From Qualitative Choice Logic to Abstract Argumentation

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Abstract

Qualitative Choice Logic (QCL) extends classical propositional formulas by a connective called ordered disjunction that is used to express preferences. We translate QCL theories to Argumentation Frameworks with Collective Attacks (SETAFs), and show that the preferred models of the original theory directly correspond to the semi-stable extensions of the target framework. This further allows us to decide the problem of preferred model entailment for QCL via SETAFs.

1 Introduction

We examine the connection between two quite different notions of knowledge representation and reasoning, namely choice logics and abstract argumentation, and show that they are more closely related than previously known.

Qualitative Choice Logic (QCL) (Brewka, Benferhat, and Berre 2004) is a formalism for preference representation that extends classical propositional logic by the connective $\vec{\times}$ called ordered disjunction. Intuitively, $A\vec{\times}B$ can be read as "A or B but preferably A". In this way, QCL enables us to express both hard- and soft-constraints, i.e., both truth and preferences, in one unified language. Choice logics have received increasing attention, with recent work ranging from computational properties (Bernreiter, Maly, and Woltran 2022) to proof systems (Bernreiter et al. 2022) to applications such as preference learning (Sedki, Lamy, and Tsopra 2020; Sedki, Lamy, and Tsopra 2022).

Abstract argumentation, on the other hand, is used to find justifiable, consistent world views when facing conflicting or inconsistent information. Especially Abstract Argumentation Frameworks (AFs) (Dung 1995), in which arguments and attacks between them are straightforwardly represented via directed graphs, have proven to be one of the most popular formalism to this end. In this work, we use the more general notion of SETAFs—argumentation frameworks with sets of attacking arguments (Nielsen and Parsons 2006). SETAFs have been in the focus of researchers recently as a more flexible and expressive formalism than "standard" AFs (Dvořák, Fandinno, and Woltran 2019), with intuitive connections to structured argumentation and other related formalisms (König, Rapberger, and Ulbricht 2022), while preserving many desired properties of regular AFs (Flouris and Bikakis 2019; Dvořák et al. 2022).

Despite the differences between choice logics and abstract argumentation, a first connection between them has been established by a translation (Sedki 2015) from Prioritized QCL-theories (Benferhat and Sedki 2008)[1] to Value-based AFs (VAFs) (Bench-Capon, Doutre, and Dunne 2007). While this translation is a valuable first step in connecting choice logics and argumentation, it leaves some issues unaddressed. Firstly, the translation is not syntactic, as each interpretation relevant to a formula is translated into an argument. This implies that the translation is not polynomial in size. Secondly, only the so-called lexicographic method of determining preferred models in QCL is considered, while other methods such as inclusion- or minmax-based approaches are not studied. Thirdly, the translation relies on a redefinition of VAF-semantics which is not commonly used elsewhere.

In this paper, we address these challenges by providing two purely syntactic and polynomial-size translations from QCL-theories to SETAFs. Depending on the translation, either the inclusion-based or the minmax preferred models of the original QCL-theory are in direct correspondence to the semi-stable extensions (Verheij 1996; Caminada, Carnielli, and Dunne 2012) of the constructed SETAF.

Our work shows that abstract argumentation is well-suited to directly capture formalisms in which hard- and soft-constraints are jointly represented. Moreover, using our translation, the problem of preferred model entailment in QCL can be solved using existing solvers for SETAFs (Dvořák, Greßler, and Woltran 2018). Choice logics thus join many other logic-based formalisms for which the connection to argumentation is well-studied (Wyner, Bench-Capon, and Dunne 2013; Modgil and Prakken 2014; Cyras and Toni 2016; Skiba and Thimm 2022).

2 Qualitative Choice Logic (QCL)

We recall the definition of QCL (Brewka, Benferhat, and Berre 2004). In the following, $\mathcal{U}$ denotes a countable infinite universe of propositional variables. An interpretation is a set $\mathcal{I} \subseteq \mathcal{U}$ of variables, where $a \in \mathcal{U}$ is true in $\mathcal{I}$ iff $a \in \mathcal{I}$.

Definition 1 (QCL-formula)**.** *The set $\mathcal{F}$ of QCL-formulas is defined inductively: $a \in \mathcal{F}$ for all $a \in \mathcal{U}$; if $\varphi \in \mathcal{F}$ then $\neg\varphi \in \mathcal{F}$; if $\varphi, \psi \in \mathcal{F}$ then $(\varphi \circ \psi) \in \mathcal{F}$ for $\circ \in \{\wedge, \vee, \vec{\times}\}$.*

[1] Prioritized QCL redefines the semantics of the classical connectives, but defines ordered disjunction in the same way as QCL.

A set of QCL-formulas is called a QCL-theory. By $var(\varphi)$ we denote the set of variables occurring in $\varphi \in \mathcal{F}$, while $sf(\varphi)$ denotes the set of all subformulas of φ.

The semantics of QCL relies on the optionality of a formula and its satisfaction degree w.r.t. an interpretation. Satisfaction degrees are either a natural number or ∞ and are used to rank interpretations (lower degrees are more preferable). Optionality is used to penalize non-satisfaction of preferable options, and can intuitively be understood as the maximum satisfaction degree a formula can be ascribed.

Definition 2 (Optionality). *The optionality of a QCL-formula is defined inductively as follows:* $opt(a) = 1$ *for all* $a \in \mathcal{U}$*;* $opt(\neg\varphi) = 1$*;* $opt((\varphi \wedge \psi)) = opt((\varphi \vee \psi)) = \max(opt(\varphi), opt(\psi))$*;* $opt(\varphi \vec{\times} \psi) = opt(\varphi) + opt(\psi)$.

Definition 3 (Satisfaction Degree). *The satisfaction degree of a QCL-formula under* $\mathcal{I} \subseteq \mathcal{U}$ *is defined inductively:*

$$deg(\mathcal{I}, a) = 1 \text{ if } a \in \mathcal{I}, \infty \text{ if } a \notin \mathcal{I} \text{ for all } a \in \mathcal{U}$$

$$deg(\mathcal{I}, \neg\varphi) = 1 \text{ if } deg(\mathcal{I}, \varphi) = \infty, \infty \text{ otherwise}$$

$$deg(\mathcal{I}, (\varphi \wedge \psi)) = \max(deg(\mathcal{I}, \varphi), deg(\mathcal{I}, \psi))$$

$$deg(\mathcal{I}, (\varphi \vee \psi)) = \min(deg(\mathcal{I}, \varphi), deg(\mathcal{I}, \psi))$$

$$deg(\mathcal{I}, (\varphi \vec{\times} \psi)) = \begin{cases} deg(\mathcal{I}, \varphi) & \text{if } deg(\mathcal{I}, \varphi) < \infty \\ opt(\varphi) + deg(\mathcal{I}, \psi) & \text{if } deg(\mathcal{I}, \varphi) = \infty \\ & \quad deg(\mathcal{I}, \psi) < \infty \\ \infty & \text{otherwise} \end{cases}$$

We also write $\mathcal{I} \models_k \varphi$ for $deg(\mathcal{I}, \varphi) = k$. If $k < \infty$ we say that $\mathcal{I}$ is a model of φ and (classically) satisfies φ. If $k = \infty$, then $\mathcal{I}$ does not (classically) satisfy φ. By $pdeg(\varphi) = \{1, \dots, opt(\varphi)\} \cup \{\infty\}$ we denote the set of possible satisfaction degrees that φ may assume. Likewise, for a QCL-theory $T = \{\varphi_1, \dots, \varphi_t\}$ we let $pdeg(T) = \{1, \dots, \max(opt(\varphi_1), \dots, opt(\varphi_t))\} \cup \{\infty\}$.

Note that $opt(\neg\varphi) = 1$ and thus $pdeg(\neg\varphi) = \{1, \infty\}$ for every $\varphi \in \mathcal{F}$. This reflects the fact that negation in QCL acts only on truth, but not on preferences (cf. Definition 3).

Preferred models of QCL-theories are defined in several ways, most notably the lexicographic (lex) and inclusion-based (inc) approaches (Brewka, Benferhat, and Berre 2004). A simpler minmax (mm) approach, where $T = \{\varphi_1, \dots, \varphi_t\}$ is semantically equivalent to $(\varphi_1 \wedge (\cdots \wedge \varphi_t))$, was introduced later (Bernreiter, Maly, and Woltran 2022). We focus on the inclusion-based and minmax approaches.

Definition 4 (Preferred Model). *Let T be a QCL-theory. $\mathcal{I} \subseteq \mathcal{U}$ is a model of T, denoted by $\mathcal{I} \in Mod(T)$, iff $deg(\mathcal{I}, \varphi) < \infty$ for all $\varphi \in T$. Moreover, let $\mathcal{I}^k(T) = \{\varphi \in T \mid deg(\mathcal{I}, \varphi) = k\}$. $\mathcal{I}$ is a preferred model of T under $\pi \in \{inc, mm\}$ iff $\mathcal{I} \in Prf^{\pi}(T)$, where*

- *$\mathcal{I} \in Prf^{inc}(T)$ iff $\mathcal{I} \in Mod(T)$ and there is no $\mathcal{J} \in Mod(T)$ such that, for some $k \in \mathbb{N}$ and all $l < k$, $\mathcal{I}^k(T) \subset \mathcal{J}^k(T)$ and $\mathcal{I}^l(T) = \mathcal{J}^l(T)$ holds.*
- *$\mathcal{I} \in Prf^{mm}(T)$ iff $\mathcal{I} \in Mod(T)$ and there is no $\mathcal{J} \in Mod(T)$ such that $\max\{deg(\mathcal{I}, \varphi) \mid \varphi \in T\} > \max\{deg(\mathcal{J}, \varphi) \mid \varphi \in T\}$.*

Definition 5 (Preferred Model Entailment). *Let T be a QCL-theory, φ a classical formula, and $\pi \in \{inc, mm\}$. $T \mid\!\sim^{\pi} \varphi$ iff $\mathcal{I} \models \varphi$ for all $\mathcal{I} \in Prf^{\pi}(T)$.*

3 Argumentation

Generalizing classical Argumentation Frameworks (AFs) (Dung 1995), Nielsen and Parsons (2006) introduced Argumentation Frameworks with *collective attacks* (SETAFs).

Definition 6 (SETAF). *A SETAF is a pair $SF = (Arg, Att)$ where Arg is a set of* arguments *and $Att \subseteq (2^{Arg} \setminus \{\emptyset\}) \times Arg$ is the* attack relation.

SETAFs $SF = (Arg, Att)$, where for all $(T, h) \in Att$ it holds that $|T| = 1$, amount to (standard Dung) AFs. We usually write (t, h) to denote the set-attack $(\{t\}, h)$. For $SF_1 = (Arg_1, Att_1), SF_2 = (Arg_2, Att_2)$ we define the *union* $SF_1 \cup SF_2$ as $(Arg_1 \cup Arg_2, Att_1 \cup Att_2)$. If there is an attack $(T, h) \in Att$ with $T \subseteq S \subseteq Arg$ and $h \in S' \subseteq Arg$, we write $S \mapsto_{Att} S'$ (or simply $S \mapsto S'$).

SETAF semantics select sets of arguments, called extensions, according to various criteria. In this work, we make use of semi-stable (*sem*) semantics for SETAFs (Flouris and Bikakis 2019). We also define conflict-free (*cf*) and admissible (*adm*) sets, as well as stable (*stb*) extensions.

Definition 7 (SETAF Semantics). *Let $SF = (Arg, Att)$ be a SETAF and $E \subseteq Arg$. E is* conflict-free *in SF, written as $E \in cf(SF)$, if $E \not\mapsto E$. An argument $a \in Arg$ is* defended *in SF by a set $S \subseteq Arg$ if $S \mapsto B$ for each $B \subseteq Arg$ such that $B \mapsto \{a\}$. A set $T \subseteq Arg$ is defended in SF by S if each $a \in T$ is defended in SF by S. $E^{\oplus} = E \cup \{a \in Arg \mid E \mapsto a\}$ is called the range of E. Let $S \in cf(SF)$. Then*

- *$S \in adm(SF)$ iff S defends itself in SF;*
- *$S \in stb(SF)$ iff $S \mapsto \{a\}$ for all $a \in Arg \setminus S$;*
- *$S \in sem(SF)$ iff $S \in adm(SF)$ and there is no $T \in adm(SF)$ such that $T^{\oplus} \supset S^{\oplus}$.*

4 Encoding QCL-formulas

We aim to capture QCL-theories via SETAFs such that the preferred models of the initial theory correspond to the extensions of the constructed framework. As a first step, we encode single QCL-formulas to obtain a correspondence between the satisfaction degree ascribed to a formula by an interpretation and the (semi-)stable extensions of the target SETAF. This intermediate step is needed to deal with the monotonic nature of satisfaction degrees, upon which the non-monotonic notion of preferred models is built. A similar intermediate step is utilized in proof systems for QCL (Bernreiter et al. 2022), where the calculus for preferred model entailment is built on a labeled monotonic calculus.

Given a QCL-formula φ, we will add arguments ψ^k for each subformula $\psi \in sf(\varphi)$ and each degree $k \in pdeg(\psi)$. Every ψ^k will attack all other ψ^{ℓ} with $\ell \neq k$ to ensure that only one of $\psi^1, \dots, \psi^{opt(\psi)}, \psi^{\infty}$ can be accepted. Moreover, we add attacks between each ψ^k and the immediate subformulas of ψ according to the degree-semantics of QCL. This will ensure that ψ^k is accepted in a (semi-)stable extension E iff ψ is satisfied to a degree of k in the interpretation $\mathcal{I}$ corresponding to E. For instance, if $\psi = (a \vec{\times} b)$ is satisfied to a degree of 2 by $\mathcal{I}$ (i.e., $\mathcal{I} \models_2 \psi$), then the argument $(a \vec{\times} b)^2$ will be accepted in the corresponding extension E, but $(a \vec{\times} b)^1$ and $(a \vec{\times} b)^{\infty}$ will be defeated. See Figure 1 for an example. We now formally specify our translation.

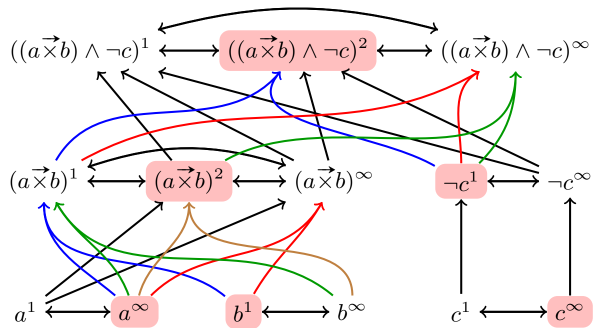

Figure 1: SF_φ for $\varphi = (a\vec{\times}b) \wedge \neg c$. Collective attacks are colored for readability. Highlighted arguments correspond to model $\mathcal{I} = \{b\}$. Note that $\mathcal{I} \notin \mathit{Prf}^\pi(\{\varphi\})$ ($\pi \in \{mm, inc\}$), since $\{a\} \models_1 \varphi$.

Definition 8. *Let φ be a QCL-formula. We define the* corresponding SETAF $SF_\varphi = (\mathit{Arg}_\varphi, \mathit{Att}_\varphi)$ *with arguments*

$$\mathit{Arg}_\varphi = \{\psi^o \mid \psi \in \mathit{sf}(\varphi), o \in \mathit{pdeg}(\varphi)\}$$
$$\mathit{Att}_\varphi = \Big(\bigcup_{\psi \in \mathit{sf}(\varphi)} \mathit{Att}^*_\psi\Big) \cup \{(\psi^o, \psi^p) \mid \psi \in \mathit{sf}(\varphi), o \neq p\}$$

where Att^*_ψ depends on the immediate subformulas of ψ. For $a \in \mathcal{U}$ we have $\mathit{Att}^*_a = \emptyset$. Otherwise, we have

$$\mathit{Att}^*_{\neg L} = \{(L^\infty, \neg L^\infty)\} \cup \{(L^\ell, \neg L^1) \mid \ell \neq \infty\};$$
$$\mathit{Att}^*_{(L \wedge R)} = \{(\{L^\ell, R^r\}, (L \wedge R)^d) \mid d > \max(\ell, r)\} \cup$$
$$\{(L^\ell, (L \wedge R)^d) \mid \ell > d\} \cup \{(R^r, (L \wedge R)^d) \mid r > d\};$$
$$\mathit{Att}^*_{(L \vee R)} = \{(\{L^\ell, R^r\}, (L \vee R)^d) \mid d < \min(\ell, r)\} \cup$$
$$\{(L^\ell, (L \vee R)^d) \mid \ell < d\} \cup \{(R^r, (L \vee R)^d) \mid r < d\};$$
$$\mathit{Att}^*_{(L\vec{\times}R)} = \{(L^\ell, (L\vec{\times}R)^d) \mid \ell \neq \infty, \ell \neq d\} \cup$$
$$\{(\{L^\infty, R^r\}, (L\vec{\times}R)^d) \mid r \neq \infty, d \neq r + \mathit{opt}(L)\} \cup$$
$$\{(\{L^\infty, R^\infty\}, (L\vec{\times}R)^d) \mid d \neq \infty\}.$$

This construction is purely syntactic, since $\mathit{opt}(\varphi)$, and therefore $\mathit{pdeg}(\varphi)$, can be computed based solely on the structure of φ (cf. Definition 2). Moreover, the construction is polynomial, since $\mathit{opt}(\varphi)$ is bounded by the number of $\vec{\times}$-occurrences in φ. Thus, SF_φ contains $O(\mathit{opt}(\varphi) \cdot |\mathit{sf}(\varphi)|)$ arguments. Moreover, attacks in SF_φ never have more than two joint attackers, hence, $|\mathit{Att}_\varphi|$ is polynomial in $|\mathit{Arg}_\varphi|$.

We now establish the semantic correspondence between a QCL-formula φ and the SETAF SF_φ. We write $\mathcal{I} \cong E$ for an interpretation $\mathcal{I}$ and an extension E if $\mathcal{I}$ corresponds to the choice of arguments in E i.e., $a^1 \in E$ iff $a \in \mathcal{I}$ and $a^\infty \in E$ iff $a \notin \mathcal{I}$. Likewise, for a set $\mathcal{M} \subseteq 2^{\mathcal{U}}$ of interpretations and a set $\sigma(SF)$ of extensions we write $\mathcal{M} \cong \sigma(SF)$ iff $\cong$ is a bijection from $\mathcal{M}$ to $\sigma(SF)$.

Lemma 1. *Let φ be a QCL-formula and SF_φ its corresponding SETAF. If $\mathcal{I} \cong E$ for some $\mathcal{I} \subseteq \mathit{var}(\varphi)$ and $E \in \mathit{stb}(SF_\varphi)$ then $\mathcal{I} \models_k \psi$ iff $\psi^k \in E$ for all $\psi \in \mathit{sf}(\varphi)$.*

As a result of the above lemma, each interpretation relevant to a formula φ corresponds to exactly one stable extension in SF_φ, and vice versa. Clearly, this implies $\mathit{stb}(SF) \neq \emptyset$ which in turn is known to yield $\mathit{stb}(SF) = \mathit{sem}(SF)$.

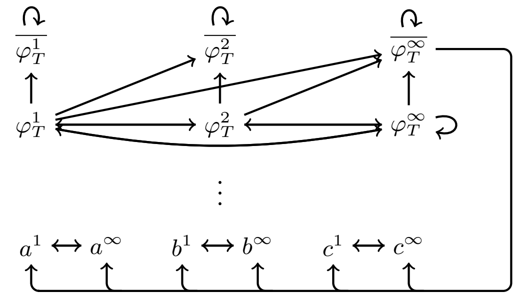

Figure 2: SF_T^{mm} from Example 1.

Proposition 2. $2^{\mathit{var}(\varphi)} \cong \mathit{stb}(SF_\varphi) = \mathit{sem}(SF_\varphi)$.

Note that we can also capture only the *models* of a formula φ by adding the attack $(\varphi^\infty, \varphi^\infty)$. If φ is (classically) unsatisfiable we will have no stable extensions.

5 Capturing Preferred Models

We now extend our construction for QCL-formulas from Section 4 to also capture QCL-theories and their preferred models. This then further allows us to decide the problem of preferred model entailment via the constructed framework.

First, we consider preferred models w.r.t. the minmax (mm) semantics (cf. Definition 4), where a theory $T = \{\varphi_1, \dots, \varphi_t\}$ is semantically equivalent to the formula $\varphi_T = (\varphi_1 \wedge (\cdots \wedge \varphi_t))$. The key idea is the following: we first construct the SETAF SF_T corresponding to φ_T (cf. Definition 8). Then, for each argument φ_T^k we introduce a self-attacking argument $\overline{\varphi_T^k}$. Each $\overline{\varphi_T^k}$ is attacked by every φ_T^ℓ such that $\ell \leq k$. As a result, if we consider two admissible sets E, E' such that $\varphi_T^\ell \in E$, $\varphi_T^k \in E'$, and $\ell < k$, then the range $E^\oplus$ of E is a superset of the range $E'^\oplus$ of E' w.r.t. to the arguments $\overline{\varphi_T^m}$, $m \in \mathit{pdeg}(\varphi_T)$. This then means that the semi-stable extensions of the constructed framework correspond to the minmax preferred models of the initial theory. Finally, we add attacks from $\overline{\varphi_T^\infty}$ to all variable-arguments a^1, a^∞ where $a \in \mathit{var}(\varphi_T)$. This ensures that, if T is not classically satisfiable, the only semi-stable extension of SF_T^{mm} is $\emptyset$. We now provide this construction formally.

Definition 9. *Let $T = \{\varphi_1, \dots, \varphi_t\}$ be a QCL-theory. Let $\varphi_T = (\varphi_1 \wedge (\cdots \wedge \varphi_t))$, and let $SF_{\varphi_T} = (\mathit{Arg}_{\varphi_T}, \mathit{Att}_{\varphi_T})$ be the SETAF corresponding to φ_T. We define $SF_T^{mm} = (\mathit{Arg}_T^{mm}, \mathit{Att}_T^{mm})$ as follows:*

$$\mathit{Arg}_T^{mm} = \mathit{Arg}_{\varphi_T} \cup \{\overline{\varphi_T^o} \mid o \in \mathit{pdeg}(\varphi_T)\}$$
$$\mathit{Att}_T^{mm} = \mathit{Att}_{\varphi_T} \cup \{(\varphi_T^\infty, \varphi_T^\infty)\}$$
$$\cup \{(\overline{\varphi_T^o}, \overline{\varphi_T^o}) \mid o \in \mathit{pdeg}(\varphi_T)\}$$
$$\cup \{(\overline{\varphi_T^\infty}, a^1), (\overline{\varphi_T^\infty}, a^\infty) \mid a \in \mathit{var}(\varphi_T)\}$$
$$\cup \{(\varphi_T^o, \overline{\varphi_T^p}) \mid o, p \in \mathit{pdeg}(\varphi_i), o \leq p\}.$$

Example 1. *Let $T = \{(a\vec{\times}c), (b\vec{\times}c), \neg(a \wedge b)\}$. Then $\varphi_T = ((a\vec{\times}c) \wedge ((b\vec{\times}c) \wedge \neg(a \wedge b)))$ with $\mathit{pdeg}(\varphi_T) = \{1, 2, \infty\}$. SF_T^{mm} is depicted in Figure 2. Arguments ψ^k corresponding to non-atomic subformulas ψ of φ_T are not depicted for the sake of succinctness.*

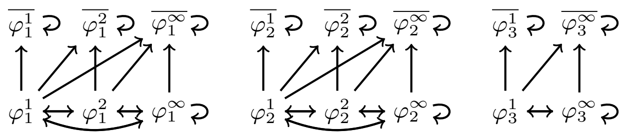

Figure 3: Minimization gadgets of SF_T^{inc} from Example 2.

There is a direct semantic correspondence between the initial theory T and the constructed framework SF_T^{mm}, namely, each preferred model of T corresponds to exactly one semi-stable extension of SF_T^{mm}, and vice versa.

Proposition 3. $\mathit{Prf}^{mm}(T) \cong \mathit{sem}(SF_T^{mm}) \setminus \{\emptyset\}$.

We now turn our attention to the inclusion-based (inc) preferred model semantics. In essence, we can build upon the tools established so far and use the same gadget as in the case of minmax semantics to minimize satisfaction degrees. However, this gadget is now constructed for every $\varphi \in T$, i.e., we add $\overline{\varphi^k}$ for each $\varphi \in T$ and each $k \in pdeg(\varphi)$.

Definition 10. *Let $T = \{\varphi_1, \dots, \varphi_t\}$ be a QCL-theory and $SF_1{=}(Arg_1, Att_1), \dots, SF_t{=}(Arg_t, Att_t)$ the SETAFs corresponding to $\varphi_1, \dots, \varphi_t$. Let $SF_T^{inc} = (Arg_T^{inc}, Att_T^{inc})$ s.t.:*

$$Arg_T^{inc} = \Big(\bigcup_{1 \le i \le t} Arg_i \Big) \cup \{\overline{\varphi_i^o} \mid \varphi_i \in T, o \in pdeg(\varphi_i)\}$$

$$\begin{aligned} Att_T^{inc} = {} & \Big(\bigcup_{1 \le i \le t} Att_i \Big) \cup \{(\varphi_i^\infty, \varphi_i^\infty) \mid \varphi_i \in T\} \\ & \cup \{(\overline{\varphi_i^o}, \overline{\varphi_i^o}) \mid \varphi_i \in T, o \in pdeg(\varphi_i)\} \\ & \cup \{(\overline{\varphi_i^\infty}, a^1), (\overline{\varphi_i^\infty}, a^\infty) \mid \varphi_i \in T, a \in var(\varphi_i)\} \\ & \cup \{(\varphi_i^o, \overline{\varphi_i^p}) \mid \varphi_i \in T, o \in pdeg(\varphi_i), o \ \le p\}. \end{aligned}$$

Example 2. *Let $T = \{\varphi_1, \varphi_2, \varphi_3\}$ with $\varphi_1 = (a \overrightarrow{\times} c)$, $\varphi_2 = (b \overrightarrow{\times} c)$, and $\varphi_3 = \neg(a \wedge b)$. To obtain SF_T^{inc} we construct a minimization gadget for each $\varphi_i \in T$, as depicted in Figure 3. For succinctness, we omit arguments corresponding to subformulas of each $\varphi_i \in T$.*

Analogously to Proposition 3, every preferred model of some QCL-theory T corresponds to exactly one semi-stable extension of SF_T^{inc}, and vice versa.

Proposition 4. $\mathit{Prf}^{inc}(T) \cong \mathit{sem}(SF_T^{inc}) \setminus \{\emptyset\}$.

We have established a semantic correspondence between the preferred models of QCL-theories (under both mm and inc semantics) and the semi-stable extensions of SETAFs. These results can now further be used to decide preferred model entailment $T \mathrel{|\!\!\sim^{\pi}} \varphi$ (cf. Definition 5). To this end, we combine the frameworks SF_T^π for the theory T and SF_φ for the entailed (classical) formula φ.

Theorem 5. *Let T be a QCL-theory and $\pi \in \{mm, inc\}$. Then $T \mathrel{|\!\!\sim^{\pi}} \varphi$ iff $\varphi^1 \in S$ for all $S \in \mathit{sem}(SF_T^\pi \cup SF_\varphi) \setminus \{\emptyset\}$.*

The above result allows us to apply fast SAT- or ASP-based argumentation solvers to reason on QCL-theories efficiently. See (Dvořák, Greßler, and Woltran 2018) for a SETAF-specific solver. For a more general overview of argumentation solvers, see (Lagniez et al. 2021). Recently, SETAFs have been investigated with focus on efficient algorithms (Dvořák, König, and Woltran 2021; 2022a; 2022b). While QCL has been encoded in ASP (Bernreiter, Maly, and Woltran 2020), to the best of our knowledge there are no implementations for preferred model entailment.

Regarding computational complexity, deciding whether an argument is contained in all semi-stable extensions (as needed in Theorem 5) is Π_2P-complete for SETAFs (Dvořák, Greßler, and Woltran 2018). Deciding $T \mathrel{|\!\!\sim^{\pi}} \varphi$ is Π_2P-complete for $\pi = inc$ and Θ_2P-complete for $\pi = mm$ (Bernreiter, Maly, and Woltran 2022). Thus, when capturing $T \mathrel{|\!\!\sim^{\pi}} \varphi$, there is a complexity gap for $\pi = mm$ but not for $\pi = inc$. Note that all discussed problems are on the second level of the polynomial hierarchy.

6 Discussion & Conclusion

We successfully mapped Qualitative Choice Logic (QCL) theories to argumentation frameworks with collective attacks (SETAFs). The preferred models of the initial QCL-theory directly correspond to the semi-stable extensions of the constructed SETAF, which further allows us to decide preferred model entailment. We consider two preferred model semantics for QCL-theories, namely the inclusion-based approach introduced in the original QCL-paper (Brewka, Benferhat, and Berre 2004) and the simpler minmax approach (Bernreiter, Maly, and Woltran 2022). Unlike the translation (Sedki 2015) from PQCL-theories to Value-based AFs, our construction is purely syntactic and polynomial in size and runtime.

Our results show that the connection between choice logics and argumentation is closer than previously known. Indeed, we find that SETAFs are well-suited for capturing languages such as QCL, where soft and hard constraints are jointly represented. Moreover, we demonstrated that semi-stable semantics are a useful tool that can handle degree-minimization in a straightforward way.

Observe that every SETAF can be translated into an equivalent Dung-style AF with only polynomial overhead (Polberg 2017). However, this requires the introduction of additional arguments. Thus, the usage of SETAFs allows us to capture QCL-formulas more directly, with each argument ψ^k corresponding to a subformula $\psi \in sf(\varphi)$.

Regarding future work, we plan to find a syntactic and polynomial translation from QCL-theories to SETAFs that respects the lexicographically preferred models of the initial theory (Brewka, Benferhat, and Berre 2004). A difficulty here is that this approach relies on *counting* how many formulas are satisfied to a certain degree.

Finally, our work can be extended to formalisms related to QCL. This includes other choice logics such as Conjunctive Choice Logic (Boudjelida and Benferhat 2016) or Lexicographic Choice Logic (Bernreiter, Maly, and Woltran 2022), both of which replace the ordered disjunction of QCL with alternative choice connectives. Note that our construction is in large parts independent of ordered disjunction, i.e., a similar construction may be possible for other choice logics. A more distantly related system is the recently introduced Lexicographic Logic (Charalambidis et al. 2021) which uses lists of truth values rather than satisfaction degrees.

Acknowledgments

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