Proceedings of the 21st International Conference on Principles of Knowledge Representation and Reasoning
Main Track

Unique Characterisability and Learnability of Temporal Queries Mediated by an Ontology

Jean Christoph Jung[1], **Vladislav Ryzhikov**[2], **Frank Wolter**[3], **Michael Zakharyaschev**[2]
[1]TU Dortmund University, Germany
[2]Birkbeck, University of London, UK
[3]University of Liverpool, UK
jean.jung@tu-dortmund.de, {v.ryzhikov, m.zakharyaschev}@bbk.ac.uk, wolter@liverpool.ac.uk

Abstract

Algorithms for learning database queries from examples and unique characterisations of queries by examples are prominent starting points for developing automated support for query construction and explanation. We investigate how far recent results and techniques on learning and unique characterisations of atemporal queries mediated by an ontology can be extended to temporal data and queries. Based on a systematic review of the relevant approaches in the atemporal case, we obtain general transfer results identifying conditions under which temporal queries composed of atemporal ones are (polynomially) learnable and uniquely characterisable.

1 Introduction

Providing automated support for constructing database queries from data examples has been an important research topic in database management, knowledge representation and computational logic, often subsumed under the query-by-example paradigm (Martins 2019). One prominent approach is based on exact learning using membership queries (Angluin 1987b), where one aims to identify a database query by repeatedly asking an oracle (e.g., domain expert) whether certain data examples are answers or non-answers to the query. Recently, the ability to uniquely characterise a database query by a finite set of positive and negative examples has been identified and investigated as a 'non-procedural' necessary condition for learnability via membership queries (Staworko and Wieczorek 2015; ten Cate and Dalmau 2022; Fortin et al. 2022). More precisely, a query $\boldsymbol{q}(x)$ is said to fit a pair $E = (E^+, E^-)$ of sets E^+ and E^- of pointed databases $(\mathcal{D}, a)$ if $\mathcal{D} \models \boldsymbol{q}(a)$ for all $(\mathcal{D}, a) \in E^+$, and $\mathcal{D} \not\models \boldsymbol{q}(a)$ for all $(\mathcal{D}, a) \in E^-$. The example set E uniquely characterises $\boldsymbol{q}$ within a class $\mathcal{Q}$ of queries if $\boldsymbol{q}$ is the only one (up to equivalence) in $\mathcal{Q}$ that fits E. The existence of (polynomial-size) unique characterisations is a necessary pre-condition for (polynomial) learnability via membership queries. Such characterisations can also be employed for explaining and synthesising queries.

Extending results on characterising and learning conjunctive queries (CQs) under the standard closed-world semantics (ten Cate and Dalmau 2022), there has recently been significant progress towards CQs mediated by a description logic (DL) ontology under the open-world semantics (Funk, Jung, and Lutz 2021; 2022b). The focus has been on ontologies in the tractable *DL-Lite* and $\mathcal{EL}$ families and tree-shaped CQs such as ELQs ($\mathcal{EL}$-concepts) and ELIQs ($\mathcal{ELI}$-concepts). In fact, even under the closed-world semantics, only acyclic queries can be uniquely characterised and, equivalently, learned using membership queries in polynomial time (ten Cate and Dalmau 2022).

In this paper, we aim to understand how far these characterisability and learnability results for atemporal queries mediated by an ontology can be expanded to the temporal case. Temporal ontology-mediated query answering provides a framework for accessing temporal data using a background ontology. It has been investigated for about a decade—see, e.g., (Artale et al. 2017) for a survey—resulting in different settings and a variety of query and ontology languages (Baader, Borgwardt, and Lippmann 2015; Borgwardt and Thost 2015; Artale et al. 2022; Gutiérrez-Basulto, Jung, and Kontchakov 2016; Artale et al. 2014; Wałęga et al. 2020). As a natural starting point, we assume that the background ontology holds at all times and does not admit temporal operators in its axioms. As a query language we consider a combination of ELIQs with linear temporal logic (*LTL*) operators. First observations on unique characterisability and learnability of plain *LTL* queries (Fortin et al. 2022) showed that, even without ontologies, a restriction to so-called *path queries* (defined below) is needed to obtain positive general and useful results. Our main contributions in this paper are general transfer theorems identifying abstract properties of query and ontology languages that are needed to lift unique characterisability and learnability from atemporal ontology-mediated queries and ontology-free path *LTL* queries to temporalised domain queries mediated by a DL ontology. To facilitate the transfer, we begin by revisiting the atemporal case. Below is an overview of the obtained results.

Atemporal case. We present and compare two approaches to finding unique (polysize) characterisations of atemporal queries mediated by an ontology: via frontiers and via split-partners (aka dualities). Both tools are developed under the condition that query containment in the respective atemporal DLs can be reduced to query evaluation. We call this condition *containment reduction*. It applies to all fragments of the expressive DL $\mathcal{ALCHI}$ and more general FO-ontologies without equality as well as to *DL-Lite* with functional roles.

It ensures that whenever a unique characterisation of a query mediated by an ontology exists, there is also one with a single positive example in E^+. These tools yield two essentially optimal unique characterisability results: frontiers give polynomial-size characterisations of ELIQs mediated by an ontology in the DLs *DL-Lite*$_\mathcal{H}$ and *DL-Lite*$^-_\mathcal{F}$ (Funk, Jung, and Lutz 2021; 2022b), while split-partners provide exponential-size characterisations of ELIQs mediated by an $\mathcal{ALCHI}$ ontology and polysize characterisations of ELQs mediated by an RDFS ontology.

Temporalising unique characterisations. We now assume that temporal data instances are finite sets of facts (ground unary and binary atoms) timestamped by the moments $i \in \mathbb{N}$ they happened and that queries are equipped with temporal operators. By combining the results from the atemporal case above with the techniques of (Fortin et al. 2022), we establish general transfer theorems on (polysize) unique characterisations of temporal queries mediated by a DL ontology.

We first consider the temporal operators $\bigcirc$ (at the next moment), $\Diamond$ (sometime later), and $\Diamond_r$ (now or later) and define, given a class $\mathcal{Q}$ of atemporal queries (say, ELIQs), the family $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ of *path queries* of the form

$$\boldsymbol{q} = \boldsymbol{r}_0 \wedge \boldsymbol{o}_1(\boldsymbol{r}_1 \wedge \boldsymbol{o}_2(\boldsymbol{r}_2 \wedge \dots \wedge \boldsymbol{o}_n \boldsymbol{r}_n)),$$

where $\boldsymbol{o}_i \in \{\bigcirc, \Diamond, \Diamond_r\}$ and $\boldsymbol{r}_i \in \mathcal{Q}$. These queries are evaluated at time 0. Even if $\mathcal{Q}$ consists of conjunctions of atoms only and no ontology is present, not all queries in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ can be uniquely characterised. A typical example of a non-characterisable query in this class is $\boldsymbol{q}(x) = \Diamond_r(A(x) \wedge B(x))$ (Fortin et al. 2022). We first give an effective syntactic criterion for an $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$-query to be 'safe' in the sense of admitting a unique characterisation. Then we prove a fully general transfer theorem stating that if a DL $\mathcal{L}$ admits containment reduction and (polysize) unique characterisations for $\mathcal{Q}$-queries mediated by an $\mathcal{L}$-ontology, then so does the class of safe temporalised queries in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$. For example, this theorem yields polysize unique characterisations of safe queries in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) mediated by a *DL-Lite*$^-_\mathcal{F}$ or *DL-Lite*$_\mathcal{H}$ ontology and exponential ones for safe $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ)-queries mediated by an $\mathcal{ALCHI}$ ontology.

Our second transfer result concerns temporal queries with the binary operator U (until) under the strict semantics and the family $\mathit{LTL}_p^{\mathsf{U}}(\mathcal{Q})$ of path queries of the form

$$\boldsymbol{q} = \boldsymbol{r}_0 \wedge (\boldsymbol{l}_1 \mathbin{\mathsf{U}} (\boldsymbol{r}_1 \wedge (\boldsymbol{l}_2 \mathbin{\mathsf{U}} (\dots (\boldsymbol{l}_n \mathbin{\mathsf{U}} \boldsymbol{r}_n) \dots)))).$$

For its subclass of so-called $\mathcal{O}$-peerless queries, in which the $\boldsymbol{r}_i, \boldsymbol{l}_i \in \mathcal{Q}$ do not contain each other wrt a given ontology $\mathcal{O}$, we prove general transfer of unique characterisations provided that unique characterisations for the atemporal class $\mathcal{Q}$ can be obtained via split-partners. For example, this result gives exponential-size unique characterisations of peerless queries in $\mathit{LTL}_p^{\mathsf{U}}$(ELIQ) mediated by any $\mathcal{ALCHI}$ ontology and polysize characterisations of peerless queries in $\mathit{LTL}_p^{\mathsf{U}}$(ELQ) mediated by any RDFS ontology. We also show that the general transfer fails if frontier-based characterisations of queries in $\mathcal{Q}$ are used in place of split-partners.

Temporalising learning. We apply our results on unique characterisations to learning a target query $\boldsymbol{q}_T$, known only to a teacher, wrt a given ontology $\mathcal{O}$ in Angluin's framework of exact learning. We allow the learner to use *membership queries*, which return in unit time whether a given example $(\mathcal{D}, a)$ is a positive one for $\boldsymbol{q}_T$ wrt to $\mathcal{O}$. Given that we always construct example sets effectively, it is not difficult to show that our exponential-size unique characterisations entail exponential learning algorithms. We are, however, mainly interested in efficient algorithms formalised as polynomial time or polynomial query learnability.

Obtaining such algorithms from polysize characterisations is more challenging and we currently only know how this can be done if the unique characterisation is based on polysize frontiers. Hence, we focus on queries in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ and show that polynomial query learnability transfers from $\mathcal{Q}$ to safe queries in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ and that polytime learnability transfers if natural additional conditions hold for $\mathcal{Q}$ and the considered ontology language.

Omitted details and proofs can be found in the full arXiv paper (Jung et al. 2023).

2 Related Work

The unique characterisation framework for temporal formulas, underpinning this paper, was originally introduced by Fortin et al. (2022). Recently, it has been generalised to finitely representable transfinite words as data examples (Sestic 2023), whose results are not directly applicable to the problems we are concerned with as the queries have no DL component and no ontology is present. It would be of interest to extend the techniques used by Sestic (2023) to the more general languages considered here.

The database and KR communities have been working on identifying queries and concept descriptions from data examples (Staworko and Wieczorek 2015; Konev et al. 2017; ten Cate, Dalmau, and Kolaitis 2013; Ozaki 2020; ten Cate and Dalmau 2022). In reverse engineering of queries, the goal is typically to decide whether there is a query separating given positive and negative examples. Relevant work includes (Arenas and Diaz 2016; Barceló and Romero 2017) under the closed world and (Lehmann and Hitzler 2010; Gutiérrez-Basulto, Jung, and Sabellek 2018; Funk et al. 2019; Jung et al. 2022) under the open world assumption.

We are not aware of any work on exact learning of temporal formulas save (Camacho and McIlraith 2019) and the related work on exact learning of finite automata starting with (Angluin 1987a). In contrast, reverse engineering of *LTL*-formulas has recently received significant attention (Lemieux, Park, and Beschastnikh 2015; Neider and Gavran 2018; Camacho and McIlraith 2019; Fijalkow and Lagarde 2021; Fortin et al. 2023).

The use of unique characterisations for explaining and constructing schema mappings was promoted and investigated by Kolaitis (2011) and Alexe et al. (2011).

Unique characterisability of DL concepts under both closed and open world assumptions has recently been studied by ten Cate, Koudijs, and Ozaki (2024).

3 Atemporal Ontologies and Queries

We assume that background knowledge about the object domain is given as a standard description logic ontology. This section recaps the relevant definitions.

As usual in DL, we work with any signature of unary and binary predicate symbols, typically denoted A, B and P, R, respectively. A *data instance* is any finite set $\mathcal{A} \neq \emptyset$ of *atoms* of the form $A(a)$ and $P(a, b)$ with *individual names* a, b, and also $\top(a)$, which simply says that a exists. We denote by $\mathit{ind}(\mathcal{A})$ the set of individuals in $\mathcal{A}$ and by P^- the *inverse* of P, assuming that $P^-(a, b) \in \mathcal{A}$ iff $P(b, a) \in \mathcal{A}$. Let S range over binary predicates and their inverses. A *pointed data instance* is a pair $(\mathcal{A}, a)$ with $a \in \mathit{ind}(\mathcal{A})$. The *size* $|\mathcal{A}|$ of $\mathcal{A}$ is the number of symbols in it.

In general, an *ontology*, $\mathcal{O}$, is a finite set of first-order (FO) sentences in the given signature. Ontologies and data instances are interpreted in structures $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$ with domain $\Delta^\mathcal{I} \neq \emptyset$, $a^\mathcal{I} \in \Delta^\mathcal{I}$, $\top^\mathcal{I} = \Delta^\mathcal{I}$, $A^\mathcal{I} \subseteq \Delta^\mathcal{I}$, and $P^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$. As usual in database theory, we assume that $a^\mathcal{I} \neq b^\mathcal{I}$ for distinct a, b; moreover, to simplify notation, we adopt the *standard name assumption* and interpret each individual name by itself, i.e., $a^\mathcal{I} = a$. Thus, $\mathcal{I}$ is a *model* of $\mathcal{A}$ if $a \in A^\mathcal{I}$ and $(a, b) \in P^\mathcal{I}$, for all $A(a) \in \mathcal{A}$ and $P(a, b) \in \mathcal{A}$. We call $\mathcal{I}$ a *model* of an ontology $\mathcal{O}$ if all sentences in $\mathcal{O}$ are true in $\mathcal{I}$, and say that $\mathcal{O}$ and $\mathcal{A}$ are *satisfiable* if they have a common model.

The ontology languages we consider here are certain members of the *DL-Lite* family, $\mathcal{ALCHI}$, and $\mathcal{ELHIF}$; we define them below as fragments of first-order logic.

DL-Lite$_\mathcal{F}$ (Calvanese et al. 2007b) aka *DL-Lite*$^\mathcal{F}_\mathit{core}$ (Artale et al. 2009) allows axioms of the following forms:

$$\forall x \big(B(x) \to B'(x)\big), \quad \forall x \big(B(x) \land B'(x) \to \bot\big), \qquad \forall x, y, z \big(S(x, y) \land S(x, z) \to (y = z)\big), \quad (1)$$

where *basic concepts* $B(x)$ are either $A(x)$ or $\exists S(x) = \exists y\, S(x, y)$. In DL parlance, the first two axioms in (1) are written as $B \sqsubseteq B'$ and $B \sqcap B' \sqsubseteq \bot$, and the third one as $\geq 2\, S \sqsubseteq \bot$ or $\mathrm{fun}(S)$, a *functionality constraint* stating that relation S is *functional*.

DL-Lite$^-_\mathcal{F}$ (Funk, Jung, and Lutz 2022b) is the fragment of *DL-Lite*$_\mathcal{F}$, in which *concept inclusions* (CIs) $B \sqsubseteq B'$ cannot have $B' = \exists S$ with functional S^-.

DL-Lite$_\mathcal{H}$ (Calvanese et al. 2007b) aka *DL-Lite*$^\mathcal{H}_\mathit{core}$ (Artale et al. 2009) is obtained by disallowing the functionality constraints in *DL-Lite*$_\mathcal{F}$ and adding axioms of the form

$$\forall x, y\, (S(x, y) \to S'(x, y)) \qquad (2)$$

known as *role inclusions* (RIs) and written as $S \sqsubseteq S'$.

RDFS[1] has CIs between concept names, RIs between role names, and CIs of the forms $\exists P \sqsubseteq A$ or $\exists P^- \sqsubseteq A$ saying that the domain of P and range of P are in A, respectively.

$\mathcal{ALCHI}$ (Baader et al. 2017) has the same RIs as in (2) but more expressive CIs $\forall x\, (C_1(x) \to C_2(x))$, in which the *concepts* C_i are defined inductively starting from atoms $\top(x)$ and $A(x)$ and using the constructors $C(x) \land C'(x)$, $\neg C(x)$, and $\exists y\, (S(x, y) \land C(y))$—or $C \sqcap C'$, $\neg C$, and $\exists S.C$ in DL terms.

$\mathcal{ELHIF}$ (Baader et al. 2017) has RIs (2), functionality constraints, and CIs with concepts built from atoms and $\bot$ using $\land$ and $\exists y\, (S(x, y) \land C(y))$ only. $\mathcal{ELHI}$ and $\mathcal{ELIF}$ are the fragments of $\mathcal{ELHIF}$ without functionality constraints and RIs, respectively.

We reserve $\mathcal{L}$ for denoting any of these ontology languages:

$$\begin{array}{ccccccccc} \text{RDFS} & \subset & \textit{DL-Lite}_\mathcal{H} & & \subset & & \mathcal{ELHI} & \subset & \mathcal{ALCHI} \\ & & & & & & & \subset & \\ & & \textit{DL-Lite}^-_\mathcal{F} & \subset & \textit{DL-Lite}_\mathcal{F} & \subset & \mathcal{ELIF} & \subset & \mathcal{ELHIF} \end{array}$$

The most general query language over the object domain we consider consists of *conjunctive queries* (CQs, for short) $\boldsymbol{q}(x)$ with a single *answer variable* x. We often think of $\boldsymbol{q}(x)$ as the set of its atoms and denote by $\mathit{var}(\boldsymbol{q})$ and $\mathit{sig}(\boldsymbol{q})$ the sets of its individual variables and predicates symbols, respectively. We say that $\boldsymbol{q}(x)$ is *satisfiable* wrt an ontology $\mathcal{O}$ if $\mathcal{O} \cup \{\boldsymbol{q}(x)\}$ has a model.

Given a CQ $\boldsymbol{q}(x)$, an ontology $\mathcal{O}$, and a data instance $\mathcal{A}$, we say that $a \in \mathit{ind}(\mathcal{A})$ is a (*certain*) *answer to* $\boldsymbol{q}$ *over* $\mathcal{A}$ *wrt* $\mathcal{O}$ and write $\mathcal{O}, \mathcal{A} \models \boldsymbol{q}(a)$ if $\mathcal{I} \models \boldsymbol{q}(a)$ for all models $\mathcal{I}$ of $\mathcal{O}$ and $\mathcal{A}$. Recall that $\emptyset, \mathcal{A} \models \boldsymbol{q}(a)$ iff there is function $h\colon \mathit{var}(\boldsymbol{q}) \to \mathit{ind}(\mathcal{A})$ such that $h(x) = a$, $A(y) \in \boldsymbol{q}$ implies $A(h(y)) \in \mathcal{A}$, and $P(y, z) \in \boldsymbol{q}$ implies $P(h(y), h(z)) \in \mathcal{A}$. Such a function h is called a *homomorphism* from $\boldsymbol{q}$ to $\mathcal{A}$, written $h\colon \boldsymbol{q} \to \mathcal{A}$; h is *surjective* if $h(\mathit{var}(\boldsymbol{q})) = \mathit{ind}(\mathcal{A})$.

We say that a CQ $\boldsymbol{q}_1(x)$ is *contained* in a CQ $\boldsymbol{q}_2(x)$ wrt an ontology $\mathcal{O}$ and write $\boldsymbol{q}_1 \models_\mathcal{O} \boldsymbol{q}_2$ if $\mathcal{O}, \mathcal{A} \models \boldsymbol{q}_1(a)$ implies $\mathcal{O}, \mathcal{A} \models \boldsymbol{q}_2(a)$, for any data instance $\mathcal{A}$ and any $a \in \mathit{ind}(\mathcal{A})$. If $\boldsymbol{q}_1 \models_\mathcal{O} \boldsymbol{q}_2$ and $\boldsymbol{q}_2 \models_\mathcal{O} \boldsymbol{q}_1$, we say that $\boldsymbol{q}_1$ and $\boldsymbol{q}_2$ are *equivalent wrt* $\mathcal{O}$, writing $\boldsymbol{q}_1 \equiv_\mathcal{O} \boldsymbol{q}_2$. For $\mathcal{O} = \emptyset$, we often write $\boldsymbol{q}_1 \equiv \boldsymbol{q}_2$ instead of $\boldsymbol{q}_1 \equiv_\emptyset \boldsymbol{q}_2$.

Two smaller query languages we need are $\mathcal{ELI}$*-queries* (or ELIQs, for short) that can be defined by the grammar

$$\boldsymbol{q} \quad := \quad \top \quad | \quad A \quad | \quad \exists S.\boldsymbol{q} \quad | \quad \boldsymbol{q} \land \boldsymbol{q}'$$

and $\mathcal{EL}$*-queries* (or ELQs), which are ELIQs without inverses P^-. Semantically, an ELIQ $\boldsymbol{q}$ has the same meaning as the *tree-shaped CQ* $\boldsymbol{q}(x)$ that is defined inductively starting from atoms $\top(x)$ and $A(x)$ and using the constructors $\exists y\, (S(x, y) \land \boldsymbol{q}(y))$, for a fresh y, and $\boldsymbol{q}(x) \land \boldsymbol{q}'(x)$. The only free (i.e., answer) variable in $\boldsymbol{q}$ is x.

We reserve $\mathcal{Q}$ for denoting a class of queries with answer variable x such that whenever $\boldsymbol{q}_1, \boldsymbol{q}_2 \in \mathcal{Q}$, then $\boldsymbol{q}_1 \land \boldsymbol{q}_2 \in \mathcal{Q}$. Some of our results require restricting $\mathcal{Q}$ to a *finite signature* σ: we denote by $\mathcal{Q}^\sigma$ the class of those queries in $\mathcal{Q}$ that are built from predicates in σ. The classes of all σ-ELIQs and σ-ELQs are denoted by ELIQ^σ and ELQ^σ, respectively.

It will be convenient to include the 'inconsistency query' $\bot$ into all of our query classes. By definition, we have $\mathcal{O}, \mathcal{A} \models \bot(a)$ iff $\mathcal{O}$ and $\mathcal{A}$ are unsatisfiable.

4 Unique Characterisability

An *example set* is a pair $E = (E^+, E^-)$, where E^+ and E^- are finite sets of pointed data instances $(\mathcal{A}, a)$. A CQ $\boldsymbol{q}(x)$ *fits* E wrt $\mathcal{O}$ if $\mathcal{O}, \mathcal{A}^+ \models \boldsymbol{q}(a^+)$ and $\mathcal{O}, \mathcal{A}^- \not\models \boldsymbol{q}(a^-)$, for all $(\mathcal{A}^+, a^+) \in E^+$ and $(\mathcal{A}^-, a^-) \in E^-$. We say that E *uniquely characterises* $\boldsymbol{q}$ *wrt* $\mathcal{O}$ *within a given class* $\mathcal{Q}$ *of*

[1] https://www.w3.org/TR/rdf12-schema/

queries if $\boldsymbol{q}$ fits E and $\boldsymbol{q} \equiv_{\mathcal{O}} \boldsymbol{q}'$, for every $\boldsymbol{q}' \in \mathcal{Q}$ that fits E. Note that, in this case, $E^+ = \emptyset$ implies $\boldsymbol{q} \equiv_{\mathcal{O}} \bot$, and so $\boldsymbol{q}$ is not satisfiable wrt $\mathcal{O}$.

We first observe that, for a large class of ontologies $\mathcal{O}$, including all those considered here, if $\boldsymbol{q}$ is uniquely characterised by some $E = (E^+, E^-)$ wrt $\mathcal{O}$, then $\boldsymbol{q}$ has a unique characterisation of the form $E' = (\{(\hat{\boldsymbol{q}}, a)\}, E^-)$ with a single positive example $(\hat{\boldsymbol{q}}, a)$. Say that an ontology $\mathcal{O}$ *admits containment reduction* if, for any CQ $\boldsymbol{q}(x)$, there is a pointed data instance $(\hat{\boldsymbol{q}}, a)$ such that the following conditions hold:

(cr$_1$) $\boldsymbol{q}(x)$ is satisfiable wrt $\mathcal{O}$ iff $\mathcal{O}$ and $\hat{\boldsymbol{q}}$ are satisfiable;

(cr$_2$) there is a surjective $h\colon \boldsymbol{q} \to \hat{\boldsymbol{q}}$ with $h(x) = a$;

(cr$_3$) if $\boldsymbol{q}(x)$ is satisfiable wrt $\mathcal{O}$, then for every CQ $\boldsymbol{q}'(x)$, we have $\boldsymbol{q} \models_{\mathcal{O}} \boldsymbol{q}'$ iff $\mathcal{O}, \hat{\boldsymbol{q}} \models \boldsymbol{q}'(a)$.

An ontology language $\mathcal{L}$ *admits containment reduction* if every $\mathcal{L}$-ontology does. If the pointed data instance $(\hat{\boldsymbol{q}}, a)$ is computable in polynomial time, for every $\mathcal{O}$ in $\mathcal{L}$, we say that $\mathcal{L}$ *admits tractable containment reduction*. The next lemma illustrates this definition by a few concrete examples.

Lemma 1. (1) *FO without equality admits tractable containment reduction*; *in particular, $\mathcal{ALCHI}$ admits tractable containment reduction.*

(2) *$\mathcal{ELIF}$ admits tractable containment reduction.*

(3) $\{\geq 3\, P \sqsubseteq \bot\}$ *does not admit containment reduction.*

Proof. For (1), one can define $\hat{\boldsymbol{q}}$ as $\boldsymbol{q}$, with the variables regarded as individual names. To show (2), $\boldsymbol{q}$ has to be factorised first to ensure functionality; (3) is shown in the full paper (Jung et al. 2023). ☐

It is readily checked that we have the following:

Lemma 2. *Suppose $\mathcal{O}$ admits containment reduction and $\boldsymbol{q} \in \mathcal{Q}$ is satisfiable wrt $\mathcal{O}$, having a unique characterisation $E = (E^+, E^-)$ wrt $\mathcal{O}$ within $\mathcal{Q}$. Then $E' = (\{(\hat{\boldsymbol{q}}, a)\}, E^-)$ is a unique characterisation of $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$, too.*

We use two ways of constructing unique characterisations: via frontiers and via split-partners. Let $\mathcal{O}$ be an ontology, $\mathcal{Q}$ a class of queries, and $\boldsymbol{q} \in \mathcal{Q}$ a satisfiable query wrt $\mathcal{O}$. A *frontier of $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$* is a set $\mathcal{F}_{\boldsymbol{q}} \subseteq \mathcal{Q}$ such that

- for any $\boldsymbol{q}' \in \mathcal{F}_{\boldsymbol{q}}$, we have $\boldsymbol{q} \models_{\mathcal{O}} \boldsymbol{q}'$ and $\boldsymbol{q}' \not\models_{\mathcal{O}} \boldsymbol{q}$;
- for any $\boldsymbol{q}'' \in \mathcal{Q}$, if $\boldsymbol{q} \models_{\mathcal{O}} \boldsymbol{q}''$, then either $\boldsymbol{q}'' \models_{\mathcal{O}} \boldsymbol{q}$ or there is $\boldsymbol{q}' \in \mathcal{F}_{\boldsymbol{q}}$ with $\boldsymbol{q}' \models_{\mathcal{O}} \boldsymbol{q}''$.

(Note that if $\boldsymbol{q} \equiv_{\mathcal{O}} \top$, then $\mathcal{F}_{\boldsymbol{q}} = \emptyset$.) An ontology $\mathcal{O}$ is said to *admit* (*finite*) *frontiers within* $\mathcal{Q}$ if every $\boldsymbol{q} \in \mathcal{Q}$ satisfiable wrt $\mathcal{O}$ has a (finite) frontier wrt $\mathcal{O}$ within $\mathcal{Q}$. Further, if such frontiers can be computed in polynomial time, we say that $\mathcal{O}$ *admits polytime-computable frontiers.*

The next theorem follows directly from the definitions:

Theorem 1. *Suppose $\mathcal{Q}$ is a class of queries, an ontology $\mathcal{O}$ admits containment reduction, $\boldsymbol{q} \in \mathcal{Q}$ is satisfiable wrt $\mathcal{O}$, and $\mathcal{F}_{\boldsymbol{q}}$ is a finite frontier of $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$. Then $(\{(\hat{\boldsymbol{q}}, a)\}, \{(\hat{\boldsymbol{r}}, a) \mid \boldsymbol{r} \in \mathcal{F}_{\boldsymbol{q}}\})$ is a unique characterisation of $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$.*

As shown by Funk, Jung, and Lutz (2022b), the two main ontology languages that admit polytime-computable frontiers within ELIQ are *DL-Lite*$_{\mathcal{H}}$ and *DL-Lite*$^-_{\mathcal{F}}$, whereas *DL-Lite*$_{\mathcal{F}}$ itself does not admit finite ELIQ-frontiers. By Theorem 1 and Lemma 1, we then obtain:

Theorem 2. *If an ELIQ $\boldsymbol{q}$ is satisfiable wrt a DL-Lite$_{\mathcal{H}}$ or DL-Lite$^-_{\mathcal{F}}$ ontology $\mathcal{O}$, then $\boldsymbol{q}$ has a polysize unique characterisation wrt $\mathcal{O}$ within ELIQ.*

We next introduce split-partners aka dualities (McKenzie 1972; ten Cate and Dalmau 2022). Let σ be a finite signature, $\mathcal{Q}^\sigma$ a class of σ-queries, $\mathcal{O}$ a σ-ontology, and $\Theta \subseteq \mathcal{Q}^\sigma$ a finite set of queries. A set $\mathcal{S}(\Theta)$ of pointed data instances $(\mathcal{A}, a)$ is called a *split-partner for* Θ *wrt* $\mathcal{O}$ *within* $\mathcal{Q}^\sigma$ if, for all $\boldsymbol{q}' \in \mathcal{Q}^\sigma$, we have

$$\mathcal{O}, \mathcal{A} \models \boldsymbol{q}'(a) \text{ for some } (\mathcal{A}, a) \in \mathcal{S}(\Theta) \quad \text{iff} \quad \boldsymbol{q}' \not\models_{\mathcal{O}} \boldsymbol{q} \text{ for all } \boldsymbol{q} \in \Theta. \tag{3}$$

Say that an ontology language $\mathcal{L}$ *has general split-partners within* $\mathcal{Q}^\sigma$ if all finite sets of $\mathcal{Q}^\sigma$-queries have split partners wrt any $\mathcal{L}$-ontology in σ. If this holds for all singleton subsets of $\mathcal{Q}^\sigma$, we say that $\mathcal{L}$ *has split-partners within* $\mathcal{Q}^\sigma$.

We illustrate the notion of split-partner by a few examples, the last of which shows that, without the restriction to a finite signature σ, split-partners almost never exist.

Example 1. (i) Let $\mathcal{O}$ be any ontology such that $\mathcal{O}$ and $\mathcal{A}$ are satisfiable for all data instances $\mathcal{A}$, say, $\mathcal{O} = \{A \sqsubseteq B\}$. Let $\mathcal{Q}^\sigma$ be any class of σ-CQs, for some signature σ. Then the split-partner $\mathcal{S}_\bot$ of the query $\bot$ wrt $\mathcal{O}$ within $\mathcal{Q}^\sigma$ is

$$\mathcal{S}_\bot = \{\mathcal{B}_\sigma\}, \text{for } \mathcal{B}_\sigma = \{R(a,a) \mid R \in \sigma\} \cup \{A(a) \mid A \in \sigma\}.$$

(Here and below we drop a from $(\mathcal{A}, a)$ if $ind(\mathcal{A}) = \{a\}$.) Clearly, $\mathcal{O}, \mathcal{B}_\sigma \models \boldsymbol{q}$, for any $\boldsymbol{q} \in \mathcal{Q}^\sigma$ different from $\bot$.

(ii) For $\mathcal{O} = \{A \sqcap B \sqsubseteq \bot\}$ and $\sigma = \{A, B\}$, we have $\mathcal{S}_\bot = \{\{A(a)\},\ \{B(a)\}\}$.

(iii) There does not exist a split-partner for $\Theta = \{A\}$ wrt the empty ontology $\mathcal{O}$ within ELIQ. To show this, observe that $B \not\models_{\mathcal{O}} A$ for any unary predicate $B \neq A$. Hence, as any data instance $\mathcal{A}$ is finite, there is no finite set $\mathcal{S}(\{A\})$ satisfying (3).

In contrast, for frontiers and unique characterisations, restrictions to sets of predicates containing all symbols in the query and ontology do not make any difference. Indeed, let σ be the signature of $\mathcal{O}$ and $\boldsymbol{q}$. Then, for any class $\mathcal{Q}$ of queries, a set $\mathcal{F}_{\boldsymbol{q}}$ is a frontier for $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$ iff it is a frontier for $\boldsymbol{q}$ wrt $\mathcal{O}$ within the restriction of $\mathcal{Q}$ to σ. The same holds for unique characterisations E of $\boldsymbol{q}$ wrt $\mathcal{O}$.

The following result is proved in the full paper (Jung et al. 2023) using a construction from the reduction of ontology-mediated query answering to constraint satisfaction (Bienvenu et al. 2014).

Theorem 3. *$\mathcal{ALCHI}$ has general split-partners within ELIQ$^\sigma$ that can be computed in exponential time.*

For ELQs, we can construct general split-partners wrt RDFS ontologies in polynomial time, provided that the number of input queries is bounded. The proof generalises the construction of split-partners for queries in ELQ wrt to the empty ontology in (Fortin et al. 2022; ten Cate et al. 2023).

Theorem 4. *Let σ be a signature, $\mathcal{O}$ a σ-ontology in RDFS, and $n > 0$. For any set $\Theta \subseteq ELQ^\sigma$ with $|\Theta| \leq n$, one can compute in polynomial time a split-partner $\mathcal{S}(\Theta)$ of Θ wrt $\mathcal{O}$ within ELQ^σ.*

Here is our second sufficient characterisability condition:

Theorem 5. *Suppose $\mathcal{Q}$ is a class of queries, an ontology $\mathcal{O}$ admits containment reduction, $\boldsymbol{q} \in \mathcal{Q}$ is satisfiable wrt $\mathcal{O}$, and σ contains the predicate symbols in $\boldsymbol{q}$ and $\mathcal{O}$. If $\mathcal{S}_{\boldsymbol{q}}$ is a split-partner for $\{\boldsymbol{q}\}$ wrt $\mathcal{O}$ within $\mathcal{Q}^\sigma$, then $(\{(\hat{\boldsymbol{q}}, a)\}, \mathcal{S}_{\boldsymbol{q}})$ is a unique characterisation of $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$.*

As a consequence of Theorems 3, 4, 5 and Lemma 1, we obtain the following:

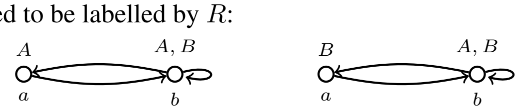
Theorem 6. *If a query $\boldsymbol{q} \in ELIQ^\sigma$ is satisfiable wrt an $\mathcal{ALCHI}$-ontology $\mathcal{O}$ in a signature σ, then $\boldsymbol{q}$ has a unique characterisation wrt $\mathcal{O}$ within $ELIQ^\sigma$.*

The sufficient conditions of Theorems 1 and 5 use the notions of frontier and split-partner, respectively. (Notice that both of them are applicable to ELIQs wrt *DL-Lite*$_\mathcal{H}$ and RDFS ontologies; however, only split-partners will give us polysize unique characterisations of temporalised ELQs wrt RDFS ontologies in Theorem 13 (ii), Section 7.) We now show examples of queries and ontologies having frontiers but no split-partners and vice versa. The query witnessing that frontiers can exist where split-partners do not exist provides a counterexample even if one admits *CQ-frontiers*, frontiers containing not only ELIQs but also CQs and defined in the obvious way in the full paper (Jung et al. 2023).

Theorem 7. *$\mathcal{EL}$ does not admit finite CQ-frontiers within ELIQ.*

Proof. The query $\boldsymbol{q} = A \wedge B$ does not have a finite CQ-frontier wrt the ontology $\mathcal{O} = \{A \sqsubseteq \exists R.A,\ \exists R.A \sqsubseteq A\}$ within ELIQs. □

Example 2. Observe that the following set of pointed data instances is a split-partner of $\{\boldsymbol{q}\}$ wrt $\mathcal{O}$ from the proof of Theorem 7 within ELIQ$^{\{A,B,R\}}$; here all arrows are assumed to be labelled by R:



Theorem 8. *There exist a DL-Lite$_\mathcal{F}^-$ ontology $\mathcal{O}$, a query $\boldsymbol{q}$ and a signature σ such that $\{\boldsymbol{q}\}$ does not have a finite split-partner wrt $\mathcal{O}$ within $ELIQ^\sigma$.*

Proof. Let $\mathcal{O} = \{\text{fun}(P), \text{fun}(P^-), B \sqcap \exists P^- \sqsubseteq \bot\}$ and $\boldsymbol{q} = A$. Then $Q = \{\boldsymbol{q}\}$ does not have a finite split-partner wrt $\mathcal{O}$ within ELIQ$^{\{A,B,P\}}$. □

Observe that $\{\top\}$ is a frontier for A wrt $\mathcal{O}$ from the proof of Theorem 8 within ELIQ and that we can combine the two proofs above to also refute the natural conjecture that frontiers and splittings together provide a 'universal tool' for constructing unique characterisations.

5 Temporal Data and Queries

We now extend the definitions of Sections 3 and 4 by adding a temporal dimension to the domain data and queries mediated by an ontology. Our definitions generalise those of (Fortin et al. 2022), where the ontology-free case was first considered.

A *temporal data instance*, denoted $\mathcal{D}$, is a finite sequence $\mathcal{A}_0, \dots, \mathcal{A}_n$ of data instances, where each $\mathcal{A}_i$ comprises the facts with timestamp i. We assume all *ind*$(\mathcal{A}_i)$ to be the same, adding $\top(a)$ to $\mathcal{A}_i$ if needed, and set *ind*$(\mathcal{D})$ = *ind*$(\mathcal{A}_0)$. The *length* of $\mathcal{D}$ is $\max(\mathcal{D}) = n$ and the *size* of $\mathcal{D}$ is $|\mathcal{D}| = \sum_{i \leq n} |\mathcal{A}_i|$. Within a temporal σ-data instance, we often denote by $\emptyset$ the instance $\{\top(a) \mid a \in ind(\mathcal{D})\}$.

Temporal queries for accessing temporal data instances we propose here are built from domain queries (with one implicit answer variable x) in a given class $\mathcal{Q}$ (say, ELIQs) using $\wedge$ and the (future-time) temporal operators of the standard linear temporal logic *LTL* over the time flow $(\mathbb{N}, <)$: unary $\bigcirc$ (next time), $\Diamond$ (sometime later), $\Diamond_r$ (now or later), and binary U (until); see below for the precise semantics. The class of such temporal queries that only use the operators from a set $\Phi \subseteq \{\bigcirc, \Diamond, \Diamond_r, \mathsf{U}\}$ is denoted by $LTL^\Phi(\mathcal{Q})$. The class $LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ comprises *path queries* of the form

$$\boldsymbol{q} = \boldsymbol{r}_0 \wedge \boldsymbol{o}_1(\boldsymbol{r}_1 \wedge \boldsymbol{o}_2(\boldsymbol{r}_2 \wedge \cdots \wedge \boldsymbol{o}_n \boldsymbol{r}_n)), \qquad (4)$$

where $\boldsymbol{o}_i \in \{\bigcirc, \Diamond, \Diamond_r\}$ and $\boldsymbol{r}_i \in \mathcal{Q}$; *path queries* in $LTL_p^{\mathsf{U}}(\mathcal{Q})$ take the form

$$\boldsymbol{q} = \boldsymbol{r}_0 \wedge (\boldsymbol{l}_1 \mathbin{\mathsf{U}} (\boldsymbol{r}_1 \wedge (\boldsymbol{l}_2 \mathbin{\mathsf{U}} (\dots (\boldsymbol{l}_n \mathbin{\mathsf{U}} \boldsymbol{r}_n) \dots)))), \qquad (5)$$

where $\boldsymbol{r}_i \in \mathcal{Q}$ and either $\boldsymbol{l}_i \in \mathcal{Q}$ or $\boldsymbol{l}_i = \bot$. We use $\mathcal{C}$ to refer to classes of temporal queries. The *size* $|\boldsymbol{q}|$ of $\boldsymbol{q}$ is the number of symbols in $\boldsymbol{q}$; the *temporal depth* *tdp*$(\boldsymbol{q})$ of $\boldsymbol{q}$ is the maximum number of nested temporal operators in $\boldsymbol{q}$.

An (atemporal) ontology $\mathcal{O}$ and temporal data instance $\mathcal{D} = \mathcal{A}_0, \dots, \mathcal{A}_n$ are *satisfiable* if $\mathcal{O}$ and $\mathcal{A}_i$ are satisfiable for each $i \leq n$. For satisfiable $\mathcal{O}$ and $\mathcal{D}$, the *entailment relation* $\mathcal{O}, \mathcal{D}, \ell, a \models \boldsymbol{q}$ with $\ell \in \mathbb{N}$ and $a \in ind(\mathcal{D})$ is defined by induction as follows, where $\mathcal{A}_\ell = \emptyset$, for $\ell > n$:

$\mathcal{O}, \mathcal{D}, \ell, a \models \boldsymbol{q}$ iff $\mathcal{O}, \mathcal{A}_\ell \models \boldsymbol{q}(a)$, for any $\boldsymbol{q} \in \mathcal{Q}$,
$\mathcal{O}, \mathcal{D}, \ell, a \models \boldsymbol{q}_1 \wedge \boldsymbol{q}_2$ iff $\mathcal{O}, \mathcal{D}, \ell, a \models \boldsymbol{q}_i$, for $i = 1, 2$,
$\mathcal{O}, \mathcal{D}, \ell, a \models \bigcirc\boldsymbol{q}$ iff $\mathcal{O}, \mathcal{D}, \ell + 1, a \models \boldsymbol{q}$,
$\mathcal{O}, \mathcal{D}, \ell, a \models \Diamond\boldsymbol{q}$ iff $\mathcal{O}, \mathcal{D}, m, a \models \boldsymbol{q}$, for some $m > \ell$,
$\mathcal{O}, \mathcal{D}, \ell, a \models \Diamond_r\boldsymbol{q}$ iff $\mathcal{O}, \mathcal{D}, m, a \models \boldsymbol{q}$, for some $m \geq \ell$,
$\mathcal{O}, \mathcal{D}, \ell, a \models \boldsymbol{q}_1 \mathbin{\mathsf{U}} \boldsymbol{q}_2$ iff $\mathcal{O}, \mathcal{D}, m, a \models \boldsymbol{q}_2$, for some $m > \ell$,
and $\mathcal{O}, \mathcal{D}, k, a \models \boldsymbol{q}_1$, for all k, $\ell < k < m$.

If $\mathcal{O}$ and $\mathcal{D}$ are not satisfiable, we set $\mathcal{O}, \mathcal{D}, \ell, a \models \boldsymbol{q}$ to hold for all $\boldsymbol{q}$, ℓ and a. Our semantics follows the well established epistemic approach to evaluating temporal queries; see (Calvanese et al. 2007a; Artale et al. 2022) and references therein. The alternative classical Tarski semantics based on temporal interpretations is equivalent to our semantics for all *Horn ontologies* whose FO-translations belong to the Horn fragment of first-order logic (Chang and Keisler 1998), and so for all DLs we consider here except $\mathcal{ALCHI}$. A detailed discussion of the relationship between the two semantics is given in the full paper (Jung et al. 2023).

We call a temporal query $\boldsymbol{q}$ *satisfiable* wrt an ontology $\mathcal{O}$ if $\mathcal{O} \cup \{\boldsymbol{q}\}$ has a model. Note that $\boldsymbol{q}$ of the form (4) or (5) is satisfiable wrt $\mathcal{O}$ iff all $\boldsymbol{r}_i$ in $\boldsymbol{q}$ are satisfiable wrt $\mathcal{O}$.

By an *example set* we now mean a pair $E = (E^+, E^-)$ of finite sets E^+ and E^- of pointed temporal data instances $\mathcal{D}, a$ with $a \in \mathit{ind}(\mathcal{D})$. We say that a query $\boldsymbol{q}$ *fits* E wrt $\mathcal{O}$ if $\mathcal{O}, \mathcal{D}^+, 0, a^+ \models \boldsymbol{q}$ and $\mathcal{O}, \mathcal{D}^-, 0, a^- \not\models \boldsymbol{q}$, for all $(\mathcal{D}^+, a^+) \in E^+$ and $(\mathcal{D}^-, a^-) \in E^-$. As before, E *uniquely characterises* $\boldsymbol{q}$ *wrt* $\mathcal{O}$ *within a class* $\mathcal{C}$ *of temporal queries* if $\boldsymbol{q}$ fits E wrt $\mathcal{O}$ and every $\boldsymbol{q}' \in \mathcal{C}$ fitting E wrt $\mathcal{O}$ is equivalent to $\boldsymbol{q}$ wrt $\mathcal{O}$.

The following lemma shows that typically queries that are not satisfiable wrt the ontology in question cannot be uniquely characterised.

Lemma 3. *Let $\mathcal{O}$ be an ontology and $\mathcal{C}$ a class of temporal queries containing, for any $n > 0$, a satisfiable wrt $\mathcal{O}$ query of temporal depth $\geq n$. If a query $\boldsymbol{q}$ is not satisfiable wrt $\mathcal{O}$, then $\boldsymbol{q}$ is not uniquely characterisable wrt $\mathcal{O}$ within $\mathcal{C}$.*

To prove Lemma 3, assume that $\boldsymbol{q}$ is not satisfiable wrt $\mathcal{O}$ but $E = (E^+, E^-)$ uniquely characterises $\boldsymbol{q}$ wrt $\mathcal{O}$ within $\mathcal{C}$. In this case $E^+ = \emptyset$. Let n be the maximal length of instances in E^-. Then any $\boldsymbol{q}' \in \mathcal{C}$ of temporal depth $> n$ fits E, which is a contradiction.

In what follows, we mostly exclude such unsatisfiable queries from consideration.

Suppose $\mathcal{C}$ is a class of queries and $\mathcal{O}$ an ontology. If each $\boldsymbol{q} \in \mathcal{C}$ satisfiable wrt $\mathcal{O}$ is uniquely characterised by some E wrt $\mathcal{O}$ within $\mathcal{C}' \supseteq \mathcal{C}$, we say that $\mathcal{C}$ is *uniquely characterisable wrt $\mathcal{O}$ within $\mathcal{C}'$*. Let $\mathcal{C}^n$ be the set of queries in $\mathcal{C}$ of temporal depth $\leq n$. We say that $\mathcal{C}$ is *polysize characterisable wrt $\mathcal{O}$ for bounded temporal depth* if there is a polynomial f such that every $\boldsymbol{q} \in \mathcal{C}^n$ is characterised by some E of size $\leq f(n)$ within $\mathcal{C}^n$, $n \in \mathbb{N}$.

Note that $\Diamond\boldsymbol{q} \equiv \bigcirc\Diamond_r\boldsymbol{q}$, so $\Diamond$ does not add any expressive power to $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ and $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q}) = \mathit{LTL}_p^{\bigcirc\Diamond_r}(\mathcal{Q})$; however, $\mathit{LTL}_p^{\bigcirc\Diamond}(\mathcal{Q}) \subsetneq \mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$. We also observe that our temporal query languages do not admit containment reduction as, for example, there is no temporal data instance $\hat{\boldsymbol{q}}$ for $\boldsymbol{q} = \bigcirc(A \wedge \Diamond B)$ because it will have to fix the number of steps between 0 and the moment of time where B holds.

We next prove general theorems lifting unique characterisability from domain queries considered above and ontology-free *LTL* queries of (Fortin et al. 2022) to temporal queries mediated by a DL ontology.

6 Unique Characterisations in $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$

The aim of this section is to give a criterion of (polysize) unique characterisability of temporal queries in the class $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ under certain conditions on the ontology and on the class $\mathcal{Q}$ of domain queries. It will be convenient to represent queries $\boldsymbol{q}$ of the form (4) as a sequence

$$\boldsymbol{q} = \boldsymbol{r}_0(t_0), R_1(t_0, t_1), \dots, R_m(t_{m-1}, t_m), \boldsymbol{r}_m(t_m), \quad (6)$$

where $R_i \in \{\mathit{suc}, <, \leq\}$, $\mathit{suc}(t, t')$ stands for $t' = t + 1$, and $\mathit{tv} = \{t_0, \dots, t_m\}$ are variables over the timeline $(\mathbb{N}, <)$.

Example 3. Below are a temporal query $\boldsymbol{q}$ and its representation of the form (6):

$$\boldsymbol{q} = \exists P.B \wedge \bigcirc(\exists P.A \wedge \Diamond A) \ \rightsquigarrow$$
$$\exists P.B(t_0), \mathit{suc}(t_0, t_1), \exists P.A(t_1), (t_1 < t_2), A(t_2) \quad (7)$$

with $\mathit{tv}(\boldsymbol{q}) = \{t_0, t_1, t_2\}$.

We divide $\boldsymbol{q}$ of the form (6) into *blocks* $\boldsymbol{q}_i$ such that

$$\boldsymbol{q} = \boldsymbol{q}_0 \mathcal{R}_1 \boldsymbol{q}_1 \dots \mathcal{R}_n \boldsymbol{q}_n, \quad (8)$$

where $\mathcal{R}_i = R_1^i(t_0^i, t_1^i) \dots R_{n_i}^i(t_{n_i-1}^i, t_{n_i}^i)$, $R_j^i \in \{<, \leq\}$ and

$$\boldsymbol{q}_i = \boldsymbol{r}_0^i(s_0^i)\mathit{suc}(s_0^i, s_1^i) \dots \mathit{suc}(s_{k_i-1}^i, s_{k_i}^i)\boldsymbol{r}_{k_i}^i(s_{k_i}^i) \quad (9)$$

with $s_{k_i}^i = t_0^{i+1}$, $t_{n_i}^i = s_0^i$. If $k_i = 0$, the block $\boldsymbol{q}_i$ is called *primitive*.

Example 4. The query $\boldsymbol{q}$ from Example 3 has two blocks

$$\boldsymbol{q}_0 = \exists P.B(t_0), \mathit{suc}(t_0, t_1), \exists P.A(t_1) \quad \text{and} \quad \boldsymbol{q}_1 = A(t_2)$$

connected by $(t_1 < t_2)$. It contains one primitive block, $\boldsymbol{q}_1$.

Suppose we are given an ontology $\mathcal{O}$ and a class $\mathcal{Q}$ of domain queries. Then a primitive block $\boldsymbol{q}_i = \boldsymbol{r}_0^i(s_0^i)$ with $i > 0$ in $\boldsymbol{q}$ of the form (8) is called a *lone conjunct wrt $\mathcal{O}$ within $\mathcal{Q}$* if $\boldsymbol{r}_0^i$ is *meet-reducible wrt $\mathcal{O}$ within $\mathcal{Q}$* in the sense that there are queries $\boldsymbol{r}_1, \boldsymbol{r}_2 \in \mathcal{Q}$ such that $\boldsymbol{r} \equiv_{\mathcal{O}} \boldsymbol{r}_1 \wedge \boldsymbol{r}_2$ and $\boldsymbol{r} \not\equiv_{\mathcal{O}} \boldsymbol{r}_i$, for $i = 1, 2$. Lone conjuncts and their impact on unique characterisability are illustrated by the next example.

Example 5. The query $\Diamond A$, which is represented by the sequence $\top(t_0), (t_0 < t_1), A(t_1)$, does not have any lone conjuncts wrt the empty ontology within ELIQ, but A is a lone conjunct of $\Diamond A$ wrt $\mathcal{O} = \{A \equiv B \sqcap C\}$ within ELIQ.

The query $\boldsymbol{q} = \Diamond A$ is uniquely characterised wrt the empty ontology within $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) by the example set $E = (E^+, E^-)$, where E^+ contains two temporal data instances $\emptyset, \{A\}$ and $\emptyset, \emptyset, \{A\}$ and E^- consists of one instance $\{A\}$. However, $\boldsymbol{q} = \Diamond A$ cannot be uniquely characterised wrt $\mathcal{O} = \{A \equiv B \sqcap C\}$ within $\mathit{LTL}_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) as it cannot be separated from queries of the form

$$\Diamond(B \wedge \Diamond_r(C \wedge \Diamond_r(B \wedge \Diamond_r(C \wedge \Diamond_r(\dots)))))$$

by a finite example set. Observe also that A is a lone conjunct in $\boldsymbol{q}' = \Diamond(A \wedge \Diamond_r D)$ wrt $\mathcal{O}' = \mathcal{O} \cup \{D \sqsubseteq A\}$ but, for the simplification $\boldsymbol{q}'' = \Diamond D$ of $\boldsymbol{q}'$, we have $\boldsymbol{q}'' \equiv_{\mathcal{O}'} \boldsymbol{q}'$ and $\boldsymbol{q}''$ does not have any lone conjuncts wrt $\mathcal{O}'$.

Example 5 shows that the notion of lone conjunct depends on the presentation of the query. To make lone conjuncts semantically meaningful, we introduce a normal form. Given an ontology $\mathcal{O}$ and a query $\boldsymbol{q}$ of the form (8), we say that $\boldsymbol{q}$ is in *normal form wrt $\mathcal{O}$* if the following conditions hold:

(n1) $\boldsymbol{r}_0^i \not\equiv_{\mathcal{O}} \top$ if $i > 0$, and $\boldsymbol{r}_{k_i}^i \not\equiv_{\mathcal{O}} \top$ if either $i > 0$ or $k_i > 0$ (thus, of all the first/last $\boldsymbol{r}$ in a block only $\boldsymbol{r}_0^0$ can be trivial);

(n2) each $\mathcal{R}_i$ is either a single $t_0^i \leq t_1^i$ or a sequence of $<$;

(n3) $\boldsymbol{r}_{k_i}^i \not\models_{\mathcal{O}} \boldsymbol{r}_0^{i+1}$ if $\boldsymbol{q}_{i+1}$ is primitive and $\mathcal{R}_{i+1}$ is $\leq$;

(n4) $\boldsymbol{r}_0^{i+1} \not\models_{\mathcal{O}} \boldsymbol{r}_{k_i}^i$ if $i > 0$, $\boldsymbol{q}_i$ is primitive and $\mathcal{R}_{i+1}$ is $\leq$;

(n5) $\boldsymbol{r}_{k_i}^i \wedge \boldsymbol{r}_0^{i+1}$ is satisfiable wrt $\mathcal{O}$ whenever $\mathcal{R}_{i+1}$ is $\leq$.

Lemma 4. *Let $\mathcal{O}$ be an FO-ontology (possibly with $=$). Then every query $\boldsymbol{q} \in LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ is equivalent wrt $\mathcal{O}$ to a query in normal form of size at most $|\boldsymbol{q}|$ and of temporal depth not exceeding $tdp(\boldsymbol{q})$. This query can be computed in polynomial time if containment between queries in $\mathcal{Q}$ wrt $\mathcal{O}$ is decidable in polynomial time.*

Note that containment between queries in $\mathcal{Q} =$ ELIQ is decidable in polynomial time for *DL-Lite*$_\mathcal{F}$-ontologies $\mathcal{O}$ but not for *DL-Lite*$_\mathcal{H}$-ontologies (unless P = NP) (Kikot, Kontchakov, and Zakharyaschev 2012).

We call a query $\boldsymbol{q} \in LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ *safe wrt $\mathcal{O}$* if it is equivalent wrt $\mathcal{O}$ to an $LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$-query in normal form that has no lone conjuncts. It follows from the proof of Theorem 9 given in the full paper (Jung et al. 2023) that, for any query satisfiable wrt $\mathcal{O}$, the normal form is unique modulo equivalence of its constituent domain queries.

We are now in a position to formulate the main result of this section.

Theorem 9. *Suppose an ontology $\mathcal{O}$ admits containment reduction and $\mathcal{Q}$ is a class of domain queries that is uniquely characterisable wrt $\mathcal{O}$. Then the following hold*:

(i) *Let $\boldsymbol{q} \in LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ be satisfiable wrt $\mathcal{O}$. Then $\boldsymbol{q}$ is uniquely characterisable within $LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ wrt $\mathcal{O}$ iff $\boldsymbol{q}$ is safe wrt $\mathcal{O}$.*

(ii) *If $\mathcal{O}$ admits polysize characterisations within $\mathcal{Q}$, then those queries that are uniquely characterisable within $LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ wrt $\mathcal{O}$ are actually polysize characterisable within $LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ wrt $\mathcal{O}$.*

(iii) *$LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ is polysize characterisable wrt $\mathcal{O}$ for bounded temporal depth if $\mathcal{O}$ admits polysize unique characterisations within $\mathcal{Q}$.*

(iv) *$LTL_p^{\bigcirc\Diamond}(\mathcal{Q})$ is uniquely characterisable wrt $\mathcal{O}$. It is polysize characterisable wrt $\mathcal{O}$ if $\mathcal{O}$ admits polysize unique characterisations within $\mathcal{Q}$.*

A detailed proof of Theorem 9 is given in the full paper (Jung et al. 2023). To explain the intuition behind it, we show and discuss the positive and negative examples that provide the unique characterisation required for (i). Suppose $\mathcal{O}$ admits containment reduction and $\mathcal{Q}$ is a class of domain queries with a unique characterisation $(\{\hat{\boldsymbol{r}}\}, \mathcal{N}_{\boldsymbol{r}})$ of $\boldsymbol{r} \in \mathcal{Q}$ wrt $\mathcal{O}$ within $\mathcal{Q}$. Assume that $\boldsymbol{q} \in LTL_p^{\bigcirc\Diamond\Diamond_r}(\mathcal{Q})$ in normal form wrt $\mathcal{O}$ takes the form (8) with $\boldsymbol{q}_i$ of the form (9). We define an example set $E = (E^+, E^-)$ characterising $\boldsymbol{q}$ under the assumption that $\boldsymbol{q}$ has no lone conjuncts wrt $\mathcal{O}$. Let b be the number of ocurrences of $\bigcirc$ and $\Diamond$ in $\boldsymbol{q}$ plus 1. For every block $\boldsymbol{q}_i$ of the form (9), let $\hat{\boldsymbol{q}}_i$ be the temporal data instance

$$\hat{\boldsymbol{q}}_i = \hat{\boldsymbol{r}}_0^i \hat{\boldsymbol{r}}_1^i \dots \hat{\boldsymbol{r}}_{k_i}^i.$$

For any two blocks $\boldsymbol{q}_i, \boldsymbol{q}_{i+1}$ such that $\boldsymbol{r}_{k_i}^i \wedge \boldsymbol{r}_0^{i+1}$ is satisfiable wrt $\mathcal{O}$, we take the temporal data instance

$$\hat{\boldsymbol{q}}_i \bowtie \hat{\boldsymbol{q}}_{i+1} = \hat{\boldsymbol{r}}_0^i \dots \hat{\boldsymbol{r}}_{k_{i-1}}^i \widehat{\boldsymbol{r}_{k_i}^i \wedge \boldsymbol{r}_0^{i+1}} \hat{\boldsymbol{r}}_1^{i+1} \dots \hat{\boldsymbol{r}}_{k_{i+1}}^{i+1}.$$

Now, the set E^+ contains the data instances given by

- $\mathcal{D}_b = \hat{\boldsymbol{q}}_0 \emptyset^b \dots \hat{\boldsymbol{q}}_i \emptyset^b \hat{\boldsymbol{q}}_{i+1} \dots \emptyset^b \hat{\boldsymbol{q}}_n$,
- $\mathcal{D}_i = \hat{\boldsymbol{q}}_0 \emptyset^b \dots (\hat{\boldsymbol{q}}_i \bowtie \hat{\boldsymbol{q}}_{i+1}) \dots \emptyset^b \hat{\boldsymbol{q}}_n$, if $\mathcal{R}_{i+1}$ is $\leq$ and
- $\mathcal{D}_i = \hat{\boldsymbol{q}}_0 \emptyset^b \dots \hat{\boldsymbol{q}}_i \emptyset^{n_{i+1}} \hat{\boldsymbol{q}}_{i+1} \dots \emptyset^b \hat{\boldsymbol{q}}_n$, otherwise.

Here, $\emptyset^b$ is a sequence of b-many $\emptyset$ and similarly for $\emptyset^{n_{i+1}}$ (intuitively, these 'paddings' of multiple $\emptyset$s are needed to ensure that queries fitting the examples have the same block structure as the target query). By the definition of $\hat{\boldsymbol{r}}$ using containment reduction, it follows that $\mathcal{O}, \mathcal{D}, 0, a \models \boldsymbol{q}$, for all $\mathcal{D} \in E^+$. Intuitively, the data instances in E^+ force any query that is entailed to be divided into blocks in a similar way as $\boldsymbol{q}$. The set E^- contains all data instances of the form

- $\mathcal{D}_i^- = \hat{\boldsymbol{q}}_0 \emptyset^b \dots \hat{\boldsymbol{q}}_i \emptyset^{n_{i+1}-1} \hat{\boldsymbol{q}}_{i+1} \dots \emptyset^b \hat{\boldsymbol{q}}_n$, if $n_{i+1} > 1$,
- $\mathcal{D}_i^- = \hat{\boldsymbol{q}}_0 \emptyset^b \dots \hat{\boldsymbol{q}}_i \bowtie \hat{\boldsymbol{q}}_{i+1} \dots \emptyset^b \hat{\boldsymbol{q}}_n$, if $\mathcal{R}_{i+1}$ is a single $<$ and $\boldsymbol{r}_{k_i}^i \wedge \boldsymbol{r}_0^{i+1}$ is satisfiable wrt $\mathcal{O}$,
- the data instances obtained from $\mathcal{D}_b$ by applying to it exactly once each of the rules (a)–(e) defined below in all possible ways.

It follows from the assumption that $\boldsymbol{q}$ is in normal form and the reduced 'gaps' between blocks in $\mathcal{D}_i^-$ that we have $\mathcal{O}, \mathcal{D}_i^-, 0, a \not\models \boldsymbol{q}$ for all $\mathcal{D}_i^-$. To obtain a unique charcaterisation, the additional data instances obtained by applying rules (a)–(e) to $\mathcal{D}_b$ are crucial. They 'weaken' $\mathcal{D}_b$ by replacing some $\hat{\boldsymbol{r}}$ by negative examples in $\mathcal{N}_{\boldsymbol{r}}$ or by introducing big 'gaps' between some $\hat{\boldsymbol{r}}$s. To make our notation more uniform, we think of the pointed data instances in $\mathcal{N}_{\boldsymbol{r}}$ as having the form $\hat{\boldsymbol{r}}'$, for a suitable CQ $\boldsymbol{r}'$ (which is not necessarily in $\mathcal{Q}$). The rules are as follows:

(a) replace some $\hat{\boldsymbol{r}}_j^i$ with $\boldsymbol{r}_j^i \not\equiv_{\mathcal{O}} \top$ by an $\hat{\boldsymbol{r}} \in \mathcal{N}_{\boldsymbol{r}_j^i}$, for i, j such that $(i, j) \neq (0, 0)$—that is, the rule is not applied to $\boldsymbol{r}_0^0$;

(b) replace some pair $\hat{\boldsymbol{r}}_j^i \hat{\boldsymbol{r}}_{j+1}^i$ within block i by $\hat{\boldsymbol{r}}_j^i \emptyset^b \hat{\boldsymbol{r}}_{j+1}^i$;

(c) replace some $\hat{\boldsymbol{r}}_j^i$ such that $\boldsymbol{r}_j^i \not\equiv_{\mathcal{O}} \top$ by $\hat{\boldsymbol{r}}_j^i \emptyset^b \hat{\boldsymbol{r}}_j^i$, where $k_i > j > 0$—that is, the rule is not applied to $\boldsymbol{r}_j^i$ if it is on the border of its block;

(d) replace $\hat{\boldsymbol{r}}_{k_i}^i$ $(k_i > 0)$ by $\hat{\boldsymbol{r}} \emptyset^b \hat{\boldsymbol{r}}_{k_i}^i$, for some $\hat{\boldsymbol{r}} \in \mathcal{N}_{\boldsymbol{r}_{k_i}^i}$, or replace $\hat{\boldsymbol{r}}_0^i$ $(k_i > 0)$ by $\hat{\boldsymbol{r}}_0^i \emptyset^b \hat{\boldsymbol{r}}$, for some $\hat{\boldsymbol{r}} \in \mathcal{N}_{\boldsymbol{r}_0^i}$;

(e) replace $\hat{\boldsymbol{r}}_0^0$ with $\boldsymbol{r}_0^0 \not\equiv_{\mathcal{O}} \top$ by $\hat{\boldsymbol{r}} \emptyset^b \hat{\boldsymbol{r}}_0^0$, for $\hat{\boldsymbol{r}} \in \mathcal{N}_{\boldsymbol{r}_0^0}$, if $k_0 = 0$, and by $\hat{\boldsymbol{r}}_0^0 \emptyset^b \hat{\boldsymbol{r}}_0^0$ if $k_0 > 0$.

The proof that (E^+, E^-) as defined above uniquely characterises $\boldsymbol{q}$ wrt $\mathcal{O}$ if $\boldsymbol{q}$ contains no lone conjuncts is nontrivial and extends ideas from the ontology-free case investigated in (Fortin et al. 2022). Claim (ii) follows from the observation that the unique characterisation constructed in (i) is polynomial in the size of the characterisations $(\{\hat{\boldsymbol{r}}\}, \mathcal{N}_{\boldsymbol{r}})$ of the domain queries used in $\boldsymbol{q}$. For (iii), assume that $tdp(\boldsymbol{q}) \leq n$. Then we add to rules (a)–(e) the following rule: if $\hat{\boldsymbol{r}}$ is a lone conjunct in $\boldsymbol{q}$, then replace $\hat{\boldsymbol{r}}$ by $(\hat{\boldsymbol{r}}_1 \emptyset^b \cdots \emptyset^b \hat{\boldsymbol{r}}_k)^n$ in $\mathcal{D}_b$ for $\mathcal{N}_{\boldsymbol{r}} = \{\hat{\boldsymbol{r}}_1, \dots, \hat{\boldsymbol{r}}_k\}$ with $\boldsymbol{r}_i \not\equiv_{\mathcal{O}} \boldsymbol{r}_j$, for $i \neq j$. As $\boldsymbol{r}$ is meet-reducible wrt $\mathcal{O}$, one can first show that $|\mathcal{N}_{\boldsymbol{r}}| \geq 2$ and then that we obtain a unique characterisation of $\boldsymbol{q}$ wrt $\mathcal{O}$ within the class of queries in $\mathcal{Q}$ of temporal depth $\leq n$. To show (iv), one can follow the proof of (i) without $\Diamond_r$ in $\boldsymbol{q}$ but possibly with lone conjuncts. Now, rules (c), (d), and (e) are not needed in the construction of E^-.

As an immediate consequence of Lemma 1 and Theorems 2, 6 and 9 we obtain:

Theorem 10. *(i) For any DL-Lite$_\mathcal{H}$ or DL-Lite$^-_\mathcal{F}$ ontology $\mathcal{O}$, the following hold*:

(i_1) *any $\boldsymbol{q} \in LTL_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) satisfiable wrt $\mathcal{O}$ is uniquely characterisable—in fact, polysize characterisable—wrt $\mathcal{O}$ within $LTL_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) iff $\boldsymbol{q}$ is safe wrt $\mathcal{O}$*;

(i_2) *$LTL_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) is polysize characterisable wrt $\mathcal{O}$ for bounded temporal depth*;

(i_3) *$LTL_p^{\bigcirc\Diamond}$(ELIQ) is polysize characterisable wrt $\mathcal{O}$.*

(ii) Let σ be a signature. Then claims (i_1)–(i_3) also hold for $\mathcal{ALCHI}$ ontologies provided that 'polysize' is replaced by 'exponential-size' and ELIQ by ELIQ$^\sigma$.

7 Unique Characterisations in $LTL_p^{\mathsf{U}}(\mathcal{Q}^\sigma)$

We next consider temporalisations by means of the binary operator U (until), which is more expressive than $\bigcirc$ and $\Diamond$ as $\bigcirc\boldsymbol{q} \equiv \bot \mathsf{U} \boldsymbol{q}$ and $\Diamond\boldsymbol{q} \equiv \top \mathsf{U} \boldsymbol{q}$ under the strict semantics. Compared to the previous section, we now have to restrict queries to a finite signature because otherwise the implicit universal quantification in U makes queries such as $\bot \mathsf{U} A$ not uniquely characterisable wrt the empty ontology (Fortin et al. 2022). For the same reason, we also have to disallow nesting of U on the left-hand side of U in queries. Finally, in the ontology-free case, polysize unique characterisations for propositional *LTL*-queries with U are only available for the so-called peerless queries (Fortin et al. 2022). These observations lead to the following classes of temporal queries, for which we are going to obtain our transfer results.

Let $\mathcal{Q}$ be a domain query language and σ a finite signature of unary and binary predicate symbols. Then $\mathcal{Q}^\sigma$ denotes the set of queries in $\mathcal{Q}$ that only use symbols in σ. The class $LTL_p^{\mathsf{U}}(\mathcal{Q}^\sigma)$ comprises temporal path queries of the form (5) where each $\boldsymbol{r}_i \in \mathcal{Q}^\sigma$ and each $\boldsymbol{l}_i$ is either in $\mathcal{Q}^\sigma$ or $\bot$ (recall that $\boldsymbol{q}$, $\boldsymbol{r}_i$, $\boldsymbol{l}_i$ have a single answer domain variable x and that we evaluate $\boldsymbol{q}$ at time point 0). Given an ontology $\mathcal{O}$, we consider the class $LTL_{pp}^{\mathsf{U}}(\mathcal{Q}^\sigma)$ of *$\mathcal{O}$-peerless queries* in $LTL_p^{\mathsf{U}}(\mathcal{Q}^\sigma)$ of the form (5), in which $\boldsymbol{r}_i \not\models_\mathcal{O} \boldsymbol{l}_i$ and $\boldsymbol{l}_i \not\models_\mathcal{O} \boldsymbol{r}_i$, for all $i \leq n$. In what follows we write $\mathcal{O}, \mathcal{D} \models \boldsymbol{q}$ instead of $\mathcal{O}, \mathcal{D}, 0, a \models \boldsymbol{q}$ when a is clear from context. We also write $\mathcal{D} \models \boldsymbol{q}$ instead of $\emptyset, \mathcal{D} \models \boldsymbol{q}$ (that is, for the empty ontology).

A fundamental difference to the previous section and Theorem 9 is that now containment reduction and unique characterisability of domain queries are not sufficient to guarantee transfer to the temporal case. Recall that *DL-Lite*$^-_\mathcal{F}$ admits polytime computable frontiers but no split-partners.

Theorem 11. *There exist a DL-Lite$^-_\mathcal{F}$ ontology $\mathcal{O}$, a signature σ and a query $\boldsymbol{q} \in LTL_{pp}^{\mathsf{U}}$(ELIQ$^\sigma$) satisfiable wrt $\mathcal{O}$ such that $\boldsymbol{q}$ is not uniquely characterisable wrt $\mathcal{O}$ within LTL_p^{U}(ELIQ$^\sigma$).*

In fact, one can take $\mathcal{O}$ and σ from the proof of Theorem 8 and set $\boldsymbol{q} = \bot \mathsf{U} A \equiv \bigcirc A$. Observe that to separate $\bigcirc A$ from $\boldsymbol{q}' \mathsf{U} A$ with a σ-ELIQ $\boldsymbol{q}'$ such that $\boldsymbol{q}' \not\models_\mathcal{O} A$, one has to add to E^- a temporal σ-data instance $\mathcal{D} = \{\top(a)\}, \mathcal{A}, \{A(a)\}$ such that $\mathcal{O}, \mathcal{A} \models \boldsymbol{q}'(a)$ but $\mathcal{O}, \mathcal{A} \not\models A(a)$. Such $\mathcal{A}$ could be provided by a finite split-partner for $\{A\}$ wrt $\mathcal{O}$ within ELIQ$^\sigma$ had it existed, but not from a frontier.

We establish the following general transfer theorem, assuming containment reduction and split-partners:

Theorem 12. *Suppose $\mathcal{Q}$ is a class of domain queries, σ a signature, an ontology language $\mathcal{L}$ has general split-partners within $\mathcal{Q}^\sigma$, and $\mathcal{O}$ is a σ-ontology in $\mathcal{L}$ admitting containment reduction. Let $\boldsymbol{q} \in LTL_{pp}^{\mathsf{U}}(\mathcal{Q}^\sigma)$ be satisfiable wrt $\mathcal{O}$. Then the following hold*:

(i) $\boldsymbol{q}$ is uniquely characterisable wrt $\mathcal{O}$ within $LTL_p^{\mathsf{U}}(\mathcal{Q}^\sigma)$.

(ii) If a split-partner for any set Θ, $|\Theta| \leq 2$, of $\mathcal{Q}^\sigma$ queries wrt $\mathcal{O}$ within $\mathcal{Q}^\sigma$ is exponential, then there is an exponential-size unique characterisation of $\boldsymbol{q}$ wrt $\mathcal{O}$.

(iii) If a split-partner of any set Θ as above is polynomial and a split-partner $\mathcal{S}_\bot$ of $\bot(x)$ within $\mathcal{Q}^\sigma$ wrt $\mathcal{O}$ is a singleton, then there is a polynomial-size unique characterisation of $\boldsymbol{q}$ wrt $\mathcal{O}$.

The detailed proof of Theorem 12 given in the full paper (Jung et al. 2023) is by reduction to the ontology-free *LTL* case, using a characterisation of (Fortin et al. 2022). Here, we define the example set that provides the characterisation for (i). Suppose a signature σ, a σ-ontology $\mathcal{O}$, and a query $\boldsymbol{q} \in LTL_{pp}^{\mathsf{U}}(\mathcal{Q}^\sigma)$ of the form (5) are given. We may assume that $\boldsymbol{r}_n \not\equiv_\mathcal{O} \top$. We obtain the set E^+ of positive examples by taking

$(\mathfrak{p}'_0)$ $\hat{\boldsymbol{r}}_0 \dots \hat{\boldsymbol{r}}_n$;

$(\mathfrak{p}'_1)$ $\hat{\boldsymbol{r}}_0 \dots \hat{\boldsymbol{r}}_{i-1} \hat{\boldsymbol{l}}_i \hat{\boldsymbol{r}}_i \dots \hat{\boldsymbol{r}}_n$;

$(\mathfrak{p}'_2)$ $\hat{\boldsymbol{r}}_0 \dots \hat{\boldsymbol{r}}_{i-1} \hat{\boldsymbol{l}}_i^k \hat{\boldsymbol{r}}_i \dots \hat{\boldsymbol{r}}_{j-1} \hat{\boldsymbol{l}}_j \hat{\boldsymbol{r}}_j \dots \hat{\boldsymbol{r}}_n$, $i < j$, $k = 1, 2$.

Here, $\hat{\boldsymbol{l}}_i^k$ is a sequence of k-many $\hat{\boldsymbol{l}}_i$. The negative examples E^- comprise the following instances $\mathcal{D}$ whenever $\mathcal{D} \not\models \boldsymbol{q}$:

$(\mathfrak{n}'_0)$ $\mathcal{A}_1, \dots, \mathcal{A}_n$ and $\mathcal{A}_1, \dots, \mathcal{A}_{n-i}, \mathcal{A}, \mathcal{A}_{n-i+1}, \dots, \mathcal{A}_n$, for $(\mathcal{A}, a) \in \mathcal{S}(\{\boldsymbol{r}_i\})$ and $(\mathcal{A}_1, a), \dots, (\mathcal{A}_n, a) \in \mathcal{S}_\bot$;

$(\mathfrak{n}'_1)$ $\mathcal{D} = \hat{\boldsymbol{r}}_0 \dots \hat{\boldsymbol{r}}_{i-1} \mathcal{A} \hat{\boldsymbol{r}}_i \dots \hat{\boldsymbol{r}}_n$, where $(\mathcal{A}, a)$ is an element of $\mathcal{S}(\{\boldsymbol{l}_i, \boldsymbol{r}_i\}) \cup \mathcal{S}(\{\boldsymbol{l}_i\}) \cup \mathcal{S}_\bot$;

$(\mathfrak{n}'_2)$ for all i and $(\mathcal{A}, a) \in \mathcal{S}(\{\boldsymbol{l}_i, \boldsymbol{r}_i\}) \cup \mathcal{S}(\{\boldsymbol{l}_i\}) \cup \mathcal{S}_\bot$, *some* data instance

$$\mathcal{D}^i_\mathcal{A} = \hat{\boldsymbol{r}}_0 \dots \hat{\boldsymbol{r}}_{i-1} \mathcal{A} \hat{\boldsymbol{r}}_i \hat{\boldsymbol{l}}_{i+1}^{k_{i+1}} \hat{\boldsymbol{r}}_{i+1} \dots \hat{\boldsymbol{l}}_n^{k_n} \boldsymbol{r}_n,$$

if any, such that $\max(\mathcal{D}^i_\mathcal{A}) \leq (n+1)^2$ and $\mathcal{D}^i_\mathcal{A} \not\models \boldsymbol{q}^\dagger$ for $\boldsymbol{q}^\dagger$ obtained from $\boldsymbol{q}$ by replacing all $\boldsymbol{l}_j$, for $j \leq i$, with $\bot$.

We have (ii) since (E^+, E^-) is at most exponential in the size of split-partners of sets with at most two queries. For (iii), observe that $(\mathfrak{n}'_1)$ is exponential in $|\mathcal{S}_\bot|$ iff $|\mathcal{S}_\bot| \geq 2$.

As a consequence of Lemma 1, Theorem 12 (ii) and (iii), and Theorems 3 and 4 we obtain the following (note that, for every RDFS ontology, the split-partner $\mathcal{S}_\bot$ of $\bot$ is a singleton by Example 1 (i)):

Theorem 13. *(i) Each $\boldsymbol{q} \in LTL_{pp}^{\mathsf{U}}$(ELIQ$^\sigma$) satisfiable wrt an $\mathcal{ALCHI}$ ontology $\mathcal{O}$ in a signature σ is exponential-size uniquely characterisable wrt $\mathcal{O}$ within LTL_p^{U}(ELIQ$^\sigma$).*

(ii) Each $\boldsymbol{q} \in LTL_{pp}^{\mathsf{U}}$(ELQ$^\sigma$) is polysize uniquely characterisable wrt any RDFS ontology in σ within LTL_p^{U}(ELQ$^\sigma$).

8 Exact Learnability

We apply the results on unique characterisability obtained in Section 6 to exact learnability of queries wrt ontologies. Given a query class $\mathcal{C}$ and an ontology $\mathcal{O}$, the *learner* aims to identify a *target query* $\boldsymbol{q}_T \in \mathcal{C}$ by means of membership queries of the form 'does $\mathcal{O}, \mathcal{D}, 0, a \models \boldsymbol{q}_T$ hold?' to the *teacher*. We call $\mathcal{C}$ *polynomial time learnable wrt $\mathcal{L}$ ontologies using membership queries* if there is a learning algorithm that given $\mathcal{O}$ constructs (up to equivalence wrt $\mathcal{O}$) any $\boldsymbol{q}_T$ satisfiable wrt $\mathcal{O}$ in time polynomial in the sizes of $\boldsymbol{q}_T$ and $\mathcal{O}$. For the weaker requirement of *polynomial query learnability*, it suffices that the total size of the examples given to the oracle be bounded by a polynomial. We start with making the following observation, where exponential query learnability is defined in the expected way.

Theorem 14. *Let $\mathcal{L}$ be an ontology language and $\mathcal{C}$ be a class of queries such that* (1) *$\mathcal{C}$ has an effective syntax,* (2) *for every $\mathcal{L}$-ontology $\mathcal{O}$, every query in $\mathcal{C}$ satisfiable wrt $\mathcal{O}$ admits effective exponential size unique characterisations wrt $\mathcal{O}$, and* (3) *satisfiability of queries in $\mathcal{C}$ wrt $\mathcal{L}$-ontologies is decidable. Then, $\mathcal{C}$ is exponential query learnable wrt $\mathcal{L}$ ontologies.*

Proof. Let $\boldsymbol{q}_T \in \mathcal{C}$ be the target query and $\mathcal{O}$ an $\mathcal{L}$ ontology. Due to (1), we can enumerate all queries from $\mathcal{C}$ in increasing size. For every enumerated $\boldsymbol{q}$, test whether $\boldsymbol{q}$ is satisfiable wrt $\mathcal{O}$, using (3). If so, then using (2), we compute its unique characterisation (E^+, E^-) wrt $\mathcal{O}$ and use membership queries to check whether all examples in E^+ are positive examples and all examples in E^- are negative examples. If so, output $\boldsymbol{q}$. □

Since our main focus in this section is, however, polynomial time and query learnability, we consider below cases which allow for polynomial size unique characterisations. As the presence of $\sqcap$ and $\bot$ in the ontology language precludes polynomial query learnability already in the atemporal case, c.f. Theorem 6 in (Funk, Jung, and Lutz 2022b), we follow their approach and assume that the learner also receives an initial positive example $\mathcal{D}, a$ with $\mathcal{D}$ and $\mathcal{O}$ satisfiable. Note that the existence of such an example implies that the target query is satisfiable wrt the ontology. In order to state our main result, we introduce one further natural condition. An ontology language $\mathcal{L}$ *admits polynomial time instance checking* if given an $\mathcal{L}$ ontology $\mathcal{O}$, a pointed instance $(\mathcal{A}, a)$, and a concept name A, it is decidable in polynomial time whether $\mathcal{O}, \mathcal{A} \models A(a)$.

Theorem 15. *Let $\mathcal{L}$ be an ontology language that contains only $\mathcal{ELHI}$ or only $\mathcal{ELIF}$ ontologies and that admits poly-size frontiers within ELIQ that can be computed. Then:*

- (i) *The class of safe LTL$_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) queries are polynomial query learnable wrt $\mathcal{L}$ ontologies using membership queries.*
- (ii) *The class LTL$_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) is polynomial query learnable wrt $\mathcal{L}$ ontologies using membership queries if the learner knows the temporal depth of the target query.*
- (iii) *The class LTL$_p^{\bigcirc\Diamond}$(ELIQ) is polynomial query learnable wrt $\mathcal{L}$ ontologies using membership queries.*

If $\mathcal{L}$ further admits polynomial time instance checking and polynomial time computable frontiers within ELIQ, then in (ii) and (iii), polynomial query learnability can be replaced by polynomial time learnability. If, in addition, meet-reducibility wrt $\mathcal{L}$ ontologies can be decided in polynomial time, then also in (i) polynomial query learnability can be replaced by polynomial time learnability.

To achieve the generality of the results independently of the exact languages, in the proof of Theorem 15 we rely on the results and techniques from Section 6 and general results proved in the context of exact learning of (atemporal) ELIQs wrt ontologies (Funk, Jung, and Lutz 2022a).

Let $\boldsymbol{q}_T$ be a target query, $\mathcal{O}$ be an ontology, and $\mathcal{D}, a$ be a positive example with $\mathcal{D} = \mathcal{A}_0 \dots \mathcal{A}_n$ and $\mathcal{D}$ and $\mathcal{O}$ satisfiable. The idea is to modify $\mathcal{D}$ in a number of steps such that, in the end, $\mathcal{D}$ viewed as temporal query is equivalent to $\boldsymbol{q}_T$.

We describe how to show (i); (ii) and (iii) are slight modifications thereof. In **Step 1**, the goal is to find a temporal data instance $\mathcal{D}$ where each $\mathcal{A}_i$ is *tree-shaped* and hence can be viewed as an ELIQ. This can be done separately for each time point using membership queries and standard unraveling techniques from the atemporal setting (Funk, Jung, and Lutz 2022a). In **Step 2**, we exhaustively apply Rules (a)-(e) from the proof of Theorem 9 to $\mathcal{D}$, as long as $\mathcal{D}, a$ remains a positive example. In **Step 3**, we take care of lone conjuncts in $\mathcal{D}$ (when viewed as a temporal query) – recall that $\boldsymbol{q}_T$ is safe and thus does not have any. For this step, we rely on a characterisation of meet-reducibility in terms of *minimal* frontiers. For computing those, we exploit the fact that containment of ELIQs wrt $\mathcal{ELHI}$ and $\mathcal{ELIF}$ ontologies is decidable (Bienvenu et al. 2016). After Step 3, $\mathcal{D}$ (viewed as query) is already very similar to $\boldsymbol{q}_T$. More precisely, when representing $\boldsymbol{q}_T$ in shape (8) as a sequence of blocks $\boldsymbol{q}_0\mathcal{R}_1\boldsymbol{q}_1 \dots \mathcal{R}_m\boldsymbol{q}_m$, then $\mathcal{D}$ has the shape $\mathcal{D}_0\emptyset^b \dots \emptyset^b\mathcal{D}_m$, for sufficiently large b, and each $\boldsymbol{q}_i$ is isomorphic to $\mathcal{D}_i$. So in **Step 4**, it remains to identify the precise separators $\mathcal{R}_i$. They can be a single $\leq$ or a sequence of $<$, and the two cases can be distinguished using suitable membership queries.

In order to show that this entire process terminates after asking polynomially many membership queries, we lift the notion of *generalisation sequences* from (Funk, Jung, and Lutz 2022a) to the temporal setting. For the sake of convenience, we treat the data instances in the time points as CQs. A sequence $\mathcal{D}_1, \dots$ of temporal data instances is a *generalisation sequence towards* $\boldsymbol{q}_T$ *wrt* $\mathcal{O}$ if for all $i \geq 1$:

- $\mathcal{D}_{i+1}$ is obtained from $\mathcal{D}_i$ by modifying one non-temporal CQ $\boldsymbol{r}_j$ in $\mathcal{D}_i$ to $\boldsymbol{r}'_j$ such that $\boldsymbol{r}_j \models_{\mathcal{O}} \boldsymbol{r}'_j$ and $\boldsymbol{r}'_j \not\models_{\mathcal{O}} \boldsymbol{r}_j$;
- $\mathcal{O}, \mathcal{D}_i, 0, a \models \boldsymbol{q}_T$ for all $i \geq 1$.

Intuitively, data instances in generalisation sequences become weaker and weaker, and based on this, we show that the length of generalisation sequences towards $\boldsymbol{q}_T$ wrt $\mathcal{O}$ is bounded by a polynomial in $\max(\mathcal{D}_1)$ and the sizes of $\boldsymbol{q}_T, \mathcal{O}$. The crucial observation is that the sequences of temporal data instances obtained by rule application are mostly generalisation sequences towards $\boldsymbol{q}_T$ wrt $\mathcal{O}$; thus the steps terminate in polynomial time. If they are not, we use a different (but usually easier) termination argument.

It remains to note that the sketched algorithm runs in polynomial time when $\mathcal{L}$ satisfies all the required criteria. $\square$

We finally apply Theorem 15 to concrete ontology languages, namely *DL-Lite*$_{\mathcal{F}}^{-}$ and *DL-Lite*$_{\mathcal{H}}$.

Theorem 16. *The following learnability results hold:*

(i) *The class of safe queries in* $LTL_p^{\bigcirc\Diamond\Diamond_r}$*(ELIQ) is polynomial query learnable wrt DL-Lite*$_{\mathcal{H}}$ *ontologies using membership queries and polynomial time learnable wrt DL-Lite*$_{\mathcal{F}}^{-}$ *ontologies using membership queries.*

(ii) *The class* $LTL_p^{\bigcirc\Diamond\Diamond_r}$*(ELIQ) is polynomial time learnable wrt both DL-Lite*$_{\mathcal{F}}^{-}$ *and DL-Lite*$_{\mathcal{H}}$ *ontologies using membership queries if the learner knows the temporal depth of the target query in advance.*

(iii) *The class* $LTL_p^{\bigcirc\Diamond}$*(ELIQ) is polynomial time learnable wrt both DL-Lite*$_{\mathcal{F}}^{-}$ *and DL-Lite*$_{\mathcal{H}}$ *ontologies using membership queries.*

Theorem 16 is a direct consequence of Theorem 15 and the fact that the considered ontology languages satisfy all conditions mentioned there. In particular, we show in the appendix that meet-reducibility of ELIQs wrt *DL-Lite*$_{\mathcal{F}}^{-}$ ontologies Turing reduces to ELIQ containment wrt *DL-Lite*$_{\mathcal{F}}^{-}$ ontologies, which is tractable (Bienvenu et al. 2013). The latter is not true for *DL-Lite*$_{\mathcal{H}}$ which explains the difference in (i). We leave it for future work whether $LTL_p^{\bigcirc\Diamond\Diamond_r}$(ELIQ) is polynomial *time* learnable wrt *DL-Lite*$_{\mathcal{H}}$ ontologies.

9 Outlook

Many interesting and challenging problems remain to be addressed. We discuss a few of them below.

(1) Is it possible to overcome our 'negative' unique characterisability results by admitting some form of infinite (but finitely presentable) examples? Some results in this direction without ontologies are obtained in (Sestic 2023).

(2) We have not considered learnability using membership queries of temporal queries with U. In fact, it remains completely open how far our characterisability results for these queries can be exploited to obtain polynomial query (or time) learnability.

(3) We only considered path queries with no temporal operator occurring in the scope of a DL operator. This is motivated by the negative results of (Fortin et al. 2022), which showed that (i) applying $\exists P$ to $\bigcirc\Diamond$-queries quickly leads to non-characterisability and that (ii) even without DL-operators and without ontology, branching $\Diamond$-queries are often not uniquely characterisable. We still believe there is some scope for useful positive characterisability results.

References

Alexe, B.; ten Cate, B.; Kolaitis, P. G.; and Tan, W. C. 2011. Characterizing schema mappings via data examples. *ACM Trans. Database Syst.* 36(4):23.

Angluin, D. 1987a. Learning regular sets from queries and counterexamples. *Inf. Comput.* 75(2):87–106.

Angluin, D. 1987b. Queries and concept learning. *Mach. Learn.* 2(4):319–342.

Arenas, M., and Diaz, G. I. 2016. The exact complexity of the first-order logic definability problem. *ACM Trans. Database Syst.* 41(2):13:1–13:14.

Artale, A.; Calvanese, D.; Kontchakov, R.; and Zakharyaschev, M. 2009. The DL-Lite family and relations. *J. Artif. Intell. Res.* 36:1–69.

Artale, A.; Kontchakov, R.; Ryzhikov, V.; and Zakharyaschev, M. 2014. A cookbook for temporal conceptual data modelling with description logics. *ACM Trans. Comput. Log.* 15(3):25:1–25:50.

Artale, A.; Kontchakov, R.; Kovtunova, A.; Ryzhikov, V.; Wolter, F.; and Zakharyaschev, M. 2017. Ontology-mediated query answering over temporal data: A survey. In *Proc. of TIME*, 1:1–1:37. Schloss Dagstuhl, Leibniz-Zentrum für Informatik.

Artale, A.; Kontchakov, R.; Kovtunova, A.; Ryzhikov, V.; Wolter, F.; and Zakharyaschev, M. 2022. First-order rewritability and complexity of two-dimensional temporal ontology-mediated queries. *J. Artif. Intell. Res.* 75:1223–1291.

Baader, F.; Horrocks, I.; Lutz, C.; and Sattler, U. 2017. *An Introduction to Description Logics*. Cambridge University Press.

Baader, F.; Borgwardt, S.; and Lippmann, M. 2015. Temporal query entailment in the description logic SHQ. *J. Web Semant.* 33:71–93.

Barceló, P., and Romero, M. 2017. The complexity of reverse engineering problems for conjunctive queries. In *Proc. of ICDT*, 7:1–7:17.

Bienvenu, M.; Ortiz, M.; Simkus, M.; and Xiao, G. 2013. Tractable queries for lightweight description logics. In *Proc. of IJCAI*, 768–774. IJCAI/AAAI.

Bienvenu, M.; ten Cate, B.; Lutz, C.; and Wolter, F. 2014. Ontology-based data access: A study through disjunctive datalog, CSP, and MMSNP. *ACM Trans. Database Syst.* 39(4):33:1–33:44.

Bienvenu, M.; Hansen, P.; Lutz, C.; and Wolter, F. 2016. First order-rewritability and containment of conjunctive queries in Horn description logics. In *Proc. of IJCAI*, 965–971.

Borgwardt, S., and Thost, V. 2015. Temporal query answering in the description logic EL. In *Proc. of IJCAI*, 2819–2825. AAAI Press.

Calvanese, D.; Giacomo, G. D.; Lembo, D.; Lenzerini, M.; and Rosati, R. 2007a. EQL-Lite: Effective first-order query processing in description logics. In *Proc. of IJCAI*, 274–279.

Calvanese, D.; Giacomo, G. D.; Lembo, D.; Lenzerini, M.; and Rosati, R. 2007b. Tractable reasoning and efficient query answering in description logics: The DL-Lite family. *J. Autom. Reasoning* 39(3):385–429.

Camacho, A., and McIlraith, S. A. 2019. Learning interpretable models expressed in linear temporal logic. In *Proc. of ICAPS*, 621–630. AAAI Press.

Chang, C., and Keisler, H. J. 1998. *Model Theory*. Elsevier.

Fijalkow, N., and Lagarde, G. 2021. The complexity of learning linear temporal formulas from examples. *CoRR* abs/2102.00876.

Fortin, M.; Konev, B.; Ryzhikov, V.; Savateev, Y.; Wolter, F.; and Zakharyaschev, M. 2022. Unique characterisability and learnability of temporal instance queries. In *Proc. of KR*.

Fortin, M.; Konev, B.; Ryzhikov, V.; Savateev, Y.; Wolter, F.; and Zakharyaschev, M. 2023. Reverse engineering of temporal queries mediated by LTL ontologies. In *Proceedings of IJCAI*.

Funk, M.; Jung, J. C.; Lutz, C.; Pulcini, H.; and Wolter, F. 2019. Learning description logic concepts: When can positive and negative examples be separated? In *Proc. of IJCAI*, 1682–1688.

Funk, M.; Jung, J. C.; and Lutz, C. 2021. Actively learning concepts and conjunctive queries under ELr-ontologies. In *Proc. of IJCAI*, 1887–1893. ijcai.org.

Funk, M.; Jung, J. C.; and Lutz, C. 2022a. Exact learning of ELI queries in the presence of dl-lite-horn ontologies. In *Proc. of DL*. CEUR-WS.org.

Funk, M.; Jung, J. C.; and Lutz, C. 2022b. Frontiers and exact learning of ELI queries under dl-lite ontologies. In *Proc. of IJCAI*, 2627–2633.

Gutiérrez-Basulto, V.; Jung, J. C.; and Kontchakov, R. 2016. Temporalized EL ontologies for accessing temporal data: Complexity of atomic queries. In *Proc. of IJCAI*, 1102–1108. IJCAI/AAAI Press.

Gutiérrez-Basulto, V.; Jung, J. C.; and Sabellek, L. 2018. Reverse engineering queries in ontology-enriched systems: The case of expressive Horn description logic ontologies. In *Proc. of IJCAI-ECAI*.

Jung, J. C.; Lutz, C.; Pulcini, H.; and Wolter, F. 2022. Logical separability of labeled data examples under ontologies. *Artif. Intell.* 313:103785.

Jung, J. C.; Ryzhikov, V.; Wolter, F.; and Zakharyaschev, M. 2023. Temporalising unique characterisability and learnability of ontology-mediated queries. *CoRR* abs/2306.07662.

Kikot, S.; Kontchakov, R.; and Zakharyaschev, M. 2012. Conjunctive query answering with owl 2 ql. In *Proc. of KR*.

Kolaitis, P. G. 2011. Schema Mappings and Data Examples: Deriving Syntax from Semantics (Invited Talk). In *Proc. of FSTTCS*, volume 13, 25–25. Dagstuhl, Germany: Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.

Konev, B.; Lutz, C.; Ozaki, A.; and Wolter, F. 2017. Exact learning of lightweight description logic ontologies. *J. Mach. Learn. Res.* 18:201:1–201:63.

Lehmann, J., and Hitzler, P. 2010. Concept learning in description logics using refinement operators. *Machine Learning* 78:203–250.

Lemieux, C.; Park, D.; and Beschastnikh, I. 2015. General ltl specification mining (t). In *Proc. of ASE*, 81–92. IEEE.

Martins, D. M. L. 2019. Reverse engineering database queries from examples: State-of-the-art, challenges, and research opportunities. *Inf. Syst.* 83:89–100.

McKenzie, R. 1972. Equational bases and nonmodular lattice varieties. *Transactions of the American Mathematical Society* 174:1–43.

Neider, D., and Gavran, I. 2018. Learning linear temporal properties. In *Proc. of FMCAD*, 1–10. IEEE.

Ozaki, A. 2020. Learning description logic ontologies: Five approaches. where do they stand? *Künstliche Intell.* 34(3):317–327.

Sestic, P. 2023. Unique characterisability of linear temporal logic. Msc, University of Amsterdam.

Staworko, S., and Wieczorek, P. 2015. Characterizing XML twig queries with examples. In *Proc. of ICDT*, 144–160.

ten Cate, B., and Dalmau, V. 2022. Conjunctive queries: Unique characterizations and exact learnability. *ACM Trans. Database Syst.* 47(4):14:1–14:41.

ten Cate, B.; Funk, M.; Jung, J. C.; and Lutz, C. 2023. Sat-based PAC learning of description logic concepts. In *Proc. of IJCAI*, 3347–3355. ijcai.org.

ten Cate, B.; Dalmau, V.; and Kolaitis, P. G. 2013. Learning schema mappings. *ACM Trans. Database Syst.* 38(4):28:1–28:31.

ten Cate, B.; Koudijs, R.; and Ozaki, A. 2024. On the power and limitations of examples for description logic concepts. In *Proc. of IJCAI*.

Wałęga, P. A.; Cuenca Grau, B.; Kaminski, M.; and V. Kostylev, E. 2020. DatalogMTL over the Integer Timeline. In *Proc. of KR*, 768–777.