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Complexity Results and Algorithms for Preferential Argumentative Reasoning in ASPIC$^+$

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Abstract

We provide complexity results and algorithms for reasoning in the central structured argumentation formalism of ASPIC$^+$. Considering ASPIC$^+$ accommodated with preferences under the last-link principle, the results are made possible by rephrasing several argumentation semantics—admissible, complete, stable, preferred and grounded—in terms of defeasible elements of an ASPIC$^+$ theory for both democratic and elitist last-link lifting. Via the rephrasing, we establish that acceptance is polynomial-time computable under grounded semantics, and complete for either NP, coNP, or Π_2^P depending on the reasoning mode and semantics. We also detail answer set programming encodings for deciding acceptance for the NP/coNP-complete reasoning tasks and empirically show that it scales significantly better than first translating ASPIC$^+$ reasoning tasks to abstract argumentation. Finally, we show that, in contrast to the last-link principle, it is NP-hard to compute the grounded extension under the weakest-link principle.

1 Introduction

Argumentation is today an important area of research in the realm of knowledge representation and reasoning (Baroni et al. 2018; Gabbay et al. 2021). We focus on the central structured argumentation (Besnard et al. 2014; Bondarenko et al. 1997; Besnard and Hunter 2008; García and Simari 2004; Garcia, Prakken, and Simari 2020) formalism ASPIC$^+$ (Modgil and Prakken 2013) which in its generality has been shown to capture a range of argumentative settings in the real world (Prakken 2020; Odekerken et al. 2022).

In ASPIC$^+$, strict inference rules—capturing deductively valid inferences—and defeasible inference rules—capturing presumptive inference—together with a knowledge base and preferential information form the basic building blocks of arguments. One approach to reasoning in ASPIC$^+$ consists of first explicitly constructing arguments from these building blocks, which gives rise to a corresponding abstract argumentation framework (Modgil and Prakken 2013), and then reasoning over the abstract argumentation framework in order to decide whether the conclusions of interest can be drawn (Dung 1995). However, this two-step approach is cumbersome in both theory and practice (Lehtonen, Wallner, and Järvisalo 2020) as the first step of argument construction may give rise to an exponentially larger abstract framework (Strass, Wyner, and Diller 2019). This makes it challenging to establish complexity results for ASPIC$^+$ reasoning and to develop practical algorithms.

Recent work on assumption-based argumentation (Bondarenko et al. 1997; Lehtonen, Wallner, and Järvisalo 2021) and specific fragments of ASPIC$^+$ (Lehtonen, Wallner, and Järvisalo 2020; Lehtonen, Wallner, and Järvisalo 2022; Odekerken et al. 2023) has established that the explicit construction of arguments can be avoided by rephrasing argumentation semantics based on defeasible elements and directly drawing conclusions on the level of these rephrasings. This allows for establishing complexity results as well as declarative algorithms based on answer set programming (ASP) (Gelfond and Lifschitz 1988; Niemelä 1999) for reasoning in ABA and ASPIC$^+$, which scale significantly better than declarative algorithms based on the two-step approach.

However, so far such rephrasings have only been established for specific fragments of ASPIC$^+$. This is mainly due to representational generality of ASPIC$^+$, with various potential ways of instantiating the formalism by considering, e.g., different types of notions of preferences, combined with the non-triviality of formally establishing rephrasings for different choices of argumentation semantics. In particular, rephrasings of several semantics have been presented for ASPIC$^+$ *without allowing preferences* (Lehtonen, Wallner, and Järvisalo 2020), while rephrasing for the more generic case of ASPIC$^+$ with preferences has been so far established only for the case of stable semantics and preferential reasoning under the so-called weakest-link principle wrt the elitist lifting (Lehtonen, Wallner, and Järvisalo 2022), and for grounded semantics in a fragment of ASPIC$^+$ allowing no strict rules or premises and only one type of defeat under the last-link principle (Odekerken et al. 2023).

In this work, we considerably extend this recent line of research from both a theoretical and practical perspective. Complementing Lehtonen, Wallner, and Järvisalo (2022), we mainly focus on ASPIC$^+$ under the last-link principle, which, along with the weakest-link principle, is one of the central preference handling mechanisms for ASPIC$^+$ (Modgil and Prakken 2013). While Lehtonen, Wallner, and Järvisalo (2022) focused on the specific case of stable semantics and the elitist lifting, we consider a range of central argumentation semantics (admissible, complete, stable, grounded and preferred) and both the elitist and democratic liftings. To cover each of these choices, we detail rephrasings of the semantics as non-trivial extensions of the previous works. Using the new rephrasings, we establish both new complexity results and algorithms. In terms of complexity of reasoning, we show that acceptance under grounded semantics is polynomial-time decidable while deciding credulously acceptance is NP-complete under admissible, complete, preferred and stable semantics; and that skeptical acceptance is (i) polynomial-time decidable under admissible and complete semantics; (ii) coNP-complete under stable semantics; and (iii) Π_2^p-complete under preferred semantics, regardless of the choice of lifting. Key to the complexity results and our algorithms is, as we show, that success of a defeat can be decided by inspecting the defeating argument and the defeated defeasible element *without* considering the full argument containing the defeated element. A consequence is that, surprisingly, inclusion of preferences under last-link does not increase computational complexity of the acceptance problems, in contrast to ASPIC$^+$ under weakest-link (Lehtonen, Wallner, and Järvisalo 2022) and ABA$^+$ (Lehtonen, Wallner, and Järvisalo 2021; Dimopoulos et al. 2024). From the algorithmic perspective, we detail how our new rephrasings allow for extending previously-proposed ASP encodings for deciding acceptance in the NP/coNP problem variants we consider, and show empirically that the approach scales significantly better than first translating ASPIC$^+$ reasoning to abstract argumentation. Finally, contrasting to polynomial-time decidability of acceptance under grounded semantics under the last-link principle, we show that grounded acceptance is NP-hard under the weakest-link principle and elitist lifting, demonstrating further the complexity discrepancy between reasoning under last-link and weakest-link. Formal proofs are provided in an online paper supplement.

2 ASPIC$^+$

We start with background on ASPIC$^+$ (Modgil and Prakken 2013; Prakken 2010). The basic notion of ASPIC$^+$ is that of an argumentation system. Following Modgil and Prakken (2013), we incorporate preorders to argumentation systems and knowledge bases to integrate preferences.

Definition 1 (Argumentation system)**.** *An* argumentation system *(AS) is a tuple $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$, where*

- *$\mathcal{L}$ is a set of atoms,*
- *$\overline{} : \mathcal{L} \mapsto 2^{\mathcal{L}}$ and $n : \mathcal{R}_d \mapsto \mathcal{L}$ are functions,*
- *$\leq$ is a partial preorder, i.e., a reflexive and transitive binary relation, on $\mathcal{R}_d$, and*
- *$\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is the union of a set $\mathcal{R}_s$ of strict and a set $\mathcal{R}_d$ of defeasible rules with $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$.*

Atom $l \in \mathcal{L}$ is a *contrary* of atom $m \in \mathcal{L}$ iff $l \in \overline{m}$ and $m \notin \overline{l}$, and a *contradictory* of m iff $l \in \overline{m}$ and $m \in \overline{l}$. The function n names defeasible rules. Strict and defeasible rules are of the form $a_1, \ldots, a_n \rightarrow c$ and $a_1, \ldots, a_n \Rightarrow c$, resp., where $a_1, \ldots, a_n, c \in \mathcal{L}$. When we do not distinguish between strict and defeasible rules, we write $a_1, \ldots, a_n \rightsquigarrow c$. For a rule r, $\mathtt{ants}(r) = \{a_1, \ldots, a_n\}$ is the set of *antecedents* and $\mathtt{cons}(r) = c$ the *consequent* of r.

Arguments are constructed from a given argumentation system with respect to a knowledge base.

Definition 2 (Knowledge base)**.** *A* knowledge base *over an argumentation system $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$ is a set $\mathcal{K} = \mathcal{K}_a \cup \mathcal{K}_p \subseteq \mathcal{L}$ with $\mathcal{K}_a \cap \mathcal{K}_p = \emptyset$, where $\mathcal{K}_a$ is a set of* axioms *and $\mathcal{K}_p$ a set of* ordinary premises.

Intuitively, axioms are true while premises can be assumed to hold but are defeasible. Argumentation theories in ASPIC$^+$ consist of an argumentation system, a knowledge base and preferences between premises.

Definition 3 (Argumentation theory)**.** *An* argumentation theory *(AT) is a triple $T = (AS, \mathcal{K}, \leq')$, where AS is an argumentation system, $\mathcal{K}$ is a knowledge base and $\leq'$ is a partial preorder on $\mathcal{K}_p$.*

Each part of an AT is assumed to be finite. As defined by Modgil and Prakken (2013), we focus on arguments of finite size, disallowing infinite rule-chaining. An argumentation theory gives rise to arguments as follows.

Definition 4 (Arguments)**.** *Let T be an AT. We define the set of arguments Arg_T inductively as follows.*

- *If $c \in \mathcal{K}$, then $c \in Arg_T$ is an (observation-based) argument for c with $conc(c) = c$.*
- *If there is a rule $r : c_1, \ldots, c_m \rightsquigarrow c$ in $\mathcal{R}$ and $A_i \in Arg_T$ with $conc(A_i) = c_i$ for each i, $1 \leq i \leq m$, then $A = A_1, \ldots, A_m \rightsquigarrow c$ is a rule-based argument for c in Arg_T with $conc(A) = c$.*

Arg_T is the smallest set containing these arguments, allowing only arguments of finite size.

We define several useful shorthands of arguments. For an observation-based argument $c \in \mathcal{K}$, the set of premises is $\mathtt{prem}(c) = \{c\}$, the set of defeasible rules $\mathtt{defrules}(c) = \emptyset$, the set of subarguments is $\mathtt{sub}(c) = \{c\}$, the top-rule is undefined, and the last defeasible rule $\mathtt{LDR}(A) = \emptyset$. For a rule-based argument A, we have $\mathtt{prem}(A) = \mathtt{prem}(A_1) \cup \cdots \cup \mathtt{prem}(A_m)$, $\mathtt{rules}(A) = \{r\} \cup \mathtt{rules}(A_1) \cup \cdots \cup \mathtt{rules}(A_m)$, $\mathtt{sub}(A) = \{A\} \cup \mathtt{sub}(A_1) \cup \cdots \cup \mathtt{sub}(A_m)$, and the top rule is $\mathtt{top\text{-}rule}(A) = r$. Moreover, we define $\mathtt{defrules}(A) = \mathtt{rules}(A) \cap \mathcal{R}_d$, if $\mathtt{top\text{-}rule}(A) \in \mathcal{R}_d$ then $\mathtt{LDR}(A) = \{r\}$, and if $\mathtt{top\text{-}rule}(A) \in \mathcal{R}_s$ then $\mathtt{LDR}(A) = \mathtt{LDR}(A_1) \cup \cdots \cup \mathtt{LDR}(A_m)$. Furthermore, for any argument, $\mathtt{prem_p}(A) = \mathtt{prem}(A) \cap \mathcal{K}_p$. For a set of arguments E, $\mathtt{conc}(E) = \{\mathtt{conc}(A) \mid A \in E\}$.

Example 1 (Running example)**.** *Let $T = (AS, \mathcal{K}, \leq')$ be an AT with $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$, where*

- *$\mathcal{L} = \{a, \neg a, b, c, d, e, \neg e, f, p, q, x, y, z, d_1, d_2, d_3, d_4\}$,*
- *$\mathcal{R}_s = \{(p, q \rightarrow a), (b, c \rightarrow e), (y \rightarrow d), (z \rightarrow d)\}$,*
- *$\mathcal{R}_d = \{(\neg a \Rightarrow b), (x \Rightarrow c), (d \Rightarrow \neg e), (\neg e \Rightarrow f)\}$,*
- *$n(\neg a \Rightarrow b) = d_1$, $n(x \Rightarrow c) = d_2$, $n(d \Rightarrow \neg e) = d_3$ and $n(\neg e \Rightarrow f) = d_4$,*
- *$\overline{a} = \{\neg a\}$, $\overline{\neg a} = \{a\}$, $\overline{e} = \{\neg e\}$ and $\overline{\neg e} = \{e\}$,*
- *$d_1 \leq d_3$ and $d_3 \leq d_2$,*

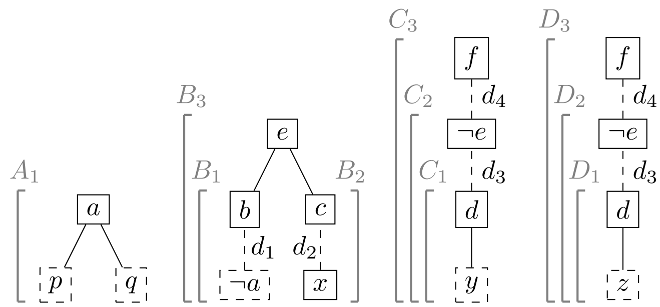

Figure 1: Running example AT. Dashed boxes and arrows signify ordinary premises and defeasible rules, and solid boxes and arrows signify axioms and strict rules, respectively.

- *$\mathcal{K}_a = \{x\}$ and $\mathcal{K}_p = \{p, q, \neg a, y, z\}$, and*
- *$p \leq' \neg a$ and $\neg a \leq' q$.*

T gives rise to 16 arguments as shown in Figure 1. An example of an observation-based argument is p. There are, for example, two (rule-based) arguments for f: $C_3 = ((y \rightarrow d) \Rightarrow \neg e) \Rightarrow f$ and $D_3 = ((z \rightarrow d) \Rightarrow \neg e) \Rightarrow f$.

The acceptability of arguments is determined by defeats between arguments. Intuitively, an argument A defeats another argument B when the conclusion of A contradicts a defeasible part of B. Additionally, certain types of defeats require that A is not less preferred than the subargument of B that A contradicts. Before discussing liftings from preferences between individual elements of $\mathcal{L}$ and $\mathcal{R}$ to preferences between arguments ($\prec$), we recall the general definitions for different types of defeats.

Definition 5 (Defeats)**.** *Given an AT $T = (AS, \mathcal{K}, \leq')$ where $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$ and two arguments A and B in Arg_T, A* defeats *B iff A* undercuts, contrary-rebuts, contrary-undermines, successfully contradictory-rebuts *or* successfully contradictory-undermines *B.*

- *A* undercuts *argument B (on B') iff for some $B' \in \mathtt{sub}(B)$ with defeasible top rule r, $\mathtt{conc}(A) \in \overline{n(r)}$.*
- *A* contrary-rebuts *argument B (on B') iff for some $B' \in \mathtt{sub}(B)$ for ϕ, B' has a defeasible top rule and $\mathtt{conc}(A)$ is a contrary of ϕ.*
- *A* successfully contradictory-rebuts *argument B (on B') iff for some $B' \in \mathtt{sub}(B)$ for ϕ, B' has a defeasible top rule, $\mathtt{conc}(A)$ is a contradictory of ϕ and $A \not\prec B'$.*
- *A* contrary-undermines *B (on B') iff for some $B' = \phi$, $\phi \in \mathtt{prem}_p(B)$ and $\mathtt{conc}(A)$ is a contrary of ϕ.*
- *A* successfully contradictory-undermines *argument B (on B') iff for some $B' = \phi$, $\phi \in \mathtt{prem}_p(B)$, $\mathtt{conc}(A)$ is a contradictory of ϕ and $A \not\prec B'$.*

For preference-dependent (contradictory-rebutting or contradictory-undermining) defeats, a lifting based on the partial preorders $\leq$ on $\mathcal{R}_d$ and $\leq'$ on $\mathcal{K}_p$ is used for comparing arguments. Four orderings were proposed by Modgil and Prakken (2013) based on combinations of the (i) elitist (ELI) and democratic (DEM) set comparators and (ii) argument comparators weakest-link and last-link principle.

Definition 6 (Ordering comparison on sets)**.** *Let Γ and Γ' be finite sets. Then $\lhd_s$ is defined as follows for $s \in \{\text{ELI}, \text{DEM}\}$.*

1. *If $\Gamma = \emptyset$ then $\Gamma \ntriangleleft_s \Gamma'$;*
2. *Else, if $\Gamma' = \emptyset$ then $\Gamma \lhd_s \Gamma'$;*
3. *Else, assuming a preordering $\leq$ (where $X < Y$ iff $X \leq Y$ and $Y \not\leq X$) over the elements in $\Gamma \cup \Gamma'$:*
   - *(a) For $s =$ ELI, $\Gamma \lhd_s \Gamma'$ if there is an $X \in \Gamma$ with $X < Y$ for all $Y \in \Gamma'$.*
   - *(b) For $s =$ DEM, $\Gamma \lhd_s \Gamma'$ if for each $X \in \Gamma$ we have $X < Y$ for some $Y \in \Gamma'$.*

Definition 7. *Let $s \in \{\text{ELI}, \text{DEM}\}$, T be an AT, and A and B be two arguments in T. We have $B \prec A$ under the* last-link *principle iff*

1. *$\mathtt{LDR}(B) \lhd_s \mathtt{LDR}(A)$, or*
2. *$\mathtt{LDR}(B) = \mathtt{LDR}(A) = \emptyset$ and $\mathtt{prem}_p(B) \lhd_s \mathtt{prem}_p(A)$.*

Under the weakest-link *principle, $B \prec A$ iff the following holds.*

1. *If all rules in B and A are strict, then $\mathtt{prem}_p(B) \lhd_s \mathtt{prem}_p(A)$.*
2. *If all premises of B and A are axioms, then $\mathtt{defrules}(B) \lhd_s \mathtt{defrules}(A)$.*
3. *Otherwise $\mathtt{defrules}(B) \lhd_s \mathtt{defrules}(A)$ and $\mathtt{prem}_p(B) \lhd_s \mathtt{prem}_p(A)$.*

Henceforth, we focus on the last-link principle, until Section 7 where we discuss the weakest-link principle.

Example 2. *Consider the running example. Under* DEM, *A_1 successfully contradictory-rebuts $\neg a$ (and thus B_1 and B_3): not all premises of A_1 are less preferred than $\neg a$, namely $q \not<' \neg a$, and so $\mathtt{prem}_p(A_1) \ntriangleleft_{\text{DEM}} \{\neg a\}$, and finally (as $\mathtt{LDR}(A_1) = \mathtt{LDR}(\neg a) = \emptyset$), $A_1 \not\prec \neg a$. On the other hand, this defeat is not present under* ELI *since a premise in A_1 is less preferred than $\neg a$: $p < \neg a$, and thus $A_1 \lhd_{\text{ELI}} \{\neg a\}$ and $A_1 \prec \neg a$. Since $\mathtt{top\text{-}rule}(A_1)$ is strict, A_1 is not defeated under either* ELI *or* DEM. *Similarly, under* DEM, *B_3 successfully contradictory-undermines C_2 and D_2: the rule $d_2 \in \mathtt{LDR}(B_3)$ is not less preferred than d_3, which constitues $\mathtt{LDR}(C_2)$ and thus $\mathtt{LDR}(B_3) \ntriangleleft_{\text{DEM}} \mathtt{LDR}(C_2)$ and $B_3 \not\prec C_2$ (similarly for D_2). On the other hand, under* ELI, *B_3 does not defeat C_2, since the rule $d_1 \in \mathtt{LDR}(B_3)$ is less preferred than d_3, so $\mathtt{LDR}(B_3) \lhd_{\text{ELI}} \mathtt{LDR}(C_2)$ and thus $B_3 \prec C_2$ (similarly for D_2).*

Argumentation theories for a given ordering can be translated into abstract argumentation frameworks.

Definition 8 (AFs defined by ATs)**.** *The abstract argumentation framework (AF) defined by an argumentation theory $T = (AS, \mathcal{K}, \leq')$ and an ordering is the pair $\langle \mathcal{A}, \mathcal{C} \rangle$, where $\mathcal{A}$ is the set of arguments in Arg_{AT} and $\mathcal{C}$ is the defeat relation on $\mathcal{A}$ determined by AT and the used preference ordering.*

The semantics of argumentation theories in ASPIC⁺ for a given ordering are defined via their corresponding abstract argumentation frameworks (Dung 1995).

Definition 9. *Let $AF = \langle \mathcal{A}, \mathcal{C} \rangle$ be an abstract argumentation framework and $S \subseteq \mathcal{A}$.*

- *S is conflict-free in F iff $(X, Y) \notin \mathcal{C}$ for each $X, Y \in S$.*
- *S defends $X \in \mathcal{A}$ in F iff for each $Y \in \mathcal{A}$ with $(Y, X) \in \mathcal{C}$, there is a $Z \in S$ with $(Z, Y) \in \mathcal{C}$.*
- *S is admissible in F iff S is conflict-free and S defends each $X \in S$.*
- *S is a complete extension of F iff S is admissible and, for each $X \in \mathcal{A}$, $X \in S$ if S defends X.*
- *The (unique) grounded extension S of F is the subset-minimal complete extension of F.*
- *S is preferred in F iff S is a subset-maximal complete extension.*
- *S is stable in F iff S is preferred and for each $Y \notin S$ there is an $X \in S$ such that $(X, Y) \in \mathcal{C}$.*

We abbreviate the semantics as $\sigma \in \{\mathsf{cf}, \mathsf{adm}, \mathsf{com}, \mathsf{grd}, \mathsf{prf}, \mathsf{stb}\}$ and refer to an extension under σ as a σ-extension (in a given AF).

An atom $x \in \mathcal{L}$ is said to be credulously accepted under semantics σ in a given AT T if there is a σ-extension E of the AF defined by T with $x \in \mathtt{conc}(E)$, and x is skeptically accepted under σ if for all σ-extensions E of the AF defined by T we find that $x \in \mathtt{conc}(E)$.

Example 3. *Returning to our running example, the AF $\langle \mathcal{A}, \mathcal{C} \rangle$ that the AT defines has 16 arguments (see also Figure 1): $\mathcal{A} = \{x, \neg a, p, q, y, z, A_1, B_1, B_2, B_3, C_1, C_2, C_3, D_1, D_2, D_3\}$. The defeat relation depends on the chosen preference ordering. Under* DEM, *$\mathcal{C} = \{(A_1, \neg a), (A_1, B_1), (A_1, B_3), (B_3, C_2), (B_3, C_3), (B_3, D_2), (B_3, D_3)\}$ and under* ELI, *$\mathcal{C} = \emptyset$. We can now see that under* DEM, *we get the unique complete, stable and preferred extension $\{p, q, x, y, z, A_1, B_2, C_1, C_2, C_3, D_1, D_2, D_3\}$. As there are no defeats under* ELI, *the whole set of arguments forms the unique complete, stable and preferred extension.*

3 Rephrasing Semantics under Last-link

In order to establish complexity results and efficient algorithms, we rephrase admissible, complete, stable, preferred and grounded semantics for last-link principle in terms of *assumptions* (sets of premises and defeasible rules), significantly extending previous work (Lehtonen, Wallner, and Järvisalo 2020; Lehtonen, Wallner, and Järvisalo 2022) to last-link and these central semantics. Working with assumptions rather than arguments enables a more efficient computation of the acceptance status of atoms than a procedure that first enumerates all arguments. In particular, for the latter, the number of arguments is not polynomially bounded in general (Strass, Wyner, and Diller 2019). Our rephrasings circumvent this blowup. Throughout this section we assume the last-link principle.

An assumption compactly represents a set of arguments. For brevity, we implicitly assume that a given AT $T = (AS, \mathcal{K}, \leq')$ is associated with $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$.

Definition 10 (Assumption (P, D))**.** *Let T be an AT. We call (P, D) with $P \subseteq \mathcal{K}_p$ and $D \subseteq \mathcal{R}_d$ an assumption in T.*

Definition 11 (Argument based on (P, D))**.** *Let T be an AT and a pair (P, D) with $P \subseteq \mathcal{L}$ and $D \subseteq \mathcal{R}_d$. An argument $A \in Arg_T$ is based on (P, D) if $\mathtt{prem}_p(A) \subseteq P$ and $\mathtt{defrules}(A) \subseteq D$. We define $Arg_T(P, D)$ to be the set of all arguments based on (P, D).*

The deductive closure of an assumption is the set of atoms that are concluded by arguments based on the assumption.

Definition 12 (Deductive closure of assumption (P, D))**.** *Let T be an AT and a pair (P, D) with $P \subseteq \mathcal{L}$ and $D \subseteq \mathcal{R}_d$. The* deductive closure *of (P, D), denoted by $Th_T(P, D)$, is $\{\mathtt{conc}(A) \mid A \in Arg_T(P, D)\}$.*

We write (P, D) *derives* ϕ iff $\phi \in Th_T(P, D)$ and omit the subscript T when it is clear from context.

For any assumption, the deductive closure can be computed in polynomial time by repeatedly applying the rules in $\mathcal{R}_s \cup D$, starting from $\mathcal{K}_a \cup P$, until a fixed point is reached (Lehtonen, Wallner, and Järvisalo 2020).

Example 4. *Consider the running example and the assumption (P, D) with $P = \{\neg a\}$ and $D = \{(\neg a \Rightarrow b), (x \Rightarrow c)\}$. The arguments based on (P, D) are $Arg_T(P, D) = \{B_1, B_2, B_3\}$ (recall Figure 1) and the deductive closure of (P, D) is $Th_T(P, D) = \{\neg a, x, b, c, e\}$. In other words, (P, D) derives each of $\neg a, x, b, c$ and e. This can be seen by noting that $\neg a$ is a premise and contained in P, $\neg x$ is an axiom, b and b can be derived from $\neg a$ and x with defeasible rules contained in D, and finally e can be derived from a and b with a strict rule.*

3.1 Rephrasing Defeat

Next, we characterize the notion of an assumption defeating a defeasible element. By Definition 5, in ASPIC+ there are various types of defeats, which in some cases are dependent on the preferences between rules and/or ordinary premises in combination with the chosen ordering. We treat each defeat type individually.

Preference-independent defeats As a first step, we consider preference-independent defeats, following (Lehtonen, Wallner, and Järvisalo 2020, Definition 10).

Definition 13 (Preference-independent defeats)**.** *Given an AT T, an assumption (P, D) in T, $r \in \mathcal{R}_d$ and $p \in \mathcal{K}_p$,*

1. *(P, D) undercuts r iff (P, D) derives an $x \in \overline{n(r)}$,*
2. *(P, D) contrary-rebuts r iff (P, D) derives a contrary of $\mathtt{cons}(r)$, and*
3. *(P, D) contrary-undermines p iff (P, D) derives a contrary of p.*

We establish correspondences between Definition 13 and preference-independent defeats on arguments (Definition 5).

Proposition 1. *Let T be an AT, (P, D) be an assumption in T. For all $B \in Arg_T$, it holds that*

- *(P, D) undercuts $\mathtt{top\text{-}rule}(B)$ iff an argument $A \in Arg_T(P, D)$ undercuts B on B in T,*
- *(P, D) contrary-rebuts $\mathtt{top\text{-}rule}(B)$ iff an argument $A \in Arg_T(P, D)$ contrary-rebuts B on B in T, and*
- *(P, D) contrary-undermines B iff an argument $A \in Arg_T(P, D)$ contrary-undermines B on B in T.*

Contradictory rebut with elitist lifting Moving on to preference-dependent defeats, we capture successful contradictory rebuts under elitist lifting as relations between assumptions (P, D) and individual defeasible rules. Towards this, we define $L^r_{(P,D)}$ which collects all consequents of defeasible rules for which all antecedents are derivable from (P, D) and that are not less preferred than r. Such rules can be used as last defeasible rules (LDR) to derive a contradictory of $\texttt{cons}(r)$ with an argument that is not less preferred than the target argument.

Definition 14. *Given an AT T, an assumption (P, D) in T, and rule $r \in \mathcal{R}$, let $L^r_{(P,D)} = \{\texttt{cons}(r') \mid r' \in D, r' \not< r$ and $\texttt{ants}(r') \subseteq Th(P, D)\}$.*

We rephrase contradictory rebuts for the elitist lifting as follows.

Definition 15 (ELI-contradictory-rebut)**.** *Given an AT T, an assumption (P, D) in T, and a defeasible rule $r \in \mathcal{R}_d$, (P, D) ELI-contradictory-rebuts r if and only if $(P \cup L^r_{(P,D)}, \emptyset)$ derives a contradictory of $\texttt{cons}(r)$.*

Example 5. *Consider assumption (P, D) with $P = \{\neg a\}$ and $D = \{(\neg a \Rightarrow b), (x \Rightarrow c)\}$ in the running example. For the rule $r = (d \Rightarrow \neg e)$ we have $L^r_{(P,D)} = \{c\}$. As $(P \cup L^r_{(P,D)}, \emptyset)$ does not derive any contradictory of $\texttt{cons}(r)$, (P, D) does not ELI-contradictory-rebut r. Thus there is no argument in $Arg_T(P, D)$ that contradictory-rebuts C_2 on C_2 in T, where $C_2 = (y \to d) \Rightarrow \neg e$.*

Contradictory rebuttals from assumptions to rules capture all contradictory rebuttals under elitist lifting.

Proposition 2. *Let T be an AT and (P, D) an assumption in T. For any argument $B \in Arg_T$, it holds that (P, D) ELI-contradictory-rebuts $\texttt{top-rule}(B)$ iff an argument $A \in Arg_T(P, D)$ contradictory-rebuts B on B in T.*

Contradictory rebut with democratic lifting We turn to contradictory rebut with democratic lifting. We capture successful contradictory rebuts under democratic lifting as relations between assumptions (P, D) and individual defeasible rules, with the help of a graph induced by $\mathcal{R}_s$ and (P, D).

Definition 16. *Let T be an AT and (P, D) an assumption in T. Then $G_{(P,D)} = (V, E)$ is a directed graph with $V = Th(P, D)$ and $E = \{(\texttt{cons}(r), a) \mid r \in \mathcal{R}_s, a \in \texttt{ants}(r)$ and $\texttt{ants}(r) \in Th(P, D)\}$.*

Intuitively, the edges of $G_{(P,D)}$ go from conclusion to antecedents of strict rules used in arguments based on (P, D).

Definition 17 (DEM-contradictory-rebut)**.** *Let T be an AT, (P, D) an assumption in T, and $r \in \mathcal{R}_d$. Then (P, D) DEM-contradictory-rebuts r iff there is some x which is a contradictory of $\texttt{cons}(r)$ and either $(P, \emptyset)$ derives x, or there is a directed path in $G_{(P,D)}$ from x to a node in $L^r_{(P,D)}$.*

Example 6. *Consider again assumption (P, D) with $P = \{\neg a\}$ and $D = \{(\neg a \Rightarrow b), (x \Rightarrow c)\}$ and rule $r = (d \Rightarrow \neg e)$ from Example 5. Then $G_{(P,D)} = (V, E)$ with $V = \{\neg a, x, b, c, e\}$ and $E = \{(e, b), (e, c)\}$. There is a directed path in $G_{(P,D)}$ from e, which is a contradictory of $\texttt{cons}(r) = \neg e$, to the node c in $L^r_{(P,D)}$. Thus (P, D) DEM-contradictory-rebuts r. Hence the argument B_3 in $Arg_T(P, D)$ contradictory-rebuts C_2 on C_2 in T.*

Contradictory rebuttals from assumptions capture all contradictory rebuttals under democratic lifting.

Proposition 3. *Let T be an AT and (P, D) be an assumption in T. For any argument $B \in Arg_T$ it holds that (P, D) DEM-contradictory-rebuts $\texttt{top-rule}(B)$ iff an argument $A \in Arg_T(P, D)$ contradictory-rebuts B on B in T.*

Contradictory undermine with elitist lifting We move on to undermining defeats, where target of a defeat is an ordinary premise. Towards rephrasing undermining defeats in terms of assumptions, given an ordinary premise $p \in \mathcal{K}_p$ and $P \subseteq \mathcal{K}_p$, we define $P_{\not< p} = \{p' \in P \mid p' \not< p\}$ as the set of all ordinary premises that are not less preferred than p,

Definition 18 (ELI-contradictory-undermine)**.** *Let T be an AT, (P, D) an assumption in T, and $p \in \mathcal{K}_p$. Then (P, D) ELI-contradictory-undermines p if and only if $(P_{\not< p}, \emptyset)$ derives a contradictory of p.*

Example 7. *In the running example, consider the assumption (P, D), where $P = \{p, q\}$ and $D = \emptyset$, and ordinary premise $\neg a$. Then $P_{\not< \neg a} = \{p' \in P \mid p' \not< \neg a\} = \{q\}$ and $(P_{\not< \neg a}, \emptyset)$ does not derive any contradictory of $\neg a$; there is no argument in $Arg_T(P, D)$ that contradictory-undermines $\neg a$ on $\neg a$ in T under the elitist last-link principle.*

Contradictory undermining from assumptions capture all contradictory undermining defeats under elitist lifting.

Proposition 4. *Let T be an AT and (P, D) an assumption in T. For any argument $B \in Arg_T$ it holds that (P, D) ELI-contradictory-undermines B iff an argument $A \in Arg_T(P, D)$ contradictory-undermines B on B in T.*

Contradictory undermine with democratic lifting To capture contradictory-undermining under democratic lifting, we use a graph induced by $\mathcal{R}_s$ and (P, D).

Definition 19. *Let T be an AT and $P \subseteq \mathcal{K}_p$. Define $G^P = (V, E)$ to be a directed graph with $V = Th(P, \emptyset)$ and $E = \{(\texttt{cons}(r), a) \mid r \in \mathcal{R}_s, a \in \texttt{ants}(r)$ and $\texttt{ants}(r) \subseteq Th(P, \emptyset)\}$.*

Definition 20 (DEM-contradictory-undermine)**.** *Let T be an AT, (P, D) an assumption in T, and $p \in \mathcal{K}_p$. Then (P, D) DEM-contradictory-undermines p iff there is an x which is a contradictory of p and either $(\emptyset, \emptyset)$ derives x or there is a directed path in G^P from x to at least one node in $P_{\not< p}$.*

Example 8. *Consider again (P, D), where $P = \{p, q\}$ and $D = \emptyset$, and ordinary premise $\neg a$ from Example 7. $G^P = (V, E)$ with $V = \{p, q, x, a\}$ and $E = \{(a, p), (a, q)\}$. There is a directed path in G^P from a to the node $q \in P_{\not< \neg a}$. By Definition 20, (P, D) DEM-contradictory-undermines $\neg a$ and thus the argument $A_1 = p, q \to a$ defeats argument $\neg a$ under democratic lifting.*

Contradictory undermining from assumptions capture all contradictory undermine defeats under democratic lifting.

Proposition 5. *Let T be an AT and (P, D) an assumption in T. For any argument $B \in Arg_T$ it holds that (P, D)* DEM*-contradictory-undermines B iff there is some argument $A \in Arg_T(P, D)$ that contradictory-undermines B on B in T.*

Defeat in general Based on our rephrasing, we introduce a general notion of defeats on assumptions.

Definition 21. *Let T be an AT, let (P, D) be an assumption in T, $x \in \mathcal{K}_p \cup \mathcal{R}_d$, and $s \in \{\text{ELI}, \text{DEM}\}$. We say that (P, D) s-defeats x (in T) if and only if*

- *(P, D) contrary-rebuts or contrary-undermines x,*
- *(P, D) undercuts x,*
- *(P, D) s-contradictory-rebuts x, or*
- *(P, D) s-contradictory-undermines x.*

The following general correspondence is a direct consequence of Propositions 1–5.

Proposition 6. *Let T be an AT, (P, D) an assumption in T and $s \in \{\text{ELI}, \text{DEM}\}$. For an argument $B \in Arg_T$, an argument $A \in Arg_T(P, D)$ defeats B on B in T under s iff*

- *if B is observation-based, then (P, D) s-defeats B, and*
- *otherwise (P, D) s-defeats $\texttt{top-rule}(B)$.*

3.2 Rephrasing Defence

Analogously to defeat, we introduce a notion of defence in terms of assumptions.

Definition 22 (Defence by assumptions)**.** *Given an AT T, let (P, D) be an assumption in T and $s \in \{\text{ELI}, \text{DEM}\}$. Let P' and D' be the sets of ordinary premises and defeasible rules not s-defeated by (P, D). For any $x \in \mathcal{K}_p \cup \mathcal{R}_d$, x is s-defended by (P, D) iff (P', D') does not s-defeat x.*

Recall that an argument A is defended by a set of arguments S if each argument B defeating A is defeated by an argument in S (Definition 9). In other words, each argument not defeated by any argument in S must not defeat A. We formally establish that a correspondence between defence by arguments and defence by assumptions holds in general.

Proposition 7. *Given an AT T, an assumption (P, D) in T, $s \in \{\text{ELI}, \text{DEM}\}$ and an argument $A \in Arg_T$, it holds that $Arg_T(P, D)$ defends A if and only if (P, D) s-defends every $x \in \texttt{prem}_p(A) \cup \texttt{defrules}(A)$.*

3.3 Rephrasing Semantics

Based on defeat and defence, we now define the considered argumentation semantics in terms of assumptions in such a way that an assumption that satisfies the criteria of a semantics compactly represents an extension of arguments under the semantics. In this section, we focus on admissible, complete, stable, and preferred semantics. The technically different treatment of grounded semantics will be discussed in the next section.

Definition 23 (Applicable rules)**.** *Let T be an AT. Given an assumption (P, D) in T, and rule $r \in D$, we say that r is* applicable *by (P, D) iff for each $a \in \texttt{ants}(r)$: $a \in Th(P, D)$.*

For comparing two assumptions (P, D) and (P', D'), we define $(P, D) \sqsubseteq (P', D')$, which holds if $P \subseteq P'$ and $D \subseteq D'$ (and $(P, D) \sqsubset (P', D')$ if one of the relations is proper).

Definition 24. *Let T be an AT and $s \in \{\text{ELI}, \text{DEM}\}$. An assumption (P, D) in T is s-conflict-free iff (P, D) does not s-defeat any $x \in P \cup D$. An s-conflict-free assumption (P, D), with all rules in D applicable by (P, D), is said to be*

- *s-admissible (in T) iff (P, D) s-defends all $x \in P \cup D$,*
- *s-complete (in T) iff (P, D) is s-admissible, P contains all ordinary premises s-defended by (P, D) and D contains all defeasible rules that are both applicable by (P, D) and s-defended by (P, D),*
- *s-stable (in T) iff each $p \in \mathcal{K}_p$ is either in P or s-defeated by (P, D) and each rule $r \in \mathcal{R}_d$ is in D, s-defeated by (P, D) or not applicable by (P', D') with P' and D' not s-defeated by (P, D), and*
- *s-preferred (in T) iff (P, D) is $\sqsubseteq$-maximally s-admissible.*

The condition for stable assumptions is slightly more involved due to the fact that a defeasible rule might be outside D and not s-defeated by (P, D), but also not applicable by undefeated defeasible elements, in which case it can not appear in an argument outside the stable extension.

Towards a semantical correspondence, we show two useful properties. Firstly, if an argument A defeats an argument B "on a defeasible element" x, then A defeats any argument C that contains x.

Proposition 8. *Let T be an AT. For any $A, B \in Arg_T$, if A defeats B on B', then*

- *if B' is observation-based, then A defeats each $C \in Arg_T$ such that $B' \in \texttt{prem}_p(C)$, and*
- *otherwise A defeats each $C \in Arg_T$ such that $\texttt{top-rule}(B) \in \texttt{defrules}(C)$.*

The other useful property is that any complete extension of arguments is "closed under defeasible elements", meaning that if a set of arguments $\mathcal{E}$ is a complete extension of an AF defined by an AT and P and D are the sets of ordinary premises and defeasible rules of arguments in $\mathcal{E}$, then any argument based on (P, D) is in $\mathcal{E}$.

Proposition 9. *Let T be an AT and F be the AF defined by T under s with $s \in \{\text{ELI}, \text{DEM}\}$. For any $\mathcal{E} \subseteq Arg_T$, if $\mathcal{E}$ is complete in F then each argument based on $(\texttt{prem}_p(\mathcal{E}), \texttt{defrules}(\mathcal{E}))$ is in $\mathcal{E}$.*

Finally, we show the correspondence between the standard definitions and our novel characterisation of semantics.

Theorem 10. *Let T be an AT, $\sigma \in \{\mathsf{adm}, \mathsf{com}, \mathsf{stb}, \mathsf{prf}\}$, $s \in \{\text{ELI}, \text{DEM}\}$, and F the AF defined by T under s.*

- *If (P, D) is an s-σ-assumption in T, then $\mathcal{E} = \{A \mid A \text{ based on } (P, D)\}$ is a σ-extension in F.*
- *If $\mathcal{E}$ is a σ-extension of F, then (P, D) is an s-σ-assumption of T with $P = \texttt{prem}_p(\mathcal{E})$ and $D = \texttt{defrules}(\mathcal{E})$.*

A direct consequence is the correspondence of acceptance problems between the standard semantics definitions and our rephrasings.

Proposition 11. *Let T be an AT, $\sigma \in \{\mathsf{adm}, \mathsf{com}, \mathsf{stb}, \mathsf{prf}\}$, $\sigma' = \{\mathsf{com}, \mathsf{stb}, \mathsf{prf}\}$, $s \in \{\textsc{Eli}, \textsc{Dem}\}$, and $x \in \mathcal{L}$. Then*

- *x is credulously accepted in T under σ and s iff there is a s-σ-assumption (P, D) in T s.t. $x \in Th_T(P, D)$, and*
- *x is skeptically accepted in T under σ' and s iff in all s-σ'-assumptions (P, D) in T we find $x \in Th_T(P, D)$.*

3.4 Fixpoint Computation for Grounded

In this section, we give a polynomial time fixpoint algorithm for computing the grounded extension based on assumptions. We first show that the defence relation is monotonous.

Lemma 12 (Monotonicity of defence)**.** *Let $T = (AS, \mathcal{K}, \leq')$ be an AT with $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$ and $s \in \{\textsc{Eli}, \textsc{Dem}\}$. For each $P \subseteq \mathcal{K}_p$, $D \subseteq \mathcal{R}_d$, $x \in \mathcal{K}_p \cup \mathcal{R}_d$, we have that if (P, D) s-defends x, then each (P', D') such that $P \subseteq P' \subseteq \mathcal{K}_p$ and $D \subseteq D' \subseteq \mathcal{R}_d$ s-defends x.*

Proposition 13 is a counterpart of Dung's fundamental lemma (Dung 1995, Lemma 10) for assumption defence.

Proposition 13. *Let $T = (AS, \mathcal{K}, \leq')$ be an AT with $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$, $s \in \{\textsc{Eli}, \textsc{Dem}\}$ and (P, D) an assumption with $P \subseteq \mathcal{K}_p$ and $D \subseteq \mathcal{R}_d$. Suppose that each rule in D is applicable by (P, D) and that $Arg_T(P, D)$ is admissible. Then each of the following holds:*

1. *For each $k \in \mathcal{K}_p$ that is s-defended by (P, D):*
   (a) *$Arg_T(P \cup \{k\}, D)$ is admissible, and*
   (b) *for each $x \in \mathcal{K}_p \cup \mathcal{R}_d$: if (P, D) s-defends x then $(P \cup \{k\}, D)$ s-defends x.*
2. *For each $r \in \mathcal{R}_d$ that is s-defended by (P, D):*
   (a) *$Arg_T(P, D \cup \{r\})$ is admissible, and*
   (b) *for each $x \in \mathcal{K}_p \cup \mathcal{R}_d$: if (P, D) s-defends x then $(P, D \cup \{r\})$ s-defends x.*

Towards defining the grounded extension without constructing arguments, we define a characteristic function for assumptions, in analogy with the "classical" characteristic function for AFs (Dung 1995, Definition 16).

Definition 25 (Characteristic function)**.** *Consider AT $T = (AS, \mathcal{K}, \leq')$ with $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$, let $s \in \{\textsc{Eli}, \textsc{Dem}\}$ and let (P, D) be an assumption. Then $def^s_T(P, D) = (P', D')$ where $P' = \{k \in \mathcal{K}_p \mid k$ is s-defended by $(P, D)\}$ and $D' = \{r \in \mathcal{R}_d \mid r$ is applicable and s-defended by $(P, D)\}$.*

For $i > 0$, we denote i applications of def^s_T on $(\emptyset, \emptyset)$ by $def^{s,i}_T(\emptyset, \emptyset)$ and define $def^{s,0}_T(\emptyset, \emptyset) = (\emptyset, \emptyset)$.

By Proposition 14, iterating the characteristic function starting from $(\emptyset, \emptyset)$ gives the grounded extension.

Proposition 14. *Given an AT $T = (AS, \mathcal{K}, \leq')$ with $AS = (\mathcal{L}, \overline{}, \mathcal{R}, n, \leq)$, let $s \in \{\textsc{Eli}, \textsc{Dem}\}$ and let (P^*, D^*) be the least fixed point of def^s_T. Then the grounded extension of the AF defined by T under s equals $Arg_T(P^*, D^*)$.*

Proposition 14 suggests a polynomial-time procedure for computing the unique grounded extension. First, compute (P^*, D^*), the least fixed point of def^s_T. An application of def^s_T takes polynomial time and at least one premise or defeasible rule is added in each iteration, so (P^*, D^*) is reached in at most $|\mathcal{K}_p| + |\mathcal{R}_d|$ iterations. Then, compute $Th(P^*, D^*)$ by repeatedly applying applicable rules in $\mathcal{R}_s \cup D^*$ until reaching a fixed point.

4 Complexity Results

We can now establish the complexity of deciding acceptance in ASPIC$^+$ under last-link principle and both elitist and democratic liftings for the central semantics of admissible, complete, stable, preferred and grounded. The characterisations of the semantics proposed in Section 3 are key to pinpointing the exact complexity, as they allow us to bypass the exponential argument construction.

It follows from Proposition 14 that acceptance under grounded semantics can be done in polynomial time.

Theorem 15. *Given an $T = (AS, \mathcal{K}, \leq')$ and an atom $x \in \mathcal{L}$, under the last-link principle and either democratic or elitist lifting, deciding whether x is accepted under grounded semantics in T can be done in polynomial time.*

The characterisations of Definition 9 along with the correspondence results of Theorem 10 and Proposition 11 imply the following complexity results.

Theorem 16. *Given an $T = (AS, \mathcal{K}, \leq')$ and an atom $x \in \mathcal{L}$, under the last-link principle and either democratic or elitist lifting, deciding whether x is credulously accepted in T under admissible, complete, preferred or stable semantics is NP-complete. Skeptical acceptance of x in T is polynomial-time decidable under admissible and complete semantics, coNP-complete under stable semantics and Π^P_2-complete under preferred semantics.*

Notably, the complexity of these problems coincides with the complexity of ASPIC$^+$ without preferences (Lehtonen, Wallner, and Järvisalo 2020) and reasoning in AFs (Dvořák and Dunne 2017). This is in contrast to the weakest-link principle under which deciding acceptance under stable semantics is known to be Σ^p_2/Π^p_2-hard (Lehtonen, Wallner, and Järvisalo 2022). We expand on this discrepancy in Section 7, showing that acceptance under grounded semantics also exhibits a complexity jump under weakest-link principle.

5 ASP Encodings

We present ASP encodings of the (co)NP-complete acceptance problems under admissible, complete and stable semantics under the last-link principle and $s \in \{\textsc{Eli}, \textsc{Dem}\}$. Concretely, the answer sets to the proposed programs correspond to admissible, complete, and stable s-assumptions. This directly extends to deciding credulous and skeptical queries. First, given an AT $T = (AS, \mathcal{K}, \leq')$, we assume a naming function n' for all rules in T (such that the image of n' does not overlap with $\mathcal{L}$) and represent T in ASP as

$$\begin{aligned}\texttt{AT}(T) = &\{\texttt{axiom}(a).\mid a \in \mathcal{K}_n\} \cup \{\texttt{premise}(a).\mid a \in \mathcal{K}_p\} \cup\\ &\{\texttt{d\_head}(n'(r), b).\mid r \in \mathcal{R}_d, b = \texttt{cons}(r)\} \cup\\ &\{\texttt{d\_body}(n'(r), b).\mid r \in \mathcal{R}_d, b \in \texttt{ants}(r)\} \cup\\ &\{\texttt{s\_head}(n'(r), b).\mid r \in \mathcal{R}_s, b = \texttt{cons}(r)\} \cup\\ &\{\texttt{s\_body}(n'(r), b).\mid r \in \mathcal{R}_s, b \in \texttt{ants}(r)\} \cup\\ &\{\texttt{contrary}(a, b).\mid b \in \overline{a}, a, b \in \mathcal{L}\} \cup\\ &\{\texttt{ctrd}(a, b).\mid b \in \overline{a}, a \in \overline{b}, a, b \in \mathcal{L}\}.\end{aligned}$$

We detail only novel parts of the programs and explain other parts as needed; the full encodings are in the supplement. As a basis we use the module Π_{common} (Lehtonen, Wallner, and Järvisalo 2020) for ASPIC+ without preferences. This encoding makes a non-deterministic guess of an assumption (P, D) (represented by the predicate `in`), derives all atoms derivable from (P, D) (`der`), using rules applicable by (P, D) (`app_by_in`) and defines preference-independent defeats of premises and rules by (P, D) (`defeated`), as well as rules applicable by premises and rules that are not defeated by (P, D) (`app_by_undefeated`). Further, Π_{common} enforces conflict-freeness and applicability of each $r \in D$. To capture the semantics of ASPIC+ with preferences, we introduce additional rules to complete `defeated` with preference-dependent defeats, detailed as the modules Π_{ELI} and Π_{DEM} (Listing 2 and 3).

We first introduce some further elements common to both liftings in Π_{prf} (Listing 1). In Line 1 a "good rule" corresponding to the rules used in deriving $L^r_{(P,D)}$ (Definition 14) is defined as a defeasible rule applicable by `in` that is not less preferred than a given defeasible rule. Line 2 enforces transitivity of preferences and Lines 3–5 derive the `no_less_pref` predicate for encoding $X \not< Y$.

For elitist lifting, Lines 1–4 of Π_{ELI} define `der_rebut`, capturing atoms derivable from (P, D) in a way that respects the conditions for contradictory rebut (recall Definition 15): premises and axioms are derivable (Lines 1–2), as well as conclusions of good rules that are applicable by (P, D) (Line 3) and conclusions of strict rules applicable in a preference-sensitive manner (Line 4). Line 5 adds contradictory rebuts to `defeated`: if `der_rebut`(y, r) holds for a rule r such that y is the contradictory of `conc`(r), then y is contradictory rebutted by (P, D). Lines 6–8 capture derivations for contradictory undermining (Definition 18), namely all atoms derivable from $(P_{\not< p}, D)$ for any given target premise p. Line 9 captures successful contradictory undermine.

For democratic lifting, Lines 1–4 of Π_{DEM} define `nodef_der`, capturing atoms derived with no defeasible rules, i.e., from $(P, \emptyset)$. Lines 5–7 encode reachability in $G_{(P,D)}$ (Definition 16), i.e., reachability via strict rules to heads of defeasible rules that are applicable by (P, D). As a special case for undermining, premises are reachable from themselves (Line 5). Line 6 states that the head of a strict rule that is applicable from $(P, \emptyset)$ reaches all body elements of the rules, and Line 7 enforces transitivity. Lines 8 and 9 correspond to the two cases in which (P, D) contradictory rebuts a rule r (recall Definition 17). The first case, on Line 8, is that (P, D) derives an atom that is a contradictory of `cons`(r) and the head of a good rule is reachable from the atom. The second, on Line 9, is that $(P, \emptyset)$ derives a contradictory of `cons`(r). Lines 10–11 encode derivability from $(\emptyset, \emptyset)$ and Lines 12–13 encode the two cases for contradictory undermining a premise p (recall Definition 20). Line 12 declares that if $(\emptyset, \emptyset)$ derives a contradictory of p, then p is undermined; and Line 13 that if a contradictory of p is derived via $(P, \emptyset)$ and the contradictory reaches a premise in P via only strict rules, then p is undermined.

Listing 1: Module Π_{prf}

```
1 good_rule(RI,R) ← app_by_in(RI), no_less_pref(RI, R).
2 preferred(X,Z) ← preferred(X,Y), preferred(Y,Z).
3 s_less_pref(X,Y) ← preferred(Y,X), not preferred(X,Y).
4 no_less_pref(X,Y) ← premise(X), premise(Y),
        not s_less_pref(X,Y).
5 no_less_pref(X,Y) ← d_head(X,_), d_head(Y,_),
        not s_less_pref(X,Y).
```

Given these modules, we are ready to complete the encodings for stable, admissible, and complete semantics defined on assumptions (recall Definition 24). The program for finding a stable assumption of $T = (AS, \mathcal{K}, \leq')$ under lifting $s \in \{\text{ELI}, \text{DEM}\}$ is

$$\begin{aligned}\Pi_{s\_stable} = \mathtt{AT}(T) \cup \Pi_{common} \cup \Pi_{prf} \cup \Pi_s \cup \\ \{ \leftarrow \mathit{not}\ \mathtt{in}(X), \mathit{not}\ \mathtt{defeated}(X), \mathtt{premise}(X). \\ \leftarrow \mathtt{app\_by\_undefeated}(R), \mathit{not}\ \mathtt{in}(R).\}\end{aligned}$$

For admissibility, in addition to the modules presented so far, an analogous module decides what is defeated by the premises and rules that are not defeated by (P, D) (predicate `defeated_by_undefeated`), which we refer to as Δ_{s_adm}. A constraint is added to enforce that not defeated premises and rules do not defeat (P, D), resulting in

$$\begin{aligned}\Pi_{s\_admissible} = \mathtt{AT}(T) \cup \Pi_{common} \cup \Pi_{prf} \cup \Pi_s \cup \\ \Delta_{s\_adm} \cup \{\leftarrow \mathtt{in}(X), \mathtt{defeated\_by\_undefeated}(X).\}\end{aligned}$$

Completeness builds on top of admissibility. We capture rules that are applicable from premises and rules that are not defeated by (P, D) (`app_by_defended`) with a module Δ_{com}, and add constraints for completeness, giving

$$\begin{aligned}\Pi_{s\_complete} = \Pi_{s\_admissible} \cup \Delta_{com} \cup \\ \{\leftarrow \mathit{not}\,\mathtt{in}(X), \mathit{not}\ \mathtt{defeated\_by\_undefeated}(X), \mathtt{premise}(X). \\ \leftarrow \mathit{not}\ \mathtt{in}(X), \mathit{not}\ \mathtt{app\_by\_defended}(X).\}\end{aligned}$$

Finally, for credulous acceptance, we add the constraint "$\leftarrow$ *not* $\mathtt{der}(q)$." stating that a given atom $q \in \Pi$ must be derivable from (P, D). The query is credulously accepted iff the program has answer sets. For skeptical acceptance, we enforce instead "$\leftarrow \mathtt{der}(q)$." stating that the query q should not be derivable from (P, D). The query is skeptically accepted iff the program has no answer sets.

Listing 2: Module Π_{ELI}

```
1 der_rebut(X,R) ← premise(X), in(X), d_head(R,_).
2 der_rebut(X,R) ← axiom(X), d_head(R,_).
3 der_rebut(X,R) ← d_head(RI,X), good_rule(RI,R).
4 der_rebut(X,R) ← s_head(RI,X), d_head(R,_),
        der_rebut(Y,R) : s_body(RI,Y).
5 defeated(R) ← d_head(R,S), der_rebut(Y,R), ctrd(S,Y).
6 der_undermine(X,Y) ← no_less_pref(X,Y), in(X),
        premise(X), premise(Y).
7 der_undermine(X,Y) ← axiom(X), premise(Y).
8 der_undermine(X,Y) ← premise(Y), s_head(R,X),
        der_undermine(Z,Y) : s_body(R,Z).
9 defeated(X)← premise(X),der_undermine(Y,X),ctrd(X,Y).
```

Listing 3: Module Π_{DEM}

```
nodef_der(X) ← axiom(X).
nodef_der(X) ← premise(X), in(X).
nodef_der(X) ← s_head(R,X), nodef_app(R).
nodef_app(R) ← s_head(R,_), nodef_der(X):s_body(R,X).
s_reach(X,X) ← premise(X).
s_reach(X,Y) ← nodef_app(R),s_head(R,X),s_body(R,Y).
s_reach(X,Y) ← s_reach(X,Z), s_reach(Z,Y).
defeated(R) ← d_head(R,S), ctrd(S,Y), der(Y),
      s_reach(Y,X), d_head(RG,X), good_rule(RG,R).
defeated(R) ← d_head(R,S), ctrd(S,Y), nodef_der(Y).
s_der(X) ← axiom(X).
s_der(X) ← s_head(R,X), s_der(Y) : s_body(R,Z).
defeated(X) ← s_der(Y), ctrd(X,Y), premise(X).
defeated(X) ← premise(X), ctrd(X,Y), nodef_der(Y),
      s_reach(Y,Z),premise(Z),no_less_pref(Z,X),in(Z).
```

6 Empirical Evaluation

We present an empirical evaluation of our ASP-based algorithms from Section 5 for the NP-hard tasks of deciding credulous stable, admissible and complete acceptance (DC-ST, DC-AD and DC-CO), and skeptical stable acceptance (DS-ST). We incorporated our approach to the ASPIC⁺ solver ASPFORASPIC, available in open source at https://bitbucket.org/coreo-group/aspforaspic/. We use CLINGO (Gebser et al. 2016) version 5.7.1 as the ASP solver. We compare our approach to PYARG (Odekerken, Borg, and Berthold 2023) which implements the two-step approach, translating an AT to an AF and then using CLINGO to reason on the AFs using the ASPARTIX ASP encodings for AF reasoning (Egly, Gaggl, and Woltran 2010). The experiments were run on 2.40-GHz Intel Xeon Gold 6148 CPUs under a per-instance time limit of 600 seconds and memory limit of 32 GB.

We generated benchmark instances following Lehtonen, Wallner, and Järvisalo (2020; 2022) for language sizes $|\mathcal{L}|$ from 50 to 2100, with 5 instances per size. We set parameters for the generators as follows based on preliminary experiments with the aim of an approximate 50%/50% split between instances with/without a stable extension. For each atom, the number of rules deriving the atom and the size of each rule were both assigned uniformly at random from $[1, 5]$. We let 1% of atoms be axioms, 20% be premises and half of the rules be strict and half defeasible. Each premise has a contrary (or contradictory), 10% of rules has a contrary, 40% of sentences are in a contrary relation, and 50% of contrary relations are symmetric (contradictories). We used a random permutation $(x_i)_{0<i\leq n}$ of both premises and defeasible rules. For $j < i$, we let x_i be preferred to x_j with 30% probability.

Table 1 compares the performance of our approach ASPFORASPIC to PYARG under elitist lifting. PYARG could not solve instances beyond 50 atoms, while our approach solves all instances with at most 1200 atoms for all tasks/semantics with mean runtimes below 20 s for instances with at most 400 atoms. Under democratic lifting (see supplement for tabulated results) ASPFORASPIC solved all instances with at most 1900 atoms, while PYARG had similar results as it did under elitist.

#solved (mean run time over solved (s))								
	PYARG							
$\lvert\mathcal{L}\rvert$	DC-ST		DC-CO		DC-AD		DS-ST	
50	2	(20.8)	2	(20.0)	1	(34.7)	2	(19.5)
>50	0	—	0	—	0	—	0	—
	ASPFORASPIC							
$\lvert\mathcal{L}\rvert$	DC-ST		DC-CO		DC-AD		DS-ST	
50	5	(0.1)	5	(0.2)	5	(0.2)	5	(0.1)
100	5	(0.3)	5	(0.5)	5	(0.5)	5	(0.3)
200	5	(1.7)	5	(2.9)	5	(3.0)	5	(1.7)
400	5	(11.6)	5	(14.7)	5	(16.9)	5	(9.6)
800	5	(64.8)	5	(87.6)	5	(97.9)	5	(59.3)
1200	5	(175.5)	5	(226.4)	5	(239.0)	5	(181.6)
1600	5	(422.6)	4	(518.9)	4	(543.4)	5	(450.0)
1700	3	(473.8)	2	(587.9)	1	(591.1)	3	(486.4)
1800	2	(569.5)	0	—	0	—	3	(568.8)
1900	0	—	0	—	0	—	0	—

Table 1: ASPFORASPIC vs PYARG under elitist lifting.

We also investigated the runtime impact of accounting for preferences on the same benchmark set. Specifically, we ran the existing ASP approach to ASPIC⁺ without preferences which we extend on in this work (Lehtonen, Wallner, and Järvisalo 2020) on the same benchmarks, ignoring the preferences. We found that accounting for preferences results in more solved instances. For example, for DC-CO, accounting for preferences resulted in 102 and 86 solved instances under democratic and elitist lifting, resp., while when ignoring the preferences, 81 instances were solved (see appendix for details). This is in line with the intuition that without preferences there are more defeats and therefore potentially more assumption sets to check.

7 Obstacles to Extending to Weakest-link

As a final contribution, we identify obstacles to capturing reasoning in ASPIC⁺ with preferences under the weakest-link principle in a similar manner as was done for last-link in this paper. We formally show, firstly, that deciding grounded acceptance under weakest-link elitist lifting has higher complexity, and secondly, that closure under defeasible elements (i.e., the property shown in Proposition 9 for last-link principle) does not hold under weakest-link principle under grounded and complete semantics. These results extend similar results previously shown for stable semantics (Lehtonen, Wallner, and Järvisalo 2022). The first is a practical and computational obstacle: extending the approach proposed in this paper would require multiple calls to an NP-solver. The latter is a theoretical one: one needs a way to bypass the fact that it is not in general possible to exactly characterize AF extensions with assumptions only.

Proposition 17. *It is NP-hard to decide whether the grounded extension contains an argument for a queried atom, under the weakest-link principle and elitist lifting.*

We now show that under weakest-link principle, closure under defeasible elements does not hold under grounded and complete semantics. Visualization of this counterexample can be found in the supplement.

Counterexample 1. *Let* $T = (\mathcal{L}, \mathcal{R}, n, ^{-}, \mathcal{K}, \leq)$ *be an AT with* $\mathcal{L} = \{a, b, c, d, e, f, p, q, q', x, y, y'\}$, $\mathcal{K}_p = \{a, b, c, d, e, f\}$, $\mathcal{K}_s = \emptyset$, $\overline{y} = \{y'\}$, $\overline{y'} = \{y\}$, $\overline{q} = \{q'\}$, $\overline{q'} = \{q\}$, $\overline{p} = \{p'\}$, $\overline{p'} = \{p\}$, *and*

$$\mathcal{R} = \{r_1 : q, p \to y',\ r_2 : a \to x,\ r_3 : b \to x,\ r_4 : x \Rightarrow y,\ r_5 : c \Rightarrow q,\ r_6 : d \Rightarrow p,\ r_7 : e \Rightarrow q',\ r_8 : y, p \to q',\ r_9 : f \Rightarrow p'\}.$$

Moreover, let $c \leq' a$, $d \leq' a$, $r_5 \leq_d r_4$, *and* $r_6 \leq_d r_4$.

Note that the argument $((c \Rightarrow q), (d \Rightarrow p)) \to y'$ *is strictly less preferred than* $((a \to x) \Rightarrow y)$ *but not less preferred than* $((b \to x) \Rightarrow y)$, *and thus defeats the latter but not the former under weakest-link and either democratic or elitist lifting. Thus the grounded extension contains, among others,* a, b *and* $((a \to x) \Rightarrow y)$. *However, from the defeasible elements of these three, one can construct* $((b \to x) \Rightarrow y)$, *which is not part of the grounded extension. As the grounded extension is complete, this counterexample applies to complete semantics as well.*

Note that taking strict rules and axioms into account does not remedy this, since $(b \to x)$ is in the grounded extension and thus all strict elements of $((b \to x) \Rightarrow y)$ are also used in the grounded extension. This result implies that given an AT, a set of arguments might be an extension, while another set using exactly the same rules and premises is not. This is problematic in terms of extending our approach to the weakest-link principle in which extensions are characterized in terms of their (defeasible) rules and premises. Lehtonen, Wallner, and Järvisalo (2022) showed that restricting to so-called well-formed theories preserves closure under defeasible elements under stable semantics. Considering similar restrictions for other semantics and other possibilities for extending the proposed approach to weakest-link principle are interesting avenues for future work.

8 Conclusions

We established the computational complexity and developed practical algorithms for reasoning in ASPIC$^+$ under the last-link principle and various central argumentation semantics. A key to the complexity results and algorithms was a formal rephrasing of argumentation semantics in terms of defeasible elements. Algorithmically, we detailed ASP encodings based on the rephrasing for the NP/coNP-complee acceptance problem variants and, relying on an ASP solver, showed empirically that our approach significanly outperforms a two-step approach to reasoning about acceptance via AF construction and ASP. Interestingly, complexity of deciding acceptance in ASPIC$^+$ does not increase with the inclusion of preferences under the last-link principle. By contrast, we established NP-hardness under grounded semantics for the weakest-link principle.

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