positive) for any of the arguments. This constitutes a significant limitation of the method, particularly in contexts such as public debates where achieving unanimity in votes is improbable. Effective semantics should be capable of discerning feasible solutions in such scenarios. For $\mathtt{COS}^{co}$ and $\mathtt{COS}^{sco}$, the result is also the empty set. That is because with *cio* we have $\mathtt{in} = \{c\}$ and $\mathtt{out} = \{d\}$, when applying down-admissibility on this labelling we get the empty set for $\mathtt{in}$ and $\mathtt{out}$, because even though $\{c\}$ is an admissible extension, the labelling is not in the set of admissible labellings: $\mathtt{out}((\{c\}, \{d\}, \{a, b, e\})) \nsubseteq \mathtt{out}((\{c\}, \{a\}, \{b, d, e\}))$. Hence, the up-complete labelling of $\mathtt{COS}^{co}$ is the empty set as well. This is a notable downfall of the method as even with OBAFs where a decision seems natural, with an overly cautious approach, it is often unable to make a decision. Under the approval ballot semantic, we obtain $\mathtt{COS}^{AB,op}_{\mathrm{pr}}(\mathcal{O}) = \{\{a, e\}\}$, where $AB \in \{u, e, h, mc\}$ and $op \in \{rep, rep^c\}$, representing the desirable extension. However, using other semantics, such as complete semantics where extensions can be subsets of each other, this method does not always yield the best solution. This is discussed further in Section 6. When considering $\mathtt{COS}^{\mathtt{AR}}$, c is the argument with the highest score, resulting in the removal of the attack (a, c). Similarly, as e is considered to be better than b and d w.r.t. $\succeq^{\tau_\epsilon}_{\mathcal{O}}$, it becomes unattacked. Then $\mathtt{COS}^{\mathtt{AR}}_{\mathrm{pr},\tau_\epsilon}(\mathcal{O}) = \{\{c, e\}\}$.

Finally, let us consider our CSS. The desired extension $\{a, e\}$ achieves the maximum score for satisfaction $\mathcal{S}$, dissatisfaction $\mathcal{D}$, and utility $\mathcal{U}$ for $\otimes \in \{\Sigma, leximin\}$, $\mathtt{CSS}^{\mathcal{M},\otimes}_{\mathrm{pr}}(\mathcal{O}) = \{\{a, e\}\}$. However, there are differences in this example when $\otimes = min$. Indeed, only $\mathcal{S}$ yields a truly egalitarian solution, resulting in $\mathtt{CSS}^{\mathcal{S},min}_{\mathrm{pr}}(\mathcal{O}) = \mathtt{CSS}^{\mathcal{M},\{\Sigma,leximin\}}_{\mathrm{pr}}(\mathcal{O}) \cup \{\{b, c\}, \{e, c\}\}$, because three of the six voters (i.e., v_4, v_5 and v_6) would not be entirely satisfied with $\{a, e\}$. In contrast, $\mathcal{D}$ diverges from this outcome, as the dissatisfaction with $\{a, e\}$ is lower than with all other extensions. Conversely, $\mathcal{U}$ aligns with $\mathcal{D}$, as overall dissatisfaction in the system outweighs satisfaction. We obtain We obtain $\mathtt{CSS}^{\{\mathcal{D},\mathcal{U}\},min}_{\mathrm{pr}}(\mathcal{O}) = \{\{a, e\}\}$. This exemplifies how our operators and aggregation methods allow for nuanced final results.



Votes	a	b	c	d	e
$v1$	1	-1	1	-1	-1
$v2$	1	-1	-1	-1	1
$v3$	1	-1	-1	1	-1

Figure 2: An OBAF showing the limitations of $\mathtt{COS}^{LA}$.

Note that all voters except one voted for argument a and against argument d, with argument e also receiving four votes. Therefore, a, e appears to be the most desirable extension in this AF with respect to the votes. Consequently, both our method and ABSAF are the only approaches that arrive at what can be considered the most reasonable outcome.

As previously discussed, the COS based on labelling judgment aggregation approaches exhibit a tendency to favor arguments with no negative votes, even if such arguments hold a weaker position in the public's opinion due to abstentions. However, this is not the only limitation of these methods. Consider the OBAF illustrated in Figure 2, for any extension-based semantics, $\mathtt{COS}^{\mathtt{AR}}$, $\mathtt{COS}^{AB}$ and $\mathtt{CSS}^{\mathcal{M}}$ return $\{\{a, c, d, e\}\}$. This is not the case for $\mathtt{COS}^{LA}$ which fails to find a non-empty solution, even with unanimity on argument a (because $\{a\} \notin \mathcal{E}_{adm}(\mathcal{F})$). This is a limitation of the labelling judgment aggregation operators in cases where unanimity on a particular argument does not guarantee its inclusion in an admissible set of the argumentation framework.

Finally, let us consider the OBAF $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$ with $\mathcal{F} = \langle \{a, b, c\}, \{(a, b), (a, c), (c, a)\} \rangle$ and $\mathcal{V}_{\mathcal{A}r} = \langle (1, -1, -1)^2, (-1, 1, -1)^3, (-1, -1, 1) \rangle$. Thus, we obtain $\mathtt{COS}^{\mathtt{AR}}_{\mathrm{pr},\tau_\epsilon}(\mathcal{O}) = \{\{a, b\}\}$. However, these drastic changes in the initial AF can no longer guarantee the conflict-freeness of the result w.r.t. the initial AF. In some real-world scenarios, such a result could be deemed unacceptable. We contend that either $\emptyset$ or $\{a\}$ could be "acceptable" results in this case. With our method, we obtain $\mathtt{CSS}^{\mathcal{M},\otimes}_{\sigma}(\mathcal{O}) = \{\{a\}\}$ with $\sigma = \mathtt{pr}$, $\otimes \in \{\Sigma, min, leximin\}$, and $\mathcal{M} \in \{\mathcal{D}, \mathcal{S}, \mathcal{U}\}$. This flexibility in selecting the baseline semantics offers a significant advantage, enabling the adaptation of the method to the individual needs of the group.

5 Properties for Collective Opinion Semantics

This section focuses on identifying properties specific to COS. We adapt established properties from related fields to the context of opinion aggregation. We showcase how our approach aligns with identified properties while discussing why certain properties remain unverified—an aspect we elaborate on in the following section to demonstrate the advantages of this approach. We use these properties to conduct an axiomatic study of our approach against the other identified methods. Furthermore, we classify these properties into two categories: essential and additional. This categorization helps prioritize the key aspects that every method should adhere to, whereas the supplementary factors are considered to enhance the overall understanding and robustness of COS in specific scenarios.

5.1 Essential Properties

Vote Anonymity, inspired by (Dunne, Marquis, and Wooldrige 2012), states that the COS should disregard the identities of the voters.

Vote Anonymity (VA). *Let $\Pi(\mathcal{V}_{\mathcal{A}r})$ denote all permutations of the votes in $\mathcal{V}_{\mathcal{A}r}$ in $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$. A COS satisfies Vote Anonymity iff $\forall \mathcal{V}'_{\mathcal{A}r} \in \Pi(\mathcal{V}_{\mathcal{A}r})$, $\texttt{COS}(\mathcal{O}) = \texttt{COS}(\mathcal{O}')$ with $\mathcal{O}' = \langle \mathcal{F}, \mathcal{V}'_{\mathcal{A}r} \rangle$.*

Neutrality, inspired by (Amgoud and Beuselinck 2021), states that a COS should not depend on the names of the arguments and the votes.

Definition 30 (Isomorphism)**.** *Let $\mathcal{O} = \langle \langle \mathcal{A}r, att \rangle, \mathcal{V}_{\mathcal{A}r} \rangle$ and $\mathcal{O}' = \langle \langle \mathcal{A}r', att' \rangle, \mathcal{V}'_{\mathcal{A}r'} \rangle$ be two OBAFs. An isomorphism from $\mathcal{O}$ to $\mathcal{O}'$ is a bijective function f from $\mathcal{A}r$ to $\mathcal{A}r'$ s.t. $\forall\, a, b \in \mathcal{A}r$, $(a, b) \in att$ iff $(f(a), f(b)) \in att'$ and a bijective function from $\mathcal{V}_{\mathcal{A}r}$ to $\mathcal{V}'_{\mathcal{A}r'}$ s.t. $\forall a \in \mathcal{A}r$, and $\forall v_i \in \mathcal{V}$, $v_i(a) = v'_i(f(a))$.*

Neutrality (N). *A COS satisfies Neutrality iff for any two OBAFs $\mathcal{O} = \langle \mathcal{F}, \mathcal{V} \rangle$, $\mathcal{O}' = \langle \mathcal{F}', \mathcal{V}' \rangle$ and an isomorphism f from $\mathcal{O}$ to $\mathcal{O}'$, it holds that $\texttt{COS}(\mathcal{O}) = \texttt{COS}(f(\mathcal{O}))$.*

Monotony, inspired by (Amgoud and Beuselinck 2021), states that adding a vote to $\mathcal{V}_{\mathcal{A}r}$ which corresponds to one of the sets of arguments belonging to $\texttt{COS}(\mathcal{O})$, should not change the fact that this set always belongs to the result.

Monotony (M). *A COS satisfies Monotony iff for any OBAF $\mathcal{O} = \langle \langle \mathcal{A}r, att \rangle, \mathcal{V} \rangle$, $\forall a, b \in \mathcal{A}r$, and $\mathcal{E} \in \texttt{COS}(\mathcal{O})$ it holds that $\mathcal{E} \subseteq \texttt{COS}(\langle \langle \mathcal{A}r, att \rangle, \mathcal{V} \cup v \rangle)$ with $v = Vec(\mathcal{E})$.*

Non-triviality, inspired by (Dunne, Marquis, and Wooldrige 2012), states that the result of a COS should contain at least one non-empty set of arguments if there exists at least one extension in $\mathcal{F}$ w.r.t. an extension-based semantics.

Non-triviality (NT). *Let σ be an extension-based semantics. A COS satisfies Non-triviality iff for any $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$ s.t. $|\mathcal{E}_\sigma(\mathcal{F})| \geq 1$ and $\emptyset \notin \mathcal{E}_\sigma(\mathcal{F})$, it holds that $|\texttt{COS}(\mathcal{O})| \geq 1$ and $\texttt{COS}(\mathcal{O}) \neq \{\emptyset\}$.*

We denote $\mathbb{F}_{NT,\sigma}$ as the set of all non trivial OBAFs for the extension-based semantics σ.

Extension Unanimity, inspired by (Dunne, Marquis, and Wooldrige 2012), states that if all the votes refer to the same extension, then this extension should be the result of the COS.

Extension Unanimity (EU). *Let σ be an extension-based semantics. A COS satisfies Extension Unanimity iff for any $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$ s.t. $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$ and $\forall v \in \mathcal{V}_{\mathcal{A}r}$, $v = Vec(\mathcal{E})$, it holds that $\texttt{COS}(\mathcal{O}) = \{\mathcal{E}\}$.*

Conflict-freeness ensures a consistent result with respect to the underlying argumentation framework. This property is adapted from (Kaci et al. 2021) to accept votes.

Conflict-freeness (CF). *A COS satisfies Conflict-freeness iff for any $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$, $\forall E \in \texttt{COS}(\mathcal{O})$, E is conflict-free w.r.t. $\mathcal{F}$.*

5.2 Additional Properties

Argument Unanimity, inspired by (Dunne, Marquis, and Wooldrige 2012), states that if all the votes are for the acceptance of an argument, then it should belong to all the sets of arguments in the result.

Argument Unanimity (AU). *A COS satisfies Argument Unanimity iff for any $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$ s.t. $\exists x \in \mathcal{A}r$, $v(x) = 1$ for all $v \in \mathcal{V}_{\mathcal{A}r}$, it holds that $\forall E \in \texttt{COS}(\mathcal{O})$, $x \in E$.*

Extension Majority, inspired by (Dunne, Marquis, and Wooldrige 2012), states that if a strict majority of votes refers to the same extension, then this extension should be in the result of the COS.

Extension Majority (EM). *Let σ be an extension-based semantics. A COS satisfies Extension Majority iff for any $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$ s.t. $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$ and $|\{v \in \mathcal{V}_{\mathcal{A}r} \mid v = Vec(\mathcal{E})\}| > \frac{|\mathcal{V}_{\mathcal{A}r}|}{2}$, it holds that $\texttt{COS}(\mathcal{O}) = \{\mathcal{E}\}$.*

Separability, inspired by (d'Aspremont and Gevers 2002), asserts that if two sets of votes yield distinct extension sets, and these extensions overlap, the common sets should be included in the final outcome.

Separability (S). *A COS satisfies Separability iff for any three OBAFs $\mathcal{O}_1 = \langle \mathcal{F}, \mathcal{V}^1_{\mathcal{A}r} \rangle$, $\mathcal{O}_2 = \langle \mathcal{F}, \mathcal{V}^2_{\mathcal{A}r} \rangle$ and $\mathcal{O}_3 = \langle \mathcal{F}, \mathcal{V}^1_{\mathcal{A}r} \cup \mathcal{V}^2_{\mathcal{A}r} \rangle$ if $\texttt{COS}(\mathcal{O}_1) \cap \texttt{COS}(\mathcal{O}_2) \neq \emptyset$ then $\texttt{COS}(\mathcal{O}_1) \cap \texttt{COS}(\mathcal{O}_2) \subseteq \texttt{COS}(\mathcal{O}_3)$.*

Continuity, inspired by (Dunne, Marquis, and Wooldrige 2012), states that it is possible that the result is exactly an extension by adding a certain number of times the same vote representing this extension. To do this, we will use the notation $\langle x \rangle^k = \underbrace{\langle x, x, x, \ldots, x \rangle}_{k \text{ times}}$ which represents a vector containing k times the element x.

Continuity (Cn). *Let σ be an extension-based semantics. A COS satisfies Continuity iff for any $\mathcal{O} = \langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \rangle$ s.t. $\mathcal{E} \in \mathcal{E}_\sigma(\mathcal{F})$ and $\mathcal{E} \notin \texttt{COS}(\mathcal{O})$, $\exists k \in \mathbb{N}$ s.t. $\texttt{COS}(\langle \mathcal{F}, \mathcal{V}_{\mathcal{A}r} \cup \langle Vec(\mathcal{E}) \rangle^k \rangle) = \{\mathcal{E}\}$.*

6 Results and Discussion

In Section 4.5, we showed that, unlike existing techniques in the literature, CSS effectively addresses dissatisfaction issues while enhancing overall voter satisfaction. Moreover, our semantics demonstrate superior decisiveness compared to other approaches, leading us to explore essential properties for effective opinion aggregation. In that sense, let us prove which properties introduced in Section 5 are satisfied by the COS listed in Section 4.

Throughout this study, we adopt three established extension-based semantics: preferred, complete, and stable semantics. These provide us control over the resulting extensions, as well as catering to diverse needs and scenarios.

Proposition 1. *Let $LA \in \{so, co, sco\}$. $\texttt{COS}^{LA}$ satisfies VA, N, M, EU, CF and S.*

	Essential Properties						Additional Properties			
Semantics	VA	N	M	NT	EU	CF	S	AU	EM	Cn
$\texttt{COS}^{so}$	✓	✓	✓	×	✓	✓	✓	×	×	×
$\texttt{COS}^{co}$	✓	✓	✓	×	✓	✓	✓	×	×	×
$\texttt{COS}^{sco}$	✓	✓	✓	×	✓	✓	✓	×	×	×
$\texttt{COS}^{\{u,h\},rep}$	✓	✓	✓	✓	$\times^{\texttt{co}}$	✓	✓	×	×	$\times^{\texttt{co}}$
$\texttt{COS}^{e,rep}$	✓	✓	✓	✓	$\times^{\texttt{co}}$	✓	✓	×	×	×
$\texttt{COS}^{mc,rep}$	✓	✓	✓	✓	$\times^{\texttt{co}}$	✓	✓	×	$\times^{\texttt{co}}$	$\times^{\texttt{co}}$
$\texttt{COS}^{\{u,h\},rep^c}$	✓	✓	✓	✓	×	✓	✓	×	×	×
$\texttt{COS}^{e,rep^c}$	✓	✓	✓	✓	×	✓	✓	×	×	×
$\texttt{COS}^{mc,rep^c}$	✓	✓	✓	✓	×	✓	✓	×	×	×
$\texttt{COS}^{\texttt{AR}}_{\sigma,\tau_\epsilon}$	✓	✓	✓	✓	✓	×	✓	✓	×	✓
$\texttt{CSS}^{\mathcal{M},\Sigma}$	✓	✓	✓	✓	✓	✓	✓	×	✓	✓
$\texttt{CSS}^{\mathcal{M},min}$	✓	✓	✓	✓	✓	✓	✓	×	×	×
$\texttt{CSS}^{\mathcal{M},leximin}$	✓	✓	✓	✓	✓	✓	✓	×	×	×

Table 2: Properties satisfied by the COS studied in this work. The symbol ✓(resp. ×) means that the property is satisfied (resp. violated) by the semantics for $\sigma \in \{\texttt{co}, \texttt{pr}, \texttt{stb}\}$. The symbol $\times^{\texttt{co}}$ means that the property is not satisfied when $\sigma = \texttt{co}$ but is satisfied when $\sigma \in \{\texttt{pr}, \texttt{stb}\}$.

Proposition 2. *Let $\sigma \in \{co, pr, stb\}$. $\texttt{COS}^{\texttt{AR}}_{\sigma,\tau_\epsilon}$ satisfies VA, N, M, NT, EU, S, AU and Cn.*

Proposition 3. *Let $\mathcal{M} \in \{\mathcal{D}, \mathcal{S}, \mathcal{U}\}$ and $\sigma \in \{co, pr, stb\}$. $\texttt{CSS}^{\mathcal{M},\Sigma}_\sigma$ satisfies VA, N, M, NT, EU, CF, S, EM and Cn.*

Proposition 4. *Let $\mathcal{M} \in \{\mathcal{D}, \mathcal{S}, \mathcal{U}\}$ and $\sigma \in \{co, pr, stb\}$. $\texttt{CSS}^{\mathcal{M},min}_\sigma$ and $\texttt{CSS}^{\mathcal{M},leximin}_\sigma$ satisfy VA, N, M, NT, EU, CF and S.*

Proposition 5. *Let $AB \in \{u, e, h, mc\}$ and $\sigma = \{co, pr, stb\}$. $\texttt{COS}^{AB,rep^c}_\sigma$ and $\texttt{COS}^{AB,rep}_{co}$ satisfy VA, N, M, NT, CF and S.*

Proposition 6. *Let $AB \in \{u, h\}$ and $\sigma \in \{pr, stb\}$. $\texttt{COS}^{AB,rep}_\sigma$ satisfies VA, N, M, NT, EU, CF, S and Cn.*

Proposition 7. *Let $\sigma \in \{pr, stb\}$. $\texttt{COS}^{e,rep}_\sigma$ satisfies VA, N, M, NT, EU, CF, and S.*

Proposition 8. *Let $\sigma \in \{pr, stb\}$. $\texttt{COS}^{mc,rep}_\sigma$ satisfies VA, N, M, NT, EU, CF, S, EM and Cn.*

Several observations can be made regarding the results reported in Table 2.

First, CSS stands out as being among the only approaches (along with $\texttt{COS}^{AB,rep}_\sigma$ for $\sigma \in \{\texttt{pr}, \texttt{stb}\}$) to adhere to all essential properties, with a particular emphasis on CF, NT, and EU. These properties are crucial as the Non-triviality property (NT) ensures decisiveness, i.e., the system is always able to arrive at some solutions. The conflict-freeness property (CF) ensures alignment with the underlying argumentation framework with respect to an extension-based semantics (i.e., the outcome is compatible with the chosen semantics). Additionally, it ensures that the framework's constraints are respected, thereby allowing for the selection of acceptable extensions that conform to the defined rules and expectations. Finally, the Extension Unanimity property (EU) is also a crucial property proving the semantics is robust and reflects true consensus. Another advantage lies in the clarity of distinction between our egalitarian (leximin) and utilitarian (sum) approaches, which is evident in the additional properties. Specifically, for an egalitarian method, EM and Cn are not desirable, while they are favorable in utilitarian methods. This adaptability proves valuable, catering to different voting scenarios. Remarkably, these properties remain consistent across all three semantics.

From the propositions, $\texttt{COS}^{AB,rep}_\sigma$ for $\sigma \in \{\texttt{pr}, \texttt{stb}\}$ is the only other approach to satisfy all essential properties. However, two significant distinctions from CSS become apparent. Firstly, the results seem to depend on the extensions-based semantics used. Indeed, we can see that the EU, EM and Cn properties are no longer satisfied when the complete semantics is used. This is due to how this approach handles extensions of different lengths. The discrepancy appears from the fact that the normalization of the operators depends on the length of the vote rather than the length of the extension (see Definition 10). For instance, consider an AF $\mathcal{F}$ with $\mathcal{E}_{\texttt{co}}(\mathcal{F}) = \{\{a\}, \{a, d\}\}$. In this case, both extensions would obtain the same score if the approval ballot were $\{a\}$. While this characteristic is not inherently disadvantageous, it may lead to unexpected behavior in certain semantics, such as the complete semantics, where extensions can be subsets of one another. Secondly, the differentiation between the utilitarian and egalitarian approaches is not as pronounced as in CSS, where EM and Cn serve as clear indicators.

We caution against unconditional satisfaction of Argument Unanimity. Indeed, this property provides a better understanding of how votes are used to select the set(s) of arguments corresponding to the result of a COS. For example, $\texttt{COS}^{\texttt{AR}}$ assumes that votes predominate over the attack relation, so an argument with all votes for acceptance will necessarily be accepted. Conversely, for CSS, votes are used to help select the extensions to the initial AF that most closely match the votes. Thus, even if all the votes are for the acceptance of an argument that does not appear in any extension, meaning that this argument is not compatible with the AF, that argument will never be in the result.

7 Related Work

Some other works have explored various aspects of aggregation within argumentation frameworks, albeit from different perspectives. While some works focus on value-based argumentation and audience preferences (Bench-Capon, Doutre, and Dunne 2007), others investigate rationalization of argumentation frameworks without considering votes or evaluations (Airiau et al. 2017). Additionally, (Croitoru 2014) proposes methods for argumentative aggregation on abstract debates, where there is no logical link bewteen the facts (i.e. no attacks between the arguments). In the context of merging argumentation frameworks, our approach bears resemblance to the work of Delobelle et al. (2016). A conversion could have been possible, where each voter corresponds to an extension (in the case voters never abstain), and the result of the merging of these extensions would be the outcome. However, we have tested this against our method, and the results differ. Tohmé et al. (2008) focuses on aggregating attack relations in Social Choice Theory, differing from our emphasis on aggregating voters' opinions. Coste-Marquis et al. (2007) center their work on merging diverse argumentation systems, which is not applicable to our study as our voters share a common underlying AF. Additionally, in (Dunne et al. 2011), the focus is on assigning weights to attack relations. And Rago et al. (2017) use quantitative argumentation debate frameworks in addition to negative and positive votes, and then score each argument is scored. This approach is reminiscent of methods proposed by Leite et al. (2011). However, we believe that aggregating votes into a single value is suboptimal for voting scenarios because it leads to the loss of crucial information about the degree of agreement or disagreement from each individual regarding potential solutions. Specifically, it forbids the pursuit of any egalitarian solutions, where nuances in individual preferences and degrees of support are crucial. We argue that retaining such information is essential for ensuring robust and fair decision-making frameworks, which is a key motivation behind our framework.

8 Conclusion

In this paper, we have conducted a comprehensive study aimed at advancing democratic resolution through opinion aggregation. To that end, we introduce a novel argumentation framework, called Opinion Based Argumentation (OBAF), where voters express their preference for the acceptability of each argument in the AF. This framework is paired with a new class of semantics, called Collective Opinion Semantics (COS), specifically tailored for efficient opinion representation and aggregation. Furthermore, we establish a set of properties and demonstrate that Collective Satisfaction Semantics consistently outperforms existing approaches, providing more precise and robust results in the challenging realm of opinion aggregation.

Theoretically, our work provides a clear framework for comparing different approaches in this field, thanks to OBAF and the axioms we propose. Practically, it shows promising results, offering a method applicable in online debates to foster democratic solutions while preserving individual liberty. Methods like our egalitarian or utilitarian approach ensure fairness and inclusivity in decision-making processes. Other methodologies can be easily integrated into our framework, as demonstrated in our paper, allowing axiomatic comparisons between them. Our method consistently shows promise, aligning with most proposed axioms with different semantics. This showcases its potential for widespread applications.

As we move forward, it is crucial to acknowledge potential limitations and chart the course for future work. This includes addressing nuanced scenarios and refining our framework to adapt to evolving challenges in the rapidly changing landscape of digital democracy.

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