Contracted Temporal Equilibrium Logic

Pedro Cabalar[1] , **Thomas Eiter**[2] , **Davide Soldà**[2]
[1]University of A Coruña, SPAIN
[2]Institute of Logic and Computation,
Technische Universität Wien, AUSTRIA
cabalar@udc.es, eiter@kr.tuwien.ac.at, davide.solda@tuwien.ac.at

Abstract

The stable model semantics of logic programs has been characterized by Equilibrium Logic, which is a non-monotonic formalism that selects models from the (monotonic) intermediate logic of Here-and-There. It provides stable models for arbitrary propositional formulas and has been fruitfully extended to different modal languages. Among them are theories in the syntax of Linear-Time Temporal Logic (LTL), giving rise to Temporal Equilibrium logic (TEL) based on Temporal Here-and-There (THT). In TEL, models are selected that minimize truth among THT traces of the same length. In this paper, we consider a selection that in addition may reduce the number of transitions in a trace, intuitively forming a contraction of it. We thus introduce *contracted* THT and *contracted* TEL on top of a model selection on a logical basis. The resulting c-stable models can be viewed as stable models in TEL that can not be summarized into a smaller trace. We illustrate contraction on several examples related to logic programming and explore several properties, like the relation to TEL and LTL, and in particular the connection to the LTL property of *stuttering*.

1 Introduction

(Linear-time) Temporal Equilibrium Logic (TEL) (Aguado et al. 2013) is a well-known extension of Equilibrium Logic (Pearce 2006), the nonmonotonic logic that characterizes answer sets of a logic program. Its semantics is defined by selecting certain models of a theory in Temporal Here-and-There (THT), a temporal extension of the intermediate logic of Here-and-There (Heyting 1930). These selected models have the form of traces that are said to be *in equilibrium* (also called *stable models* or *stable traces*) when a certain kind of minimality holds, obtaining in this way a non-monotonic entailment relation.

A TEL theory may have many stable models, such as the formula $\varphi = \Box(a \vee b)$, where each trace $\mathbf{T} = T_0{\cdot}T_1{\cdot}T_2\cdots$ with T_i being either $\{a\}$ or $\{b\}$ is a stable model; in particular $\mathbf{T} = \{a\}{\cdot}\{a\}{\cdot}\{b\}{\cdot}\{b\}{\cdot}\{b\}{\cdot}\{a\}{\cdot}\{a\}{\cdot}\{a\}\cdots$ and $\mathbf{T}' = \{a\}{\cdot}\{b\}{\cdot}\{a\}$ are stable models. The is intuitively more succinct and may be preferred over $\mathbf{T}$. A natural question then is how to select stable models with relevant state transitions.

In the literature, trace selection by length has been considered; e.g., in (Schuppan and Biere 2005) the authors select models on the basis of the shortest counterexamples for model checking purposes. For planning problems, `ASP` solvers are usually run up to a certain plan horizon, aiming at the computation of shortest plans. Other approaches for selection over traces involve the use of minimization criteria with weighted atoms, see (Dodaro, Fionda, and Greco 2022) for LTL over finite traces among others.

However, rather than simply imposing a selection function on stable models, we are interested in providing, in the spirit of TEL, a semantics that selects models on a logical basis. The idea is that not only the truth of atoms is minimized, but in addition segments of a trace are summarized.

To illustrate this superficially on the formula from above, by contracting in the trace $\mathbf{T}$ the initial segment $\{a\}{\cdot}\{a\}$ into $\{a\}$, and similarly $\{b\}{\cdot}\{b\}{\cdot}\{b\}$ and $\{a\}{\cdot}\{a\}{\cdot}\{a\}\cdots$ into $\{b\}$ and $\{a\}$, respectively, we obtain $\mathbf{T}'$ which preserves φ under contraction, as b resp. a is true over the segment associated with each position. The trace $\mathbf{T}'$ is a model of φ that can not be contracted, and is thus selected. To see why taking the stable models of minimum length does not suffice, consider the following example.

Example 1 *Suppose that, to move to the airport from our office, we may go by bus or take a taxi. If we go by bus, we must make two bus stops, bs_1 and bs_2 before arriving whereas, if we go by taxi, we always have to stop at a crossroad c. The number of transitions we may take between two stops is not predetermined. The following* TEL *theory is one possible simplified formalization of this example (recall that $\Box$, $\Diamond$, $\circ$ stand for always, eventually, and next time, respectively):*

$$bus \vee taxi \tag{1}$$
$$\Box(bus \rightarrow \circ\Diamond bs_1) \tag{2}$$
$$\Box(bs_1 \rightarrow \circ\Diamond bs_2) \tag{3}$$
$$\Box(bs_2 \rightarrow \circ\Diamond airport) \tag{4}$$
$$\Box(taxi \rightarrow \circ\Diamond c) \tag{5}$$
$$\Box(c \rightarrow \circ\Diamond airport) \tag{6}$$

The stable models of (1)-(6) follow two different patterns:

1. $\{bus\} \cdot \emptyset^* \cdot \{bs_1\} \cdot \emptyset^* \cdot \{bs_2\} \cdot \emptyset^* \cdot \{airport\} \cdot \emptyset^*$
2. $\{taxi\} \cdot \emptyset^* \cdot \{c\} \cdot \emptyset^* \cdot \{airport\} \cdot \emptyset^*$

where, in both cases, we may replace the last $\emptyset^*$ by $\emptyset^\omega$, dealing with traces of infinite length. The shortest stable model corresponds to $\{taxi\} \cdot \{c\} \cdot \{airport\}$, where we take the taxi and it arrives in the fastest possible way, without any delay in each trip segment. We claim that the stable model

$\{bus\} \cdot \{bs_1\} \cdot \{bs_2\} \cdot \{airport\}$, although longer, should be incomparably minimal as well, as it corresponds to the shortest trace we may get *when we decide to take the bus*.

For developing contraction, we consider furthermore the following desiderata as a guidance: (D1) A contracted trace should be in equilibrium, that is, contraction selects from the stable traces. (D2) Consecutively repeated states, known as *stuttering*, should preferably be eliminated, if possible. (D3) Prevailing semantics such as LTL and TEL should be recoverable by including axioms into a theory.

Our main contributions are then as follows.

- We introduce contraction THT (cTHT), in which interpretations are structures $\langle \mathbf{T}', \mathbf{T}, \mu \rangle$ where μ maps segments of $\mathbf{T}$ to $\mathbf{T}'$, in a way such that the contracted (summarized) trace $\mathbf{T}'$ is sound with respect to inferences that could be made in the trace $\mathbf{T}$; that is, while inferences might be dropped, no new formulas are derivable in a summarized segment of $\mathbf{T}$. For the definition of entailment, we resort to a temporal version of the intermediate logic known as *Bounded Depth 2*.
- On top of contracted THT, we then define contracted TEL (cTEL) by model selection according to a preference relation. Intuitively, a trace is in equilibrium, if no proper summarization can be made. The resulting equilibrium models, called c-stable models, obey D1 because they are also regular stable models, and D2 for meaningful language fragments. For instance, for formulas without the next-operator ($\circ$) and without nested implication, we are able to prove that c-stable models coincide exactly with the regular stable models (D1) that are stutter-free (D2).
- Both LTL and TEL can be recovered from cTEL by adding suitable axioms; the well-known property of LTL that for $\circ$-free formulas states can be stuttered is then a corollary.
- We show that satisfiability (stable model existence) has in cTEL the same complexity as in TEL, which is EXPSPACE-complete (Bozzelli and Pearce 2015), and that for cTEL fragments, standard reasoning tasks can be modularly translated into TEL.

We believe that our work provides a basic framework for defining contraction and summarization of (stable) traces that can be utilized in various contexts, such as for generating example traces, condensing given traces, analyzing minimal plans, and many further applications.

2 Preliminaries

The syntax of THT (and TEL) is the same as for LTL. In particular, in this paper, we use the following notation. Given a (countable, possibly infinite) set $\mathcal{A}$ of propositional variables (called *alphabet*), *temporal formulas* φ are defined by the grammar:

$$\varphi ::= a \mid \top \mid \bot \mid \varphi_1 \otimes \varphi_2 \mid \circ\varphi \mid \varphi_1 \,\mathbf{U}\, \varphi_2 \mid \varphi_1 \,\mathbf{R}\, \varphi_2$$

where $a \in \mathcal{A}$ is an atom and $\otimes$ is any binary Boolean connective $\otimes \in \{\to, \wedge, \vee\}$. The last four cases correspond to the temporal connectives whose names are listed as follows: $\circ$ for *next*; $\mathbf{U}$ for *until*; and $\mathbf{R}$ for *release*. A formula φ is said to be $\otimes$*-free* if it does not contain any occurrence of some connective $\otimes$. We also define several common derived operators like the Boolean connectives $\neg\varphi =_{\text{def}} \varphi \to \bot$, $\varphi \leftrightarrow \psi =_{\text{def}} (\varphi \to \psi) \wedge (\psi \to \varphi)$, and the following temporal operators: *always* as $\Box\varphi =_{\text{def}} \bot \,\mathbf{R}\, \varphi$, *eventually* as $\Diamond\varphi =_{\text{def}} \top \,\mathbf{U}\, \varphi$, *final* as $\mathbf{F} =_{\text{def}} \neg\circ\top$, and *weak next* as $\widehat{\circ}\varphi =_{\text{def}} \circ\varphi \vee \mathbf{F}$. A *(temporal) theory* is a (possibly infinite) set of temporal formulas.

Although THT and LTL share the same syntax, they have different semantics, the former being a weaker logic than the latter. The semantics of THT relies on the concept of pairs of traces. In LTL, a *trace* $\mathbf{T}$ of length $\lambda \geq 1$ (possibly infinite, $\lambda = \omega$) is a sequence $\mathbf{T} = (T_i)_{[0..\lambda)}$ of sets $T_i \subseteq \mathcal{A}$. A THT-trace $\mathbf{M}$ is a pair $\mathbf{M} = \langle \mathbf{H}, \mathbf{T} \rangle$ where $\mathbf{H}$ and $\mathbf{T}$ are LTL traces of the same length, $\mathbf{H} = H_0 \cdot H_1 \cdot \ldots$ and $\mathbf{T} = T_0 \cdot T_1 \cdot \ldots$, and we additionally require $H_i \subseteq T_i \subseteq \mathcal{A}$. We sometimes use the notation $|\mathbf{M}| =_{\text{def}} \lambda$ to stand for the length of the trace. We say that $\mathbf{T}$ is *infinite* if $|\mathbf{T}| = \omega$ and *finite* if $|\mathbf{T}| \in \mathbb{N}$. Given $a \in \mathbb{N}$ and $b \in \mathbb{N} \cup \{\omega\}$, we let $[a..b)$ stand for the set $\{i \in \mathbb{N} \mid a \leq i < b\}$ and, analogously, $(a..b]$ when $b \neq \omega$ stands for $\{i \in \mathbb{N} \mid a < i \leq b\}$.

Definition 1 (THT**-satisfaction)** *Let* $\mathbf{M}$ *be a* cTHT*-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T} \rangle$ *over alphabet* $\mathcal{A}$ *and let* $\lambda = |\mathbf{H}| = |\mathbf{T}|$. *Then* $\mathbf{M}$ satisfies *a temporal formula* φ *at step* $k \in [0..\lambda)$, *written* $\mathbf{M}, k \models \varphi$, *if the following recursive conditions hold:*

1. $\mathbf{M}, k \models \top$ *and* $\mathbf{M}, k \not\models \bot$*;*
2. $\mathbf{M}, k \models p$ *if* $p \in H_k$ *for any atom* $p \in \mathcal{A}$*;*
3. $\mathbf{M}, k \models \varphi \wedge \psi$ *iff* $\mathbf{M}, k \models \varphi$ *and* $\mathbf{M}, k \models \psi$*;*
4. $\mathbf{M}, k \models \varphi \vee \psi$ *iff* $\mathbf{M}, k \models \varphi$ *or* $\mathbf{M}, k \models \psi$*;*
5. $\mathbf{M}, k \models \varphi \to \psi$ *iff* $\langle \mathbf{H}', \mathbf{T} \rangle, k \not\models \varphi$, *or* $\langle \mathbf{H}', \mathbf{T} \rangle, k \models \psi$ *for all* $\mathbf{H}' \in \{\mathbf{H}, \mathbf{T}\}$*;*
6. $\mathbf{M}, k \models \circ\varphi$ *iff* $k{+}1 < \lambda$ *and* $\mathbf{M}, k{+}1 \models \varphi$*;*
7. $\mathbf{M}, k \models \varphi \mathbf{U} \psi$ *iff for some* $j \in [k..\lambda)$, *we have* $\mathbf{M}, j \models \psi$ *and* $\mathbf{M}, i \models \varphi$ *for all* $i \in [k..j)$*;*
8. $\mathbf{M}, k \models \varphi \,\mathbf{R}\, \psi$ *iff for all* $j \in [k..\lambda)$, *we have* $\mathbf{M}, j \models \psi$ *or* $\mathbf{M}, i \models \varphi$ *for some* $i \in [k..j)$. ☐

If $\mathbf{H} = \mathbf{T}$ in Definition 1, then we obtain that $\langle \mathbf{T}, \mathbf{T} \rangle, 0 \models \varphi$ iff $\mathbf{T}$ is an LTL model of φ.

Definition 2 (TEL) *A total* cTHT*-trace* $\langle \mathbf{T}, \mathbf{T} \rangle$ *is a* temporal equilibrium model *(or* stable model*) of a theory* Γ *if* $\mathbf{T}, 0 \models \Gamma$ *and there is no* $\mathbf{H} \neq \mathbf{T}$ *such that* $\langle \mathbf{H}, \mathbf{T} \rangle \models \Gamma$.

3 Contracted THT

To compare two different traces, we start introducing the concept of a *contractor* function μ, a mapping that transforms indices $i \in [0..\lambda)$ from an interval of length λ into new positions $\mu(i)$ inside an interval of length $\lambda' \leq \lambda$.

Definition 3 (Contractor) *Let* $\lambda, \lambda' \in \mathbb{N} \cup \{\omega\}$ *be two trace lengths. A* contractor *function* μ *from* λ *to* λ' *is any surjective function of type* $\mu : [0..\lambda) \to [0..\lambda')$ *that satisfies* $\mu(0) = 0$ *and is monotonic, i.e.,* $\mu(i+1) \leq \mu(i) + 1$ *for all* $i \in [0..\lambda)$ *and* $i + 1 < \lambda$. ☐

Note first that, by monotonicity, $\mu(i+1) \geq \mu(i)$ and so, $\mu(i+1)$ can only be either $\mu(i)$ or $\mu(i) + 1$. Second, as μ is surjective, all elements in $[0..\lambda')$ have a preimage in $[0..\lambda)$,

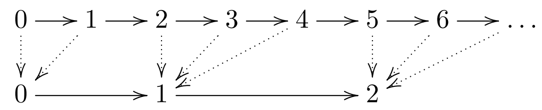

Figure 1: Example of contractor function μ_1.

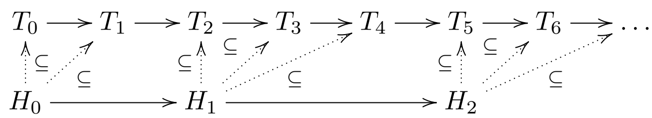

Figure 2: Subset relations imposed by $\mathbf{T} \downarrow^{\mu_1} \mathbf{H}$.

so $\lambda' \leq \lambda$, that is, μ generally produces an image interval of a smaller (or equal) length λ' than the original one λ (hence the name "contractor"). Moreover, for each point $i \in \big[0..\lambda'\big)$ we can define its (non-empty) preimage set as usual:

$$\mu^-(i) =_{\text{def}} \{j \in \big[0..\lambda\big) \mid \mu(j) = i\}.$$

The identity function $id(i) =_{\text{def}} i$ is the only case of contractor in which the preimage $id^-(i)$ is a singleton – in other words, id^- constitutes an inverse *function*. An example of a contractor μ_1 for $\lambda = \omega$ and $\lambda' = 3$ is shown in Figure 1, where $\mu_1(i) = 2$ for all $i \geq 5$.

We can also see a contractor function μ as a way to organize the points in $\big[0..\lambda\big)$ into a sequence of λ' consecutive intervals so that $\mu^-(i)$ denotes the i-th interval (starting in 0). For instance, for μ_1 in Figure 1 we have the intervals ${\mu_1}^-(0) = \big[0..2\big)$, ${\mu_1}^-(1) = \big[2..5\big)$ and ${\mu_1}^-(2) = \big[5..\omega\big)$.

As we can see, we may have cases in which the contractor just leaves the same length $\lambda' = \lambda$, that is, there is no length contraction at all. If $\lambda \in \mathbb{N}$ this only happens with the identity function id: any other contractor will necessarily reduce the length $\lambda' < \lambda$. However, when $\lambda = \omega$ we have (infinitely many) other contractors μ different from id that do not reduce the interval length, leaving $\lambda' = \omega$ as well. As an example, we may take the function $\mu_2(i) =_{\text{def}} i \div 2$ for all $i \in \mathbb{N}$.

Definition 4 (Trace Contraction) *Let* $\mathbf{H}$ *and* $\mathbf{T}$ *be two traces of lengths* $\lambda_h = |\mathbf{H}|$ *and* $\lambda_t = |\mathbf{T}|$ *respectively. We say that a contractor* μ *from* λ_t *to* λ_h contracts $\mathbf{T}$ *to* $\mathbf{H}$*, written* $\mathbf{T} \downarrow^\mu \mathbf{H}$*, when* $T_i \supseteq H_{\mu(i)}$ *for all* $i \in \big[0..\lambda_t\big)$. ☐

Definition 5 (Summarization $\preceq$) *We say that trace* $\mathbf{H}$ summarizes *trace* $\mathbf{T}$*, written* $\mathbf{H} \preceq \mathbf{T}$*, when there exists some contractor* μ *such that* $\mathbf{T} \downarrow^\mu \mathbf{H}$. ☐

Figure 2 shows the inclusion relations (in dashed lines) imposed by $\mathbf{T} \downarrow^{\mu_1} \mathbf{H}$ using the contractor μ_1 in Figure 1 for two traces $\mathbf{T}$ of length $\lambda_t = \omega$ and $\mathbf{H}$ of length $\lambda_h = 3$. Informally speaking, when we jump from H_i to H_{i+1} we allow trace $\mathbf{T}$ to make any finite number of additional transitions, but all those new states must be supersets of H_i. In other words, all those new T_j are allowed to include more information than H_i, but never to remove any atom that is true at H_i. When $\mathbf{H}$ is shorter than $\mathbf{T}$, as in the example, the last state in $\mathbf{H}$, in the example H_2, must be a subset of *all* the remaining states in $\mathbf{T}$, even if the latter is infinite.

To introduce the relation with THT, we notice that:

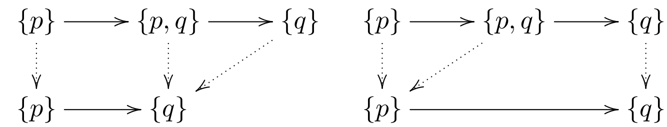

Figure 3: Two possible contractors to prove that $\mathbf{H} \prec \mathbf{T}$ for traces $\mathbf{H} = \{p\} \cdot \{q\}$ and $\mathbf{T} = \{p\} \cdot \{p, q\} \cdot \{q\}$.

Proposition 1 *The condition from Defn. 4 for* $\mathbf{T} \downarrow^{id} \mathbf{H}$ *amounts to:*

- $\lambda_h = \lambda_t$ *and, for all* $i \in \big[0..\lambda_h\big)$*,* $H_i \subseteq T_i$. ☐

In fact, the condition we obtained above with $\mu = id$ amounts to the ordering relation among traces used in standard TEL (Aguado et al. 2021). Moreover, we will also use the previous notation $\mathbf{H} \leq \mathbf{T}$ to mean $\mathbf{T} \downarrow^{id} \mathbf{H}$. Note that $\mathbf{H} \leq \mathbf{T}$ implies $\mathbf{H} \preceq \mathbf{T}$ but the opposite is not true, for instance, $\preceq$ allows now comparing traces of different lengths.

Proposition 2 $\preceq$ *is a preorder relation among traces.*

In other words, $\preceq$ is reflexive and transitive but, in general, anti-symmetry may not hold (at least, for pairs of infinite traces). As a counterexample, consider the traces $\mathbf{T} = \emptyset \cdot \emptyset \cdot \{a\}^\omega$ and $\mathbf{T}' = \emptyset \cdot \{a\}^\omega$. We can observe that $\mathbf{T}' \downarrow^{id} \mathbf{T}$ whereas we can also contract $\mathbf{T} \downarrow^\mu \mathbf{T}'$ using the function $\mu(0) = 0$ and $\mu(i) = i - 1$ for all $i > 0$. This means we have both $\mathbf{T} \preceq \mathbf{T}'$ and $\mathbf{T}' \preceq \mathbf{T}$ but $\mathbf{T} \neq \mathbf{T}'$. However, in the finite case, anti-symmetry holds, and we thus have:

Proposition 3 $\preceq$ *is an order relation among finite traces.*

In the sequel, we write $\mathbf{H} \prec \mathbf{T}$ if $\mathbf{H} \preceq \mathbf{T}$ and $\mathbf{H} \neq \mathbf{T}$. It must be observed that, given two traces $\mathbf{H} \prec \mathbf{T}$, we may have more than one contractor function μ for which $\mathbf{T} \downarrow^\mu \mathbf{H}$. As a simple example, take $\mathbf{T} = \{p\} \cdot \{p, q\} \cdot \{q\}$ and $\mathbf{H} = \{p\} \cdot \{q\}$. To prove that these two traces satisfy $\mathbf{H} \prec \mathbf{T}$ we can use contractor μ with $\mu(1) = \mu(2) = 1$ or contractor μ' with $\mu'(1) = 0$ and $\mu'(2) = 2$ as shown in Figure 3.

Definition 6 (cTHT-trace) *A* cTHT*-trace for alphabet* $\mathcal{A}$ *is a triple* $\langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *satisfying* $\mathbf{T} \downarrow^\mu \mathbf{H}$.

A cTHT-trace $\langle \mathbf{H}, \mathbf{T}, \mu \rangle$ is called *integral* when $\mu = id$ (there is no contraction) and *contracted* otherwise. An integral trace where we further have $\mathbf{H} = \mathbf{T}$ is said to be *total*.

Given a cTHT-trace $\langle \mathbf{H}, \mathbf{T}, \mu \rangle$, we call each $k \in \big[0..|\mathbf{H}|\big)$ a trace *step*. Moreover, step k is said to be *integral* when $\mu(k) = \{i\}$ is a singleton and when this happens, by abuse of notation, we may sometimes use $\mu^-(k)$ as a function denoting the element i. We say that step k is *contracted* when it is not integral, i.e., $|\mu^-(k)| > 1$. To put an example, for the contractor on the left of Figure 3, step 0 is integral because $\mu^-(0) = 0$ while step 1 is contracted as $\mu^-(1) = \{1, 2\}$. The opposite happens for the contractor on the right: in that case 0 is contracted $\mu^-(0) = \{0, 1\}$ and 1 is integral $\mu^-(1) = 2$.

We define next a particular kind of traces that we will consider later on.

Definition 7 (cTHT-satisfaction) *Let* $\mathbf{M}$ *be a* cTHT*-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *over alphabet* $\mathcal{A}$ *and let* $\lambda = |\mathbf{H}|$*. Then* $\mathbf{M}$ satisfies *a temporal formula* φ *at step* $k \in \big[0..\lambda\big)$*, written*

$\mathbf{M}, k \models \varphi$, *if the same conditions 1–8 as in Defn. 1 hold excepting 5 and 6, which are modified as follows:*

5. $\mathbf{M}, k \models \varphi \to \psi$ *iff both:*
   (a) $\mathbf{M}, k \not\models \varphi$ *or* $\mathbf{M}, k \models \psi$*;*
   (b) $\langle \mathbf{T}, \mathbf{T}, id \rangle, j \not\models \varphi$ *or* $\langle \mathbf{T}, \mathbf{T}, id \rangle, j \models \psi \;\; \forall j \in \mu^-(k)$
6. $\mathbf{M}, k \models \circ\varphi$ *iff* $|\mu^-(k)| = 1$*,* $k{+}1 < \lambda$*, and* $\mathbf{M}, k{+}1 \models \varphi$ □

The intuitive meaning of the condition for the next operator $\circ\varphi$ is that the current step k must not be a final state $k{+}1 < \lambda$, it must be integral ($|\mu^-(k)| = 1$) and φ must hold at $k{+}1$. The fact that k is integral means that H_k cannot be an "abbreviation" of a sequence of $\mathbf{T}$ states above. That is, to satisfy $\circ\varphi$ at k we force the existence of a state at $k{+}1$ (as usual) but also that the transition from k to $k{+}1$ is integral for $\mathbf{T}$. To put an example, $\mathbf{M}, 0 \models \circ q$ in the interpretation $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ corresponding to the left diagram of Fig. 3, but not in the right as the transition from 0 to 1 is contracting in that case. The reason for this restriction has to do with persistence (anything satisfied in $\mathbf{H}$ must be satisfied in $\mathbf{T}$ too) as we will prove later on. Informally speaking, in the right diagram, from H_0 we can see that q holds at the next state H_1. But when we move above, we should check $\circ q$ both at T_0 and T_1 as $\mu^-(0) = \{0, 1\}$. Here this holds, but if T_1 were $\{p\}$ instead, then T_0 does not satisfy $\circ q$ and we would have a case where $\circ q$ is true in $\mathbf{H}$ but not in $\mathbf{T}$. If the transition is integral, as in the left diagram, we can guarantee that T_1 satisfies q because it is restricted by $H_1 = \{q\}$.

Proposition 4 *Satisfaction* $\langle \mathbf{H}, \mathbf{T}, id \rangle, k \models \varphi$ *is equivalent to satisfaction* $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi$ *in* THT*.*

Corollary 1 $\langle \mathbf{T}, \mathbf{T}, id \rangle, k \models \varphi$ *is equivalent to* $\mathbf{T}, k \models \varphi$ *in* LTL*.*

Due to these results, we may replace an integral trace $\langle \mathbf{H}, \mathbf{T}, id \rangle$ by the THT-trace $\langle \mathbf{H}, \mathbf{T} \rangle$ and $\langle \mathbf{T}, \mathbf{T}, id \rangle$ by $\mathbf{T}$ when using them in satisfaction relations. Given an interval or set S of time steps, we will also write $\mathbf{T}, S \models \varphi$ to stand for $\mathbf{T}, j \models \varphi$ for all $j \in S$. Using this result and notation, we can replace item 7(b) in Definition 7 by the simpler condition:

7(b′) $\mathbf{T}, \mu^-(k) \models \varphi \to \psi$.

The following result lifts an essential property of THT to the contracted setting: that every formula that is satisfied by a THT-trace $\langle \mathbf{H}, \mathbf{T} \rangle$ must be satisfied by $\mathbf{T}$ viewed as LTL-interpretation. It reflects the intuitionistic view that when moving from a state $\mathbf{H}$ to a state $\mathbf{T}$ with more truth information, inferences made will be preserved.

Theorem 1 (Persistence) *For every* cTHT*-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *with* $\lambda = |\mathbf{H}|$ *and every* $k \in [0..\lambda)$*:* $\mathbf{M}, k \models \varphi$ *implies* $\mathbf{T}, \mu^-(k) \models \varphi$*.*

As in THT, the satisfaction of negation $\neg\varphi$ at point k amounts to an LTL check on the $\mathbf{T}$ component, but in this case, we must make that check on all the preimage points $\mu^-(k)$. Formally:

Proposition 5 *For every* cTHT*-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$*, formula* φ*, and position* k*,* $\mathbf{M}, k \models \neg\varphi$ *iff* $\mathbf{T}, \mu^-(k) \models \neg\varphi$ *(in LTL).*

As usual, given a temporal formula φ for alphabet $\mathcal{A}$, we write $\models \varphi$ to represent that φ is a *tautology*, that is, $\mathbf{M}, k \models \varphi$ for every cTHT-trace $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ over $\mathcal{A}$ and every $k \in [0..|H|)$.

Definition 8 (entailment/equivalence) *Let* φ *and* ψ *be two temporal formulas over alphabet* $\mathcal{A}$*. We say that* φ entails ψ*, written* $\varphi \models \psi$*, when* $\mathbf{M}, k \models \varphi$ *implies* $\mathbf{M}, k \models \psi$*, for any trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *over* $\mathcal{A}$ *and every* $k \in [0..|\mathbf{H}|)$*. We say that* φ *and* ψ *are* equivalent*, written* $\varphi \equiv \psi$*, when both* $\varphi \models \psi$ *and* $\psi \models \varphi$*.*

Proposition 6 $\varphi \models \psi$ *iff* $\models \varphi \to \psi$*.*

Corollary 2 $\varphi \equiv \psi$ *iff* $\models \varphi \leftrightarrow \psi$*.*

A cTHT-trace $\mathbf{M}$ is a *model* of a theory Γ if $\mathbf{M}, 0 \models \varphi$ for all $\varphi \in \Gamma$. The following property from LTL and THT also holds in cTHT (yet, it is known to be false once we introduce past operators).

Proposition 7 $\varphi \equiv \psi$ *iff* φ *and* ψ *have the same models.*

Proposition 8 *The semantics induced for derived operators is the following. Let* $\mathbf{M}$ *be a* cTHT*-trace* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *over alphabet* $\mathcal{A}$ *and let* $\lambda = |\mathbf{H}|$*.*

1. $\mathbf{M}, k \models \Diamond\varphi$ *iff* $\mathbf{M}, j \models \varphi$ *for some* $j \in [k..\lambda)$
2. $\mathbf{M}, k \models \Box\varphi$ *iff* $\mathbf{M}, j \models \varphi$ *for all* $j \in [k..\lambda)$
3. $\mathbf{M}, k \models \mathsf{F}$ *iff* $|\mu^-(k)| = 1$ *and* $k{+}1 = \lambda$
4. $\mathbf{M}, k \models \widehat{\circ}\varphi$ *iff* $|\mu^-(k)| = 1$ *and either* $k{+}1 = \lambda$ *or* $\mathbf{M}, k{+}1 \models \varphi$*.*

It is well-known that THT is a strictly weaker logic than LTL (Aguado et al. 2021), that is THT $\subset$ LTL. Proposition 4 allows proving that cTHT $\subseteq$ THT, namely, that any cTHT-tautology is also an THT-tautology. We may also observe that this relation is strict, cTHT $\subset$ THT. For instance, while $\widehat{\circ}\top \equiv \circ\top \vee \neg\circ\top$ is a tautology in LTL and in THT, it is not a tautology any more in cTHT. Indeed, from Proposition 8.4 we conclude that $\mathbf{M}, k \models \widehat{\circ}\top$ iff $|\mu^-(k)| = 1$, that is, satisfying $\widehat{\circ}\top$ at point k just means requiring that k is an integral transition. Thus, we can take any interpretation where k is contracting, such as $k = 0$ in Figure 2, to falsify $\widehat{\circ}\top$. Furthermore, including the axiom:

$$\Box\widehat{\circ}\top \qquad \textbf{(INT)}$$

forces all steps to be integral (no contraction), and so, $\mu = id$ collapsing to THT. In other words, cTHT+ **(INT)** = THT.

Similarly, we may also observe that the THT-equivalent formulas $\circ\top$ and $\neg\neg\circ\top (= \neg\mathsf{F})$ are not equivalent in cTHT either. While satisfying $\circ\top$ at k asserts that k is integral and jumps to a state $k{+}1$, $\neg\mathsf{F}$ just means the preimage of k does not contain the last position in $\mathbf{T}$.

Proposition 9 *Let* $\mathbf{M} = \langle \mathbf{H}, \mathbf{T}, \mu \rangle$ *be a* cTHT*-trace. Then,* $\mathbf{M}, k \models \neg\mathsf{F}$ *iff* $\max(\mu^-(k)) + 1 < |\mathbf{T}|$*.*

One important observation is that the non-temporal fragment of cTHT is actually weaker than HT. If we restrict to propositional connectives $\vee, \wedge, \to, \bot, \top$ and atoms, the satisfaction relation collapses to an intermediate logic whose Kripke models have the form of "forks", namely, one point (or world) H_0 that can see a group of worlds T_i for $i \in \mu^-(0)$

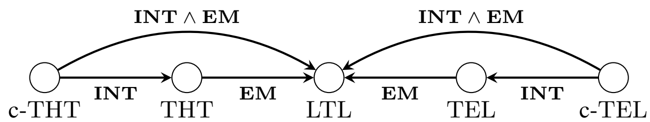

Figure 4: Relations between LTL, THT, and cTHT.

with no (intuitionistic) accessibility among them. This structure corresponds to the intermediate logic of Bounded Depth 2 (BD2), one of the seven interpolable intermediate logics (Maksimova 1977), like HT itself, although BD2 is also strictly weaker. For instance, BD2 does not satisfy the principle of *weak excluded middle* $\neg\varphi \vee \neg\neg\varphi$, which is an HT-tautology. In some sense, cTHT can be seen as one of the (possible) temporal extensions of BD2.

We have seen that axiom (**INT**) allows collapsing cTHT into THT. It is also known that THT collapses to LTL by the addition of the *(temporal) excluded middle axiom* (**EM**) scheme. We also introduce its weaker version (**WEM**):

$$\mathbf{EM} := \Box(p \vee \neg p) \tag{7}$$
$$\mathbf{WEM} := \Box(\neg\neg p \vee \neg p) \tag{8}$$

for every $p \in \mathcal{A}$. Figure 4 depicts some reductions among different logics obtained by the inclusion of axioms.

Although cTHT is strictly weaker than LTL, there are syntactic fragments on which LTL-equivalences are still applicable for cTHT. For instance:

Proposition 10 *Let φ, ψ be a pair of $\rightarrow$-free, $\circ$-free formulas. Then, the formula $\varphi \leftrightarrow \psi$ is a* cTHT*-tautology iff it is an* LTL*-tautology.*

As an illustration, the LTL-tautology

$$p \,\mathbf{U}\, \Diamond q \quad \leftrightarrow \quad \Diamond q \tag{9}$$

is also a cTHT-tautology because the formulas on the two sides of the double implication are $\rightarrow$-free and $\circ$-free. Note that we can still exploit this result to prove properties about formulas with $\circ$ or $\rightarrow$. To put an example, the formula

$$\circ\varphi \,\mathbf{U}\, \Diamond(\psi \rightarrow \gamma) \quad \leftrightarrow \quad \Diamond(\psi \rightarrow \gamma) \tag{10}$$

is still a cTHT-tautology because cTHT satisfies the law of uniform substitution, whose validity can be proved by contradiction, and we can replace p by $\circ\varphi$ and q by $(\psi \rightarrow \gamma)$ in (9) to obtain (10).

4 Contracted Temporal Equilibrium Logic

We are now introducing a selection criterion over cTHT models, which requires the nonexistence of a proper logical summarization. As pointed out in (Lamport 1983), standard temporal logics like LTL cannot express *possibilities* (or the absence of possibilities) over different behaviors; for that, a second-order logic is needed. TEL already provides a notion of possibility, in the sense that a selection criteria over models is employed, but its purpose is to simulate the stable semantics on a temporal setting only.

Definition 9 (cTEL) *A total* cTHT*-trace $\langle \mathbf{T}, \mathbf{T}, id\rangle$ is a* contracted temporal equilibrium model *(or* c-stable model*) of a theory Γ if it is a model of Γ (that is $\mathbf{T}, 0 \models \Gamma$ in* LTL*) and there is no model $\langle \mathbf{H}, \mathbf{T}, \mu\rangle$ of Γ with $\mathbf{H} \prec \mathbf{T}$.*

If we constrain μ in Definition 9 to identity id, then we obtain the definition of (integral) temporal equilibrium model as in (Aguado et al. 2013) for infinite and finite traces, which we call *stable models*. A first observation is then the following.

Theorem 2 *Any c-stable model of a theory Γ is also a (standard) stable model of Γ.*

Hence, the desidered condition (D1) is satisfied. To see how the semantics works, let us see some examples.

Example 2 *Consider the theory $\Gamma = \{\Diamond p\}$. Its stable models are the traces $\mathbf{T}$ with p at a single state, i.e., $\mathbf{T} = \emptyset^m \cdot \{p\} \cdot \emptyset^n$ with $m, n \geq 0$ and, possibly, $n = \omega$. Let us try to build an $\mathbf{H}$ and μ such that $\langle \mathbf{H}, \mathbf{T}, \mu\rangle \models \Diamond p$ and $\mathbf{H} \prec \mathbf{T}$.*

- *Consider $\mathbf{T} = \{p\}$. Then $|\mathbf{H}| = 1$ and $\mu = id$ must hold; as $\mathbf{T}$ is stable, $\mathbf{H} \prec \mathbf{T}$ is not possible, so $\mathbf{T}$ is c-stable.*
- *Consider $\mathbf{T} = \emptyset \cdot \{p\}$. Then $H_0 = \{p\}$ is not possible as $\mu(0) = 0$, which forces $H_0 \subseteq T_0 = \emptyset$. So, we must have $H_0 = \emptyset$; as $\langle \mathbf{H}, \mathbf{T}, \mu\rangle, 0 \models \Diamond p$ must hold, this forces $H_1 = \{p\}$, $\mu(1) = 1$ and so $\mathbf{H} = \mathbf{T}$. Again $\mathbf{T}$ is c-stable.*
- *Also trace $\mathbf{T} = \{p\} \cdot \emptyset$ is c-stable: $|\mathbf{H}| = 1$ would force $p \in H_0$ and $H_0 \subseteq T_1 = \emptyset$, while $|\mathbf{H}| = 1$ forces $\mu = id$; then $H_1 \subseteq T_1 = \emptyset$ and so $H_0 = \{p\}$ but then $\mathbf{H} = \mathbf{T}$.*
- *The trace $\mathbf{T} = \emptyset \cdot \{p\} \cdot \emptyset$ is c-stable: $|\mathbf{H}| = 3$ would force $\mu = id$, and as $\mathbf{T}$ is stable, $\mathbf{H} \prec \mathbf{T}$ is not possible. Otherwise, $\mu(2) = 1$ would force $H_1 = \emptyset$ thus $H_0 = \{p\} \subseteq T_0 = \emptyset$; $\mu(2) = 0$ would force $\mathbf{H} = H_0 = \emptyset$ but then $\langle \mathbf{H}, \mathbf{T}, \mu\rangle, 0 \models \Diamond p$ is not possible.*
- *Take any trace $\mathbf{T} = \emptyset^m \cdot \{p\} \cdot \emptyset^n$ where $m > 1$. Then, we can build $\mathbf{H} = \emptyset \cdot \{p\} \cdot \emptyset^n$ and use $\mu(i) = m + i - 1$ for all $i \geq 1$ for the model $\mathbf{H} \prec \mathbf{T}$, so $\mathbf{T}$ is not c-stable.*
- *Finally for any trace $\mathbf{T} = \emptyset^m \cdot \{p\} \cdot \emptyset^n$ with $n > 1$ we similarly build $\mathbf{H} = \emptyset^m \cdot \{p\} \cdot \emptyset$ and use $\mu(i) = i$ for all $i \in \big[0..m\big)$ (remember the last state forces $\mu(m+1) = \omega$) for the desired model $\mathbf{H} \prec \mathbf{T}$, so $\mathbf{T}$ is not c-stable.*

In conclusion, this theory has only four *c-stable models: $\{p\}$, $\{p\} \cdot \emptyset$, $\emptyset \cdot \{p\}$, and $\emptyset \cdot \{p\} \cdot \emptyset$, which are compactly represented with the regular expression as $\mathbf{T} = \emptyset^? \cdot \{p\} \cdot \emptyset^?$.* $\Box$

Example 3 *Consider the dual theory $\Gamma = \{\,\Box p\,\}$. Its stable models are $\mathbf{T} = \{p\}^\lambda$ where $\lambda \geq 1$ and possibly $\lambda = \omega$. The only c-stable stable model is $\mathbf{T} = \{p\}$: for any $\lambda > 1$ we can use $\mathbf{H} = \{p\}$ and readily show that $\mathbf{T}$ is not c-stable.*

Example 4 *Consider next the theory Γ with the formulas*

$$\Diamond p \tag{11}$$
$$\Box(p \rightarrow \circ p \vee q). \tag{12}$$

To satisfy (11)*, any stable model $\mathbf{T}$ must make p true at some point k, and by* (12) *p must be true forever, i.e., $T_i = \{p\}$ for $i \in \big[k..\lambda\big)$, or until both p and q are true at some point $k' \geq q$, i.e., $T_i = \{p\}$ for $i \in \big[k..k'\big)$ and $T_{k'} = \{p, q\}$. By the minimality condition of $\mathbf{T}$, no p or q can appear before k or after k', i.e., $T_i = \emptyset$ for $i \in \big[0..k\big)$ and $i \in \big[k'{+}1..\lambda\big)$, as we could make them all false in a smaller $\mathbf{H}$ for an* HT*-model; likewise, we could make all p at $i \in \big[k..\lambda\big)$ resp. $i \in \big[k..k'\big)$ false. Thus, $\mathbf{T}$ must be, written as regular expression, of the form $\mathbf{T} = \emptyset^* \cdot \{p, q\} \cdot (\emptyset^* + \emptyset^\omega)$.*

It is not hard to check that any nonempty subsequence $\emptyset^k$ in $\mathbf{T}$*, including $\emptyset^\omega$, can be contracted into $\emptyset$, leading to four c-stable models: $\{p,q\}$, $\emptyset \cdot \{p,q\}$, $\{p,q\} \cdot \emptyset$, and $\emptyset \cdot \{p,q\} \cdot \emptyset$, which are compactly represented as $\mathbf{T} = \emptyset^? \cdot \{p,q\} \cdot \emptyset^?$.* $\square$

Example 5 *Let then $\Gamma = \{\, \Diamond(p \wedge \Box(p \rightarrow \circ p \vee q)) \,\}$. In* LTL*, this theory is weaker than Γ in Example 4, which we rename to Γ', as* (12) *is nested into* (11)*; thus we have $\Gamma' \models_{\mathrm{LTL}} \Gamma$; the same holds in* THT *and in c*THT.

Consequently, any stable model $\mathbf{T}$ of Γ must be a stable model of Γ'. Conversely, any stable model $\mathbf{T}$ of Γ' is an LTL *model of Γ, and by its particular form, we can not form a smaller $\mathbf{H}$ such that $\langle \mathbf{H}, \mathbf{T} \rangle \models \Gamma$; hence $\mathbf{T}$ is also a stable model of Γ. This likewise holds for c-stable models. Thus Γ and Γ' have the same stable resp. c-stable models.* $\square$

We next consider occurrence of negation.

Example 6 *Let $\Gamma = \{\Box(\neg p \vee p)\}$. As* THT *plus* **EM** *collapses to* LTL*, any trace is a stable model of Γ that intuitively represents a choice for p or $\neg p$ at each point.*

To see which of them are c-stable, whenever we have a state repetition in $\mathbf{T}$, i.e., $T_i = T_{i+1}$ for some $i \geq 0$, then we can contract $\mathbf{T}$ to $\mathbf{T}'$ by leaving out T_{i+1}, i.e., we let $\mu(j) = j$ for $j \in [0..i]$ and $\mu(j) = j-1$ for $j \in [i..|\mathbf{T}|)$, and obtain $\langle \mathbf{T}', \mathbf{T}.\mu \rangle \models \Gamma$; if all T_j, $j > i$, are the same, we can also set $\mu(j) = i$. As this can be repeated, the c-stable models of Γ are all traces that alternate between $\{p\}$ and $\emptyset$; formally, they are captured by the regular expression

$$\emptyset \cdot (\{p\} \cdot \emptyset)^* \cdot \{p\}^? + \{p\} \cdot (\emptyset \cdot \{p\})^* \cdot \emptyset^? + (\emptyset \cdot \{p\})^\omega + (\{p\} \cdot \emptyset)^\omega$$

$\square$

In the examples above, repeated states have been eliminated, as desired by condition (D2). Clearly, this is not always possible.

Example 7 *Take $\Gamma = \{p, \circ p\}$. Its stable models are $\mathbf{T} = \{p\} \cdot \{p\}(\emptyset^* + \emptyset^\omega)$, and the formula $\circ p$ forces any contraction μ to be integral at step 0; the c-stable models are $\{p\} \cdot \{p\}$ and $\{p\} \cdot \{p\} \cdot \emptyset$.*

The $\circ$-operator can be seen as a way to state that a given *transition cannot be contracted*. Thus, when a $\circ$-formula is derived, we may have repetitions of states that cannot be removed in c-stable models; as we shall see in the next section, we can safely remove stuttering when we deal with $\circ$-free formulas, so (D2) will be satisfied.

Regarding (D3), we readily obtain from the discussion about LTL, THT and cTHT in the previous section that $c\mathrm{TEL}+(\mathbf{INT}) = \mathrm{TEL}$ and $\mathrm{TEL}+(\mathbf{EM}) = \mathrm{LTL}$, completing the diagram in Figure 4.

5 Characterising c-Stable Models

We notice that c-stable models are ω-regular languages, which follows from an automata construction for deciding the satisfiability problem in cTEL. Intuitively, this can be already seen from Defn. 9, as we need to produce an LTL automaton for the *guess* $\mathbf{T}$, and a THT automaton for producing a *defeater* $\mathbf{H}$ and μ. After completing the defeater automaton, we project away all the atoms referring to $\mathbf{H}$ and μ, and we compute the intersection of the two automata. All the above mentioned automata operators are closed under ω-regular languages (Büchi 1960).

We further note that for $\circ$-free formulas, an alphabet of size at least 2 is needed for having an aperiodic c-stable trace. For instance, the trace $\mathbf{T} = \emptyset \cdot a \cdot \emptyset \cdot ab \cdot \emptyset \cdot (ab)^2 \cdot \emptyset \cdot (ab)^3 \cdot \ldots$ is a c-stable trace for the formula $\Box((a \vee \neg a) \wedge (b \vee \neg b))$.

5.1 $\circ$-free Formulas

We now turn our attention to $\circ$-free formulas, i.e., formulas without the $\circ$-operator. The absence of the intricate semantic behavior of the latter allows us to identify sufficient conditions for the existence of c-stable models as well as characterizations for classes of $\circ$-free theories, and under restricted contractions for all such theories.

A key notion for this endeavor is stuttering of traces.

Definition 10 (Stuttering) *A trace $\mathbf{T}$ is a* stuttering *of a trace $\mathbf{T}'$ if $\mathbf{T} \downarrow^\mu \mathbf{T}'$ for some μ such that $T_i = T'_{\mu(i)}$, for all $i \in [0..|\mathbf{T}|)$; it is* proper *if, in addition, $\mathbf{T} \neq \mathbf{T}'$.*

That is, in a stuttering the same state is repeated, possibly multiple times or even infinitely often; properness ensures that $\mathbf{T}$ must have some repetition that is not in $\mathbf{T}'$.

Let us consider what happens when we "pump" LTL and THT models of a set Γ of $\circ$-free formulas.

Lemma 1 (Stutter Equivalences) *Suppose $\mathbf{T}$ (resp. $\mathbf{H}$) is a stuttering of $\mathbf{T}'$ (resp. $\mathbf{H}'$) via contraction μ. If φ is $\circ$-free, then for each $j \in [0, \lambda')$,*

1. *$\mathbf{T}', j \models \varphi$ iff $\mathbf{T}, \mu^{-1}(j) \models \varphi$, and*
2. *$\langle \mathbf{H}', \mathbf{T}' \rangle, j \models \varphi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, \mu^{-1}(j) \models \varphi$.*

Item 1 of Lemma 1 is a well-known result of the LTL $\circ$-free fragment, while 2 is an immediate generalization of 1. Notably, we can summarize stuttered intervals in the There-trace using contractors while preserving THT satisfaction:

Proposition 11 (T-stutter Equivalence) *Let $\mathbf{M} = \langle \mathbf{T}', \mathbf{T}, \mu \rangle$ where $T_i = T_j$ for every $i, j \in \mu^-(k)$ and $k \in [0, |\mathbf{T}'|)$, and let $\mathbf{H} \downarrow^\mu \mathbf{T}'$ be a stuttering of $\mathbf{T}'$. If φ is a $\circ$-free formula, then for each $k \in [0, |\mathbf{T}|)$, we have $\mathbf{M}, \mu(k) \models \varphi$ iff $\langle \mathbf{H}, \mathbf{T} \rangle, k \models \varphi$.*

Proposition 11 allows us to link summarisation inference to ordinary HT-inference. Armed with this proposition, we then show the following property. Let us call a trace $\mathbf{T}$ *stutter-free*, if it is not a proper stuttering of any sequence $\mathbf{T}'$.

Proposition 12 *For any set Γ of $\circ$-free formulas, $\mathbf{T}$ is a c-stable model of Γ only if $\mathbf{T}$ is a stable model of Γ such that $T_i \neq T_{i+1}$ for all $i \in [0, \lambda)$, i.e., $\mathbf{T}$ is stutter-free.*

Thus, stutter-freeness is a necessary condition for c-stability in the absence of the $\circ$-operator. On the other hand this condition is not sufficient in general, as shown by the following example.

Example 8 *Consider the theory $\Gamma = \{\neg p \rightarrow \Diamond p\}$, which has the stable model $\mathbf{T} = \emptyset \cdot \{p\}$: at $i = 0$, p is false and thus p must be true at $i = 1$. However, while $\mathbf{T}$ is stutter-free, it is not a c-stable model. Indeed, for $\mathbf{M} = \langle \mathbf{T}', \mathbf{T}, \mu \rangle$ where $T' = \emptyset$ and $\mu(0) = \mu(1) = 0$, we have $\mathbf{M} \models \Gamma$ as $\mathbf{M} \not\models \neg p$ and $\mathbf{T} \models \Diamond p$. Intuitively, contraction of $\mathbf{T}$ into $\mathbf{T}'$ affects stability as the antecedent $\neg p$ is no longer provable.* $\square$

To obtain a fragment of TEL for which stutter-freeness is a sufficient condition for summarization, we thus have to impose some restrictions. This intuitively regards negation respectively implication, as summarization does not affect provability of formulas without implication, which we call *positive formulas*. As it turns out, by excluding nested implication we achieve this goal.

Let THT^1 denote the class of all formulas without nested implication. We then obtain the following result, which informally generalizes the only-if direction of Proposition 11 for THT^1 theories.

Proposition 13 *Let* $\mathbf{M} = \langle \mathbf{T}', \mathbf{T}, \mu \rangle$ *and* $\mathbf{H} \downarrow^{\mu} \mathbf{T}'$ *be a stuttering of* $\mathbf{T}'$. *If* φ *is a* $\circ$*-free formula from* THT^1*, then for each* $k \in [0, \lambda')$, $\mathbf{M}, k \models \varphi$ *implies* $\langle \mathbf{H}, \mathbf{T} \rangle, \mu^{-}(k) \models \varphi$.

The converse direction does not hold, as shown by $\mathbf{T}' = \emptyset$, $\mathbf{T} = \{p\}\cdot\emptyset$, μ as obvious, $\mathbf{H} = \emptyset\cdot\emptyset$, and $\Gamma = \{\Diamond(p \vee \neg p)\}$.

From Proposition 13, we obtain the converse of Proposition 12 for THT^1:

Proposition 14 *Suppose* Γ *is a set of* $\circ$*-free formulas from* THT^1. *Then* $\mathbf{T}$ *is a c-stable model of* Γ *if* $\mathbf{T}$ *is a stable model of* Γ *such that* $T_i \neq T_{i+1}$ *for all* $i \in [0, \lambda)$*, i.e.,* $\mathbf{T}$ *is not a proper stuttering of any sequence* $\mathbf{T}'$.

From Propositions 12 and 14, we then obtain the characterization of the c-stable models in terms of stable models.

Theorem 3 *For any* $\circ$*-free* THT^1 *theory* Γ*, (i) the c-stable models coincide with the stutter-free stable models; (ii) a c-stable model exists iff a stable model exists; and (iii) every stable model becomes c-stable by removing all repetitions.*

Revisiting Example 6, we see that the c-stable models of Γ are captured by the characterization in Theorem 3.

Even $\circ$-free THT^1 formulas allows us to enforce infinite models sensitive to c-stability.

Example 9 *By adding the formula* $\Box(\circ\top)$ *to the theory* $\Gamma = \{\Box(\neg p \vee p)\}$ *in Example 6, as well as to any theory, all models of* Γ *must be infinite, and only the two infinite traces*

$$(\emptyset \cdot \{p\})^{\omega} \text{ and } (\{p\} \cdot \emptyset)^{\omega})$$

would remain as c-stable models.

Infinite models may be also enforced by adding the formula

$$\Box((q \vee \neg q) \wedge \Diamond q \wedge \Diamond \neg q) \tag{13}$$

where is q *is an auxiliary atom. While* $\Box(\circ\top)$ *restricts any mapping* $\mathbf{T} \downarrow^{\mu} \mathbf{T}'$ *between traces to identity (*$\mu = id$*) and thus c-stability falls back to stability, formula (13) preserves the full mappability.*

We note that Theorem 3 can be extended to non-THT^1 theories by imposing syntactic conditions. In particular, it continues to hold if Γ contains for each non-THT^1 formula φ in Γ also $(\mathbf{EM})$ for each variable p that occurs in φ; this enforces totality of models on the variables $\mathcal{A}_{\varphi}$ in φ, and thus the condition in Proposition 11 applies relative to $\mathcal{A}_{\varphi}$. As LTL equals $\mathrm{THT} + (\mathbf{EM})$ (cf. Figure 4), the stutter-free LTL-models of any $\circ$-free theory Γ are thus characterized by the c-stable models of $\Gamma \cup (\mathbf{EM})$, i.e., by the contractions of the stable models of Γ under classical semantics.

We further remark that $(\mathbf{EM})$ also belong to the syntactic fragment THT^1 whereas its inclusion captures full LTL: we can convert each formula φ in LTL into negation normal form in polynomial , by rewriting implication to disjunction and moving negations inside formulas – in particular, using $\neg(\varphi \,\mathbf{U}\, \psi) \equiv \neg\varphi \,\mathbf{R}\, \neg\psi$ and $\neg(\varphi \,\mathbf{R}\, \psi) \equiv \neg\varphi \,\mathbf{U}\, \neg\psi$ in LTL, where double negation cancels.

5.2 GP-formulas

While the class THT^1 excludes nested implication, it is still expressive and allows for encoding EXPSPACE-complete problems, even if $\Diamond$ and $\Box$ are the only temporal operators available (Bozzelli and Pearce 2015). In particular, it embraces theories that include statements of the form

$$\alpha_1 \wedge \cdots \wedge \alpha_m \rightarrow \beta_1 \vee \cdots \vee \beta_n, \tag{14}$$

$$\Box(\alpha_1 \wedge \cdots \wedge \alpha_m \rightarrow \beta_1 \vee \cdots \vee \beta_n) \tag{15}$$

$$\Box(\Box\alpha_1 \rightarrow \beta_1) \tag{16}$$

$$\Box(\alpha_1 \rightarrow \Diamond\beta_1) \tag{17}$$

$$\alpha_1 \vee \neg\alpha_1 \tag{18}$$

$m, n \geq 0$, where all α_i and β_j are positive formulas. They are rules of temporal logic programs (Aguado et al. 2021) if the formulas are atoms in (16)-(17) and atoms or formulas $\circ p$ in (14)-(15) where p is an atom. For $m = 0$ these are (disjunctive) facts, and for $n = 0$ constraints where the consequent is $\bot$). The formula (18) is a *guessing rule* which makes in stable semantics α_1 either true or false, which then leads to two different scenarios reflected in different stable models (so they exist).

It is well-known that for *positive disjunctive logic programs*, which are sets of formulas (14) where all α_i and β_j are atoms, the stable models coincide with the $\subseteq$-minimal (in short, minimal) models. We now present the class GP (standing for *Generalized Positive*) of formulas with an analogous property for theories Γ over this class. The c-stable models of $\circ$-free Γ theories are then the minimal stutter-free models of Γ; furthermore, for no different c-stable models $\mathbf{T}'$ and $\mathbf{T}$ of Γ, we can have $\mathbf{T}' \prec \mathbf{T}$ (which for arbitrary $\circ$-free theories is possible, cf. Example 6).

Definition 11 (GP formulas) *The class* GP *consists of all* THT^1 *formulas* φ *where each subformula* $\varphi_1 \vee \varphi_2$, $\varphi_1 \,\mathbf{U}\, \varphi_2$, *or* $\varphi_1 \,\mathbf{R}\, \varphi_2$ *of* φ *is positive unless* $\varphi_1 = \bot$.

Clearly GP properly generalizes positive formulas; e.g. $p \rightarrow q$ is positive and an admissible GP formula, but $\Box(p \wedge \Diamond r \rightarrow q \vee \Box s)$ is only admissible GP. The restriction on $\mathbf{U}$ and $\mathbf{R}$ subformulas mirrors the restriction on disjunction $\vee$, since both operators involve temporal disjunction. i.e., over time instants. Exempting the case $\varphi_1 = \bot$ means that $\varphi_1 \vee \varphi_2$ amounts to φ_2, which then simply must be from THT^1, $\varphi_1 \,\mathbf{U}\, \varphi_2$ amounts similarly to φ_2, and $\varphi_1 \,\mathbf{R}\, \varphi_2$ amounts to $\Box\varphi_2$, which intuitively is a temporal conjunction of φ_2 over the timeline.

Notably, positive (negation-free) temporal logic programs with rules of the form (14)–(17) fall into the class GP, and several examples considered above involve GP formulas.

Furthermore, with $\Box\circ\top$ but also with $\circ$-free GP formulas we may enforce infinite LTL models, such as with $\Box((p \rightarrow$

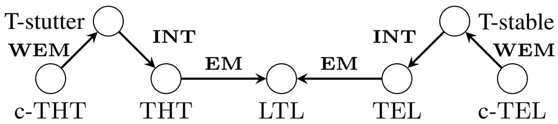

Figure 5: T-stuttering and T-stable semantics vs. the other semantics.

$\Diamond q) \wedge (q \rightarrow \Diamond p))$; by further adding $\Box((p \vee q) \wedge (p \wedge q \rightarrow \bot))$, each infinite stable trace gives rise to some c-stable model.

Based on Theorem 3, we then obtain the following characterization of c-stable models.

Theorem 4 *For any theory Γ of $\circ$-free* GP *formulas, the following conditions are equivalent:*

(1) $\mathbf{T}$ *is a c-stable model of* Γ*;*

(2) $\mathbf{T}$ *is a stutter-free $\subseteq$-minimal model of* Γ*, i.e., $T_i \neq T_{i+1}$ for all $i \in [0, \lambda)$;*

(3) $\mathbf{T}$ *is a ($\subseteq$-minimal) model of* Γ *and no model* $\mathbf{T}' \prec \mathbf{T}$ *of* Γ *exists.*

We remark that Theorem 4 does not hold for THT^1 formulas, which is easily witnessed by $\Gamma = \{p \vee \neg p\}$ and $\mathbf{T} = \{p\}$: $\mathbf{T}$ is a c-stable model but the conditions (2) and (3) are not satisfied. As this example shows, requiring in (2) and (3) that $\mathbf{T}$ is an stable model would not change this. However, the conditions (2) and (3) are sufficient for c-stability.

Proposition 15 *For any $\circ$-free theory Γ of THT^1 formulas, $\mathbf{T}$ is a c-stable model of Γ if (1) $\mathbf{T}$ is a stutter-free $\subseteq$-minimal model of Γ, or (2) $\mathbf{T}$ is a model of Γ and no model $\mathbf{T}' \prec \mathbf{T}$ of Γ exists.*

5.3 Taming Summarization

As we illustrated with Example 8, if our theory is not in the fragment THT^1, stutter-free stable models may not c-stable, even in the absence of the next-operator. An intuitive explanation for this is that arbitrary contractions can be aggressive and compromise stability of formulas, as negation has to be evaluated over a segment in the There-trace. Specifically, this happens in Example 8 for the contraction of the stable model $\mathbf{T} = \emptyset \cdot \{p\}$ of $\Gamma = \{\neg p \rightarrow \Diamond p, \neg p \rightarrow \circ p\}$ to $\mathbf{T}' = \emptyset$, where the antecedent $\neg p$ of the implications has to be evaluated over $\emptyset \cdot \{p\}$, while for stability, it is only evaluated over $T_0 = \emptyset$.

Similarly, aggressive summarization may eliminate stable models if a change of axioms should only affect local stability, as for the theories $\Gamma_1 = \{\Box(p \vee q)\}$ and $\Gamma_2 = \{\Box(\neg p \rightarrow q), \Box(\neg q \rightarrow p)\}$, which have the same stable models. However, while the c-stable models of Γ_1 are its (infinitely many) stutter-free stable models, which are all the finite and infinite traces $\mathbf{T}$ that alternate between $\{p\}$ and $\{q\}$, Γ_2 has only $\{p\}$ and $\{q\}$ as c-stable models: each different stable model $\mathbf{T}$ can be contracted by μ to $\mathbf{T}' = \emptyset$, for which $\langle \mathbf{T}', \mathbf{T}, \mu \rangle \models \Gamma_2$ holds.

Aggressive summarization can be avoided by restricting contractions. In particular, the elimination of repetitions, which is a necessary feature of c-stable models, would be sufficient. To this end, we introduce the following notion.

Definition 12 (T-stutter,T-stable model) *We call a* cTHT-*trace $\langle \mathbf{H}, \mathbf{T}, \mu \rangle$* T-stutter, *if $T_i = T_j$ whenever $\mu(i) = \mu(j)$, for every $i, j \in [0..|\mathbf{T}|)$. Furthermore, a trace $\mathbf{T}$ is a* T-stable model *of a theory Γ if $\mathbf{T} \models \Gamma$ and no T-stutter $\mathbf{M} = \langle \mathbf{T}', \mathbf{T}, \mu \rangle$ exists such that $\mathbf{M} \models \Gamma$ and $\mathbf{T}' \neq \mathbf{T}$.*

The less aggressive summarization allows us to recover all the $\circ$-free THT tautologies.

Proposition 16 *Let φ, ψ be a pair of $\circ$-free formulas. Then, $\varphi \leftrightarrow \psi$ is a T-stuttering tautology iff it is a* LTL*-tautology.*

In Example 8, we thus can't contract $\mathbf{T} = \emptyset \cdot \{p\}$ to $\mathbf{T}'{=}\emptyset$, and $\mathbf{T}$ is T-stable; similarly, no stable model $\mathbf{T}$ of Γ_2 with alternating $\{p\}$ and $\{q\}$ can be contracted to any trace $\mathbf{T}' \neq \mathbf{T}$ such that $\mathbf{M} = \langle \mathbf{T}', \mathbf{T}, \mu \rangle$ is T-stutter and $\mathbf{M} \models \Gamma_2$. Clearly,

Proposition 17 *Every c-stable model $\mathbf{T}$ of a theory Γ is a T-stable model of Γ.*

It is not hard to see that we can constrain models $\mathbf{M}$ to be T-stutter by the temporal weak excluded middle axiom. We thus obtain the following characterization.

Theorem 5 *For any theory Γ, the T-stable models of Γ coincide with the c-stable models of $\Gamma \cup (\mathbf{WEM})$.*

The refined picture of semantics is shown in Figure 5. Many of the results for c-stable models of THT^1 theories similarly hold for T-stable models. The main result is an analogon of Theorem 3 for all $\circ$-free theories.

Theorem 6 *For any $\circ$-free theory Γ, (i) the T-stable models coincide with the stutter-free stable models; (ii) some T-stable model exists iff some stable model exists; and (iii) every stable model becomes T-stable by removing all repetitions.*

In other words, the T-stable models capture the stutter-free stable models of any $\circ$-free theory. Furthermore,

Corollary 3 *For any $\circ$-free THT^1 theory Γ, the T-stable models of Γ coincide with the c-stable models of Γ.*

As a final remark, when we consider the non-temporal fragment of cTHT under T-stutter traces, it is easy to see that we return back to the HT intermediate logic – although the current world in the BD2 structure may still see multiple accessible worlds in $\mathbf{T}$, all of them have the same valuation of atoms and collapse to a single point.

6 Computational Complexity and Reasoning

The satisfiability problem for LTL is well-known to be PSPACE-complete, while Bozzelli and Pearce (2015) showed that for TEL it is EXPSPACE-complete. They gave a detailed complexity picture for syntactic fragments $\mathrm{THT}^n_m(Op_1, \ldots, Op_k)$, where n is the implication depth, m the temporal operator depth, and (optional) the $Op_1, \ldots, Op_k$ are the admitted temporal operators, focusing on infinite traces but with corollaries for arbitrary and finite traces. We present the following novel results ('c' stands for 'complete').

Theorem 7 (TEL **complexity)** *Deciding* TEL *satisfiability of a theory, i.e., whether it has some stable model, from*

- $\mathrm{GP}_2(\Box, \Diamond)$ *is EXPSPACE-c for infinite traces;*
- $\mathrm{THT}^2_2(\mathsf{U})$ *is EXPSPACE-c for finite and for infinite traces.*
- $\mathrm{THT}^1_2(\circ, \Box, \Diamond)$ *is PSPACE-hard for finite traces, and* $\mathrm{THT}^1_2(\circ, \Box, \Diamond, \widehat{\circ}, \mathsf{F})$ *is in PSPACE for finite traces.*

- $\mathrm{THT}^1_2(\Box, \Diamond, \mathbf{F})$ *is NP-c for finite traces.*

Notably, as a corollary we get that deciding TEL satisfiability of $\neg$-free (and $\circ$-free) temporal programs from (Cabalar and Schaub 2019) is PSPACE-c (is NP-c), while it remains EXPSPACE-c in the general case.

For c-stable semantics, we easily obtain from our discussion about enforcing infinite traces (see Section 5) and the results in (Bozzelli and Pearce 2015) some lower bounds, as we can block non-id mappings by adding the formula $\Box\circ\top$.

Lemma 2 *Deciding whether a given theory Γ has some c-stable model is*

1. *EXPSPACE-hard in general and for $\mathrm{THT}^1_{m+1}(\Diamond, \Box)$, $\mathrm{GP}_2(\Diamond, \Box, \circ)$, and $\mathrm{THT}^2_2(\mathbf{U})$ and*
2. *PSPACE-hard for $\mathrm{THT}^1(\circ, \mathbf{R})$ and THT^0.*

The general case has a matching upper bound, which can be shown using automata-theoretic techniques, similar as for the result that satisfiability for TEL is in EXPSPACE. Hence, cTEL does not have higher complexity than TEL.

Theorem 8 (cTEL-complexity) *Deciding cTEL satisfiability of a theory Γ, i.e., whether Γ has a c-stable model, is EXPSPACE-complete.*

Combining our results in Section 5 with results in (Bozzelli and Pearce 2015), we obtain some upper bounds for infinite models (remind the non-duality between **U** and **R** in TEL):

Proposition 18 *Let $\mathbf{INF} =_{\mathrm{def}} \Box\Diamond\circ\top$ denote an operator expressing infinity. Then deciding whether a given theory Γ has an infinite c-stable model is*

1. *in PSPACE for $\mathrm{THT}^1(\mathbf{R})$ and $\mathrm{THT}^1(\mathbf{U}, \mathbf{INF})$, and*
2. *in Σ^p_2 for $\mathrm{THT}(\Diamond, \mathbf{INF})$.*

Notably, in many cases, the existence of a stable model for a formula φ is, as shown by (Bozzelli and Pearce 2015), equivalent to the existence of a stable model $\mathbf{T}$ that is *strongly ultimately periodic*, i.e., that some $j \geq k$ exists such that $T_i = T_j$ for all $i \geq j$, where j is bounded in the size of the formula φ. For c-stability, the ultimate periodic part may either remain or be reduced to a finite suffix of the trace; thus, finite c-stable model existence is covered as well.

A detailed study of the complexity of cTHT and cTEL is beyond this paper. We remark, however, that are also low complexity fragments of cTEL semantics. In particular,

Proposition 19 *Deciding whether a given $\circ$-free theory Γ has some c-stable model is NP-complete for THT^1_1 and GP_1.*

This holds as some c-stable model exists in case of THT^1_1 (which includes GP_1) by Theorem 3 iff some stable model exists iff some infinite stable model exists (as states can be stuttered), which is NP-complete to decide (Bozzelli and Pearce 2015). The NP-hardness holds for GP_1, as it includes positive disjunctive logic programs (empty rule heads permitted), for which deciding stable model existence is well-known to be NP-complete (Eiter and Gottlob 1995). Proposition 19 thus shows that the benign complexity of a major class of logic programs extends to a meaningful temporal analogue.

6.1 Reasoning

Theorem 6 can be exploited to obtain the T-stable models of Γ from the stable models of an extension of Γ.

Let *diff* be a fresh atom and let Γ_{diff} be defined as

$$\Gamma_{\mathit{diff}} = \Gamma \cup \{\, \mathit{diff},\ \Box(\neg \mathit{diff} \to \bot)\} \cup \{\Box((p \wedge \circ\neg p) \vee (\neg p \wedge \circ p) \to \circ \mathit{diff}) \mid p \in \mathcal{A}\}.$$

That is, *diff* must be derived at each position; it is a fact at position 0, but at later positions can be derived iff adjacent positions are different. In case $\mathcal{A}$ or Γ is finite, we can also eliminate the auxiliary atom. Formally, we obtain:

Proposition 20 *The T-stable models $\mathbf{T}$ of any theory Γ of $\circ$-free THT^1 formulas correspond 1-1 to the stable models $\mathbf{T}'$ of Γ_{diff}, where $T'_i = T_i \cup \{\mathit{diff}\}$ for all $i \in [0, \lambda)$.*

From Theorem 6, We obtain that inference from the T-stable models of Γ is captured by inference from the stable models of Γ_{diff}. Furthermore, by Corollary 1, for $\circ$-free formulas, inference from the stable models is preserved:

Theorem 9 *Let Γ be a $\circ$-free theory and φ be a formula over $\mathcal{A}$. Then the following conditions are equivalent:*

(1) $\mathbf{T} \models \varphi$ *for every / some T-stable model $\mathbf{T}$ of Γ;*
(2) $\mathbf{T} \models \varphi$ *for every / some stable model $\mathbf{T}$ of Γ_{diff};*
(3) $\mathbf{T} \models \varphi$ *for every / some stable model $\mathbf{T}$ of Γ, if φ is $\circ$-free.*

Since by Corollary 3 the T-stable models of $\circ$-free THT^1 theories coincide with the c-stable models, we obtain for this syntactic fragment an analogous result to Theorem 9 with with c-stable models in place of T-stable models.

7 Applications

We briefly discuss two applications of contractions. The first one is a query-based application. Let TS be the transition system depicted in Figure 6, representing an environment model. Given a query $\varphi = \Box(p_1 \vee p_0)$, we may be interested in the stable models of φ which are consistent with the possible evolutions in TS. The respective c-stable models are $\mathbf{T}_1 = \{p_1\}$ and $\mathbf{T}_2 = \{p_1\}\cdot\{p_0\}\cdot\{p_1\}$, which represent two qualitatively different fulfillments of φ consistent with TS. Applying the *shortest* selection criterion to the possible evolutions as in (Schuppan and Biere 2005) would single out $\mathbf{T}_1$; for a set of samples that cover all qualitative-different evolutions, $\mathbf{T}_2$ should also be selected. This feature may be exploited in Runtime Verification (Falcone, Havelund, and Reger 2013; Leucker and Schallhart 2009), where given a prefix with all transitions integral, a complete sample of different possible evolutions can be provided; a starting point may be progression-based TEL monitoring (Soldà et al. 2023).

At the same time, given a finite trace $\mathbf{T}$, we may want a stable trace $\mathbf{T}'$ of φ such that $\langle \mathbf{T}', \mathbf{T}, \mu\rangle \models \varphi$ for a contraction μ, where φ is a formula that concerns only some aspects of the examined behavior. Suppose $\mathbf{T} = \{p, q\}\cdot\{p, r\}\cdot\{q, s\}\cdot\{r, s\}\cdot\{q\}$ and that we focus on $\varphi = p\mathbf{U}s$. The unique c-stable model $\mathbf{T}'$ such that $\langle \mathbf{T}', \mathbf{T}, \mu\rangle \models \varphi$ is $\mathbf{T}' = \{p\}\cdot\{s\}\cdot\emptyset$. Note that the trace $\mathbf{T}$ could be also infinite, but finitely represented in a prefix plus loop form. Therefore, if φ represents a set of faulty behaviors, in this way, we can obtain a logical succinct representation of them.

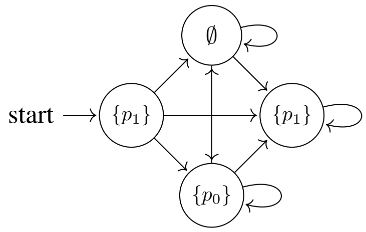

Figure 6: Transition System TS

8 Related Work and Conclusion

As mentioned in the Introduction, other works such as (Schuppan and Biere 2005; Dodaro, Fionda, and Greco 2022) aim at cost-based trace selection in LTL, using length or weighted atoms. Our setting relies on a non-monotonic logic, and contraction may preserve patterns that are eliminated by cost based selection, e.g. for $\Gamma = \{\, \Box(p \vee \neg p) \}$.

Stutter-invariance of $\circ$-free formulas in LTL is widely used, and our results generalize it to cTEL thanks to the recoverage of LTL. Extensions to nesting of $\circ$ and patterns in LTL have been studied, cf. (Kucera and Strejcek 2002), which would be interesting to explore for cTEL as well.

In planning, some approaches render simplified plans, such as CEGAR planning, (Seipp and Helmert 2013) or hierarchical planning, where macro-actions are composed of concrete actions. However, both are different from summarization in c-stable models: the former focuses on sets of states whereas the latter has an I/O flavor, disregarding intermediate states.

Another related line is the HyperLTL extensions of LTL using sets of traces for modeling concurrent processes. This was recently enriched with control of moving/stuttering traces (Baumeister et al. 2021) and lockstepwise traversal of subtraces removing "redundant" positions HyperLTL (Bozzelli, Peron, and Sánchez 2021). While our contraction establishes some asynchronous relationship between traces, it aims to support non-monotonic inference rather than to control execution traces; possible connections remain for study.

Outlook Our core work can be continued in several directions. Regarding logic and semantics, normal forms or equivalence in cTHT and cTEL may be investigated. Further characterizations of the c-stable models, in particular in the presence of the next-operator, are an intriguing issue.

For computation, refining the complexity picture is suggestive and will help in guiding the development of suitable algorithms and implementations, especially for finite traces, where existing solvers such as telingo may be used.

Finally, we also plan to explore the application of c-stable models in the context of planning and explanation finding, such as for constructing plans or counterexamples.

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