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On Conflict-free Labellings – Realizability, Construction and Patterns of Redundancy

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Abstract

The paper deals with the topic of realizability in abstract argumentation. More precisely, we consider the most basic kind of labellings, so-called conflict-free labellings. The understanding of these labellings is essential as any mature labelling-based semantics selects its output among these labellings. We show how to decide whether a given set of labellings is the conflict-free outcome of a certain Dungean framework. To this end we introduce several new criteria like L-tightness, reject-witness and reject-compositionality. These properties play a decisive role in the central characterization theorem. Moreover, we present a construction method, showing how to realize a conflict-free realizable labelling-set. Finally, we study the representational freedom in case of such labellings. This leads to a uniqueness result for a certain sub-class and a surprising connection to strong equivalence in case of extension-based semantics.

1 Introduction

Consider a logical formalism $\mathcal{L}$ together with its semantics σ. Depending on the application in mind, it might be interesting to know which set of interpretations $\mathcal{I}$ are actually expressible in $\mathcal{L}$. More formally, how to determine whether $\mathcal{I} \in \{\sigma(T) \mid T \text{ is a } \mathcal{L}\text{-theory}\}$. For instance, in case of propositional logic any finite set of two-valued interpretations is *realizable*. Differently, in case of normal logic programs under stable model semantics we have that any finite candidate set is realizable if and only if it forms a $\subseteq$-antichain (Eiter et al. 2013). One major application of realizability issues are dynamic evolvements like in case of belief revision (Alchourrón, Gärdenfors, and Makinson 1985; Delgrande, Peppas, and Woltran 2013). Here, we are typically faced with the problem of modifying a given $\mathcal{L}$-theory T, s.t. the revised version S of it satisfies $\sigma(S) = \mathcal{I}$ for some desired set $\mathcal{I}$. Now, before trying to do this revision in a certain minimal way it is essential to know whether $\mathcal{I}$ is realizable at all, i.e. $\mathcal{I} \in \{\sigma(T) \mid T \text{ is a } \mathcal{L}\text{-theory}\}$.

The first formal treatment of realizability in *formal argumentation* (Atkinson et al. 2017) was given in (Dunne et al. 2013). They considered the leading abstract formalism, so-called *argumentation frameworks* (Dung 1995). They provided locally verifiable necessary as well as sufficient properties for realizability w.r.t. *extension-based semantics*. In this paper we begin to complement the existing study by shifting the focus to the more involved *labelling-based semantics* (Baroni, Caminada, and Giacomin 2018). In a nutshell, we present an in-depth study for the building blocks of any mature semantics, so-called *conflict-free labellings*.

2 Formal Preliminaries

2.1 Argumentation Frameworks and Semantics

An *argumentation framework* (AF) is a pair $F = (A, R)$ where A (set of arguments) is a subset of a fixed infinite background set $\mathcal{U}$. Moreover, R (set of attacks) is a subset of $A \times A$ (Dung 1995). The set of all finite AFs is denoted by $\mathcal{F}$. An *extension-based semantics* $\mathcal{E}_\sigma : \mathcal{F} \to 2^{2^{\mathcal{U}}}$ assigns to each AF $F = (A, R)$ a set of sets of arguments with $\mathcal{E}_\sigma(F) \subseteq 2^A$. Each one of them, a so-called σ-extension, is considered to be acceptable with respect to F. The most basic requirement underlying almost any semantics is *conflict-freeness*.

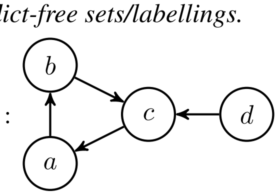
Definition 1. *Given $F = (A, R)$. A set $E \subseteq A$ is conflict-free ($E \in \mathcal{E}_{cf}(F)$) if there are no $a, b \in E$, s.t. $(a, b) \in R$.*

A *labelling-based semantics* $\mathcal{L}_\sigma : \mathcal{F} \to 2^{\left(2^{\mathcal{U}}\right)^3}$ assigns to any AF $F = (A, R)$ a set of triples of sets of arguments denoted by $\mathcal{L}_\sigma(F) \subseteq \left(2^A\right)^3$. Each one of them, a so-called σ*-labelling* of F, is a triple $L = (I, O, U)$ indicating that arguments in I, O or U are considered to be *accepted (in)*, *rejected (out)* or *undecided* with respect to F. We further assume $I \cap O = I \cap U = O \cap U = \emptyset$ (pairwise disjointness) and $I \cup O \cup U = A$ (covering). We use L^{I} to refer to the first component of the labelling L, analogously for L^{O} and L^{U}. We proceed with the central notion of *conflict-free labellings* (Caminada 2011; Arieli 2012).

Definition 2. *A labelling L of $F = (A, R)$ is called conflict-free ($L \in \mathcal{L}_{cf}(F)$) whenever:*

1. *If $a, b \in L^{\mathrm{I}}$, then $(a, b) \notin R$, and (no internal conflicts)*
2. *If $a \in L^{\mathrm{O}}$, then there is an $b \in L^{\mathrm{I}}$ with $(b, a) \in R$. (reason for rejection)*

Example 1. *Consider the following illustrating AF F and its associated conflict-free sets/labellings.*

F:


- $\mathcal{E}_{cf}(F) = \{\{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, d\}, \emptyset\}$
- $\mathcal{L}_{cf}(F) = \{(\{a\}, \{b\}, \{c, d\}), (\{a\}, \emptyset, \{b, c, d\}),$
$(\{b\}, \{c\}, \{a, d\}), (\{b\}, \emptyset, \{a, c, d\}),$
$(\{c\}, \{a\}, \{b, d\}), (\{c\}, \emptyset, \{a, b, d\}),$
$(\{d\}, \{c\}, \{a, b\}), (\{d\}, \emptyset, \{a, b, c\}),$
$(\{a, d\}, \{b, c\}, \emptyset), (\{a, d\}, \{c\}, \{b\}),$
$(\{a, d\}, \{b\}, \{c\}), (\{a, d\}, \emptyset, \{b, c\}),$
$(\{b, d\}, \{c\}, \{a\}), (\{b, d\}, \emptyset, \{a, c\}),$
$(\emptyset, \emptyset, \{a, b, c, d\})\}$

We observe that there is not necessarily a match between the numbers of conflict-free sets and conflict-free labellings. However, both notions are intimately connected via the following relations (Baroni, Caminada, and Giacomin 2018).

Proposition 1. *Given $F = (A, R)$ and $E \subseteq A$, we use $E^{\mathcal{L}}$ as shorthand for $(E, E^+, A \setminus (E \cup E^+))$.*[1]

1. *If $E \in \mathcal{E}_{cf}(F)$, then $E^{\mathcal{L}} \in \mathcal{L}_{cf}(F)$, and*
2. *If $L \in \mathcal{L}_{cf}(F)$, then $L^I \in \mathcal{E}_{cf}(F)$.*

2.2 Realizability in Abstract Argumentation

The first formal treatment of expressibility issues in abstract argumentation was given in (Dunne et al. 2013). They considered extension-based semantics and provided simple criteria deciding whether a certain set can be the semantical outcome of a Dungean AF. In the following we introduce the central notions of *realizability* and *signature*. In a nutshell, a set $\mathbb{X}$ is realizable w.r.t. σ, if there is an AF F such that its set of σ-extensions/σ-labellings coincides with $\mathbb{X}$. Collecting all realizable sets defines the concept of a σ-signature.

Definition 3. *Given a semantics $\sigma : \mathcal{F} \to 2^{(2^{\mathcal{U}})^n}$. A set $\mathbb{X} \subseteq (2^{\mathcal{U}})^n$ is σ-realizable if there is an AF $F \in \mathcal{F}$, s.t. $\sigma(F) = \mathbb{X}$. The σ-signature is defined as $\Sigma_\sigma = \{\sigma(F) \mid F \in \mathcal{F}\}$.*

Extension-based Semantics We proceed with further notation as well as the central notions of *downward-closedness* and *tightness* (Dunne et al. 2015).

Definition 4. *A finite $\mathbb{S} \subseteq 2^{\mathcal{U}}$ is called extension-set. We use $Args_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}} S$, $Pairs_{\mathbb{S}} = \{(a, b) \mid \exists S \in \mathbb{S} : \{a, b\} \subseteq S\}$ and $dcl(\mathbb{S}) = \{S' \subseteq S \mid S \in \mathbb{S}\}$ (downward-closure of $\mathbb{S}$).*

The following example illustrates the introduced concepts.

Example 2. *Let $\mathbb{S} = \{\{a, d\}, \{b, d\}, \{c\}\}$. We have: $Args_{\mathbb{S}} = \{a, b, c, d\}$, $Pairs_{\mathbb{S}} = \{(a, a), (b, b), (c, c), (d, d), (a, d), (d, a), (b, d), (d, b)\}$, and $dcl(\mathbb{S}) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, d\}\}$.*

Definition 5. *Given an extension-set $\mathbb{S}$. $\mathbb{S}$ is downward-closed if $\mathbb{S} = dcl(\mathbb{S})$. It is called tight whenever: for all $S \in \mathbb{S}$ and $a \in Args_{\mathbb{S}}$ we have: if $S \cup \{a\} \notin \mathbb{S}$, then there exists an $s \in S$ such that $(a, s) \notin Pairs_{\mathbb{S}}$.*

Example 3 (Example 2 cont.)**.** *$\mathbb{S}$ is not downward-closed as $dcl(\mathbb{S}) \neq \mathbb{S}$. However, $\mathbb{S}$ is tight. To exemplify consider $S = \{b, d\}$. First, in case of $x \in \{b, d\}$ we have nothing to show as $S \cup \{x\} = S \in \mathbb{S}$. Secondly, for $y \in \{a, c\}$ we have $S \cup \{y\} \notin \mathbb{S}$ and find $b \in S$ with $(y, b) \notin Pairs_{\mathbb{S}}$.*

[1] E^+ is the *range* of E defined as $\{b \mid (a, b) \in R, a \in E\}$.

Consider the central characterization (Dunne et al. 2013).

Theorem 2. *Given an extension-set $\mathbb{S}$, then*

$$\mathbb{S} \in \Sigma_{\mathcal{E}_{cf}} \Leftrightarrow \mathbb{S} \text{ is tight, non-empty and downward-closed.}$$

Labelling-based Semantics We start with some notational shorthands. Moreover, we extend the notion of an extension-set to labelling-based semantics, a so-called *labelling-set*.

Definition 6. *Given a finite set $\mathbb{L} \subseteq (2^{\mathcal{U}})^3$, we set $Args(\mathbb{L}) = \bigcup_{L \in \mathbb{L}} (L^I \cup L^O \cup L^U)$ and use $\mathbb{L}^I$, $\mathbb{L}^O$ and $\mathbb{L}^U$ to denote $\{L^I \mid L \in \mathbb{L}\}$, $\{L^O \mid L \in \mathbb{L}\}$ or $\{L^U \mid L \in \mathbb{L}\}$, respectively. Moreover, we say that $\mathbb{L}$ is a labelling-set if*

1. *$L_1^I \cup L_1^O \cup L_1^U = L_2^I \cup L_2^O \cup L_2^U$ for any $L_1, L_2 \in \mathbb{L}$ and, (same arguments)*
2. *$L_1^I \cap L_1^O = L_1^I \cap L_1^U = L_1^O \cap L_1^U = \emptyset$ for each $L_1 \in \mathbb{L}$. (disjointness)*

Finally, for a fixed set of arguments $E \subseteq \mathcal{U}$ we use

- *$\mathbb{L}_{I=E} = \{L \mid L \in \mathbb{L}, L^I = E\}$, and (corr. labellings)*
- *$\mathbb{L}^O_{I=E} = \{L^O \mid L \in \mathbb{L}, L^I = E\}$. (corr. out-labels)*

Let us illustrate the introduced concepts.

Example 4. *Let $\mathbb{L} = \{(\{a, d\}, \{b\}, \{c\}), (\{b, d\}, \{a, c\}, \emptyset), (\{b, d\}, \{a\}, \{c\})\}$. We observe that $\mathbb{L}$ is indeed a labelling-set as all triples refer to the same arguments, namely a, b, c, d, and for any triple we have that each argument occurs in one of the three sets only. We obtain the following sets:*

- *$\mathbb{L}^I = \{\{a, d\}, \{b, d\}\}$, $\mathbb{L}^O = \{\{b\}, \{a, c\}, \{a\}\}$ and $\mathbb{L}^U = \{\{c\}, \emptyset\}$,*
- *$\mathbb{L}^O_{I=\{a,d\}} = \{\{b\}\}$ and $\mathbb{L}^O_{I=\{b,d\}} = \{\{a, c\}, \{a\}\}$, and*
- *$\mathbb{L}_{I=\{a,d\}} = \{(\{a, d\}, \{b\}, \{c\})\}$ and $\mathbb{L}_{I=\{b,d\}} = \{(\{b, d\}, \{a, c\}, \emptyset), (\{b, d\}, \{a\}, \{c\})\}$.*

3 Conflict-free Labellings

3.1 L-tightness and Rejection Properties

The presence of characterization theorems for extension-based semantics is of little help in characterizing the corresponding labelling-based version. This is due to the fact that the latter provides one with strictly more information. First of all, they are more restrictive as they assign a status to any argument. Consequently, the possible number of realizing frameworks is limited from the start. Secondly, they are more fine-grained as they explicitly distinguish two non-acceptance cases, namely *rejected* and *undecided*.

In this section we introduce three new properties relevant for characterizing conflict-free labellings. We start with so-called *L-tightness*. A labelling-set $\mathbb{L}$ is L-tight if: First, greatest out-labels exist and secondly, the union of two in-labels I_1, I_2 is an in-label too if and only if the greatest out-label regarding I_1 does not share elements with I_2 and vice versa. Intuitively, L-tightness fulfills a similar purpose for labellings as tightness for extensions since it gives a reason why certain sets are not in-labels.

Definition 7. *A labelling-set $\mathbb{L}$ is called L-tight, if*

1. *for each $E \in \mathbb{L}^I$: $\mathbb{L}^O_{I=E}$ possesses a $\subseteq$-greatest element. (Notation: We use $\overline{\mathbb{L}}^O_{I=E}$ for the $\subseteq$-greatest element and $\overline{\mathbb{L}}_{I=E}$ for the associated labelling.)*

2. *for all $I_1, I_2 \in \mathbb{L}^I$ we have:*
$$I_1 \cup I_2 \in \mathbb{L}^I \Leftrightarrow (\overline{\mathbb{L}}^O_{I=I_1} \cap I_2) \cup (\overline{\mathbb{L}}^O_{I=I_2} \cap I_1) = \emptyset.$$

Example 5 (Example 1 cont.). *First note, that $\subseteq$-greatest elements do not exist in general witnessed by $\{(\{a\},\{b\},\{c\}),(\{a\},\{c\},\{b\})\}$. Let us verify the L-tightness of the labelling-set $\mathbb{L} = \mathcal{L}_{cf}(F)$. We have:*

- $\mathbb{L}^I = \mathcal{E}_{cf}(F) = \{\{a\},\{b\},\{c\},\{d\},\{a,d\},\{b,d\},\emptyset\}$.
- *For each $E \in \mathbb{L}^I$ we have a $\subseteq$-greatest element in $\mathbb{L}^O_{I=E}$:* $\overline{\mathbb{L}}^O_{I=\{a\}} = \{b\}$, $\overline{\mathbb{L}}^O_{I=\{b\}} = \{c\}$, $\overline{\mathbb{L}}^O_{I=\{c\}} = \{a\}$, $\overline{\mathbb{L}}^O_{I=\{d\}} = \{c\}$, $\overline{\mathbb{L}}^O_{I=\{a,d\}} = \{b,c\}$, $\overline{\mathbb{L}}^O_{I=\{b,d\}} = \{c\}$ *and* $\overline{\mathbb{L}}^O_{I=\emptyset} = \emptyset$.
- *For the second item of L-tightness we have to consider each possible pairing of in-labels. We consider two pairings only and leave the remaining combinations for the reader.*
  1. *Consider $I_1 = \{a\}$ and $I_2 = \{d\}$.*
     We have $I_1 \cup I_2 = \{a,d\} \in \mathbb{L}^I$ and moreover, $\overline{\mathbb{L}}^O_{I=I_1} \cap I_2 = \{b\} \cap \{d\} = \emptyset = \{c\} \cap \{a\} = \overline{\mathbb{L}}^O_{I=I_2} \cap I_1$.
  2. *Consider $I_1 = \{a\}$ and $I_2 = \{b\}$.*
     We have $I_1 \cup I_2 = \{a,b\} \notin \mathbb{L}^I$ and the corresponding non-emptiness of the union witnessed by $\overline{\mathbb{L}}^O_{I=I_1} \cap I_2 = \{b\} \cap \{b\} = \{b\} \neq \emptyset$.

The second property is called *reject-compositionality*. In a nutshell, a labelling-set $\mathbb{L}$ is reject-compositional, if the out-labelled arguments for a given in-labelled set E can be found in the union of out-labels of single arguments in E.

Definition 8. *A labelling-set $\mathbb{L}$ is called reject-compositional, if for each $E \in \mathbb{L}^I$, we have:*
$$\bigcup \mathbb{L}^O_{I=E} = \bigcup_{a \in E} \bigcup \mathbb{L}^O_{I=\{a\}}.$$

In case of L-tight labelling-sets the equation can be replaced with the more convenient $\overline{\mathbb{L}}^O_{I=E} = \bigcup_{a \in E} \overline{\mathbb{L}}^O_{I=\{a\}}$.

Before turning to an example we introduce the third new concept. A labelling-set $\mathbb{L}$ is *reject-witnessing* if each out-labelled argument possesses a witnessing "basic" labelling.

Definition 9. *A labelling-set $\mathbb{L}$ is called reject-witnessing, if for each $L \in \mathbb{L}$ we have:*
$$\forall o \in L^O \ \exists i \in L^I, \text{ s.t. } (\{i\},\{o\}, Args_{\mathbb{L}} \setminus \{i,o\}) \in \mathbb{L}.$$

Example 6 (Example 1 cont.). *Consider again $\mathbb{L} = \mathcal{L}_{cf}(F)$. Regarding reject-witnessing the only non-trivial labels are $L_1 = (\{a,d\},\{b,c\},\emptyset) \in \mathbb{L}$ and $L_2 = (\{b,d\},\{c\},\{a\})) \in \mathbb{L}$. For L_1 the rejections are witnessed by $(\{a\},\{b\},\{c,d\}) \in \mathbb{L}$ and $(\{d\},\{c\},\{a,b\}) \in \mathbb{L}$. The latter basic labelling also serves as a witness for L_2.*

Now, for reject-compositionality. We have already seen that $\mathbb{L}$ is L-tight (Example 5). This means, it suffices to show $\overline{\mathbb{L}}^O_{I=E} = \bigcup_{a \in E} \overline{\mathbb{L}}^O_{I=\{a\}}$. We only have to consider both two-element sets. Let us start with $E = \{a,d\}$. We have, $\overline{\mathbb{L}}^O_{I=\{a,d\}} = \{b,c\}$ and the matching sets, $\overline{\mathbb{L}}^O_{I=\{a\}} = \{b\}$ and $\overline{\mathbb{L}}^O_{I=\{d\}} = \{c\}$. Finally, for $E = \{b,d\}$ we get $\overline{\mathbb{L}}^O_{I=\{b,d\}} = \{c\}$ and, $\overline{\mathbb{L}}^O_{I=\{b\}} = \{c\}$ and $\overline{\mathbb{L}}^O_{I=\{d\}} = \{c\}$ as required.

Finally, we mention that none of the properties can be derived from the remaining two. This means, for example, that there are labelling-sets satisfying both rejection properties without being L-tight.

3.2 Characterization Theorem

In the following we present the central characterization theorem for conflict-free labellings. It can be seen that the newly introduced properties play a central role here. Please note that any of the five properties can be decided by looking at the labelling-set in question only.

Theorem 3. *Given a labelling-set $\mathbb{L}$, then*

$\mathbb{L} \in \Sigma_{\mathcal{L}_{cf}} \Leftrightarrow$
1. *$\mathbb{L}^I$ is downward-closed and non-empty,*
2. *$\mathbb{L}^O_{I=E}$ is downward-closed for all $E \in \mathbb{L}^I$,*
3. *$\mathbb{L}$ is L-tight,*
4. *$\mathbb{L}$ is reject-compositional, and*
5. *$\mathbb{L}$ is reject-witnessing.*

We mention that the characterization theorem can be presented in a more compact way since the presented set of properties is not independent. More precisely, reject-witness is redundant as it is implied by reject-compositionality (4.) and requiring downward-closedness for each in-associated out-label set (2.). Confer the subsequent proposition.

Proposition 4. *Let a labelling-set $\mathbb{L}$ be given. If $\mathbb{L}$ is reject-compositional and for each $E \in \mathbb{L}^I$, $\mathbb{L}^O_{I=E}$ is downward-closed, then $\mathbb{L}$ is reject-witnessing.*

Let us compare the achieved characterization theorem with the already existing one regarding conflict-free sets (Theorem 2). In light of Proposition 1 one can see that labelling-based *cf*-realizability of $\mathbb{L}$ requires extension-based *cf*-realizability of the corresponding set of in-labelled arguments.[2] This means, $\mathbb{L}^I$ has to be tight, non-empty and downward-closed. The last two are explicitly given in Theorem 3 (1.) and one may wonder about the "missing" tightness property. This property of $\mathbb{L}^I$ is implicit as shown next.

Proposition 5. *Let $\mathbb{L}$ be a labelling-set. Given L-tightness and reject-compositionality of $\mathbb{L}$ and downward-closedness of $\mathbb{L}^I$, then $\mathbb{L}^I$ is tight.*

3.3 Standard Construction

As a matter of fact, knowing that a certain set is realizable or not is a valuable feature. However, for many applications it is not only of interest *whether* a certain set is realizable, but also *how* to realize it. In the following we introduce a standard construction witnessing the realizability of a considered labelling-set. First, regarding arguments, we simply collect any argument occurring somewhere in a labelling. Secondly, regarding the attack relation, we set a self-loop for an argument a, whenever $\{a\}$ does not occur as an in-labelled set. Moreover, a attacks an other argument b, if $\{a\}$ can be found as an in-labelled set and b is contained in at least one corresponding out-labelled set.

Definition 10. *Given a labelling-set $\mathbb{L}$, we define $F^{cf}_{\mathbb{L}} = (A_{\mathbb{L}}, R_{\mathbb{L}})$ with $A_{\mathbb{L}} = Args(\mathbb{L})$ and*

[2]This relation is a general interaction between both kinds of semantics. Confer (Baumann 2018, Theorem 7) for more information.

1. $\forall a \in A_{\mathbb{L}}$: $(a,a) \in R_{\mathbb{L}}$ *if and only if* $\{a\} \notin \mathbb{L}^I$, *and*
2. $\forall a, b \in A_{\mathbb{L}}$: *If* $a \neq b$, *then* $(a,b) \in R_{\mathbb{L}}$ *iff* $\{a\} \in \mathbb{L}^I$ *and* $b \in \bigcup \mathbb{L}^O_{I=\{a\}}$.

Theorem 6. *For any labelling-set* $\mathbb{L}$, *we have:*

$$\mathbb{L} \in \Sigma_{\mathcal{L}_{cf}} \Leftrightarrow \mathbb{L} = \mathcal{L}_{cf}\left(F^{cf}_{\mathbb{L}}\right).$$

We emphasize two important points: First, the construction is well-defined. This means, the AF $F^{cf}_{\mathbb{L}}$ can be built for any labelling-set $\mathbb{L}$, even if the considered set is not cf-realizable. Secondly, the above theorem can in a sense also be seen as an characterization theorem. However, in contrast to Theorem 3 it requires the construction of an AF and the computation of semantics. We proceed with an illustrating example.

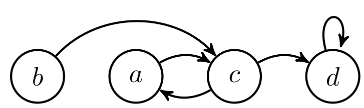
Example 7. *Consider the following labelling-set* $\mathbb{L}$ *with*

$$\begin{aligned}\mathbb{L} = \{&(\{a,b\},\{c\},\{d\}),(\{a,b\},\emptyset,\{c,d\}),\\&(\{a\},\{c\},\{b,d\}),(\{a\},\emptyset,\{b,c,d\}),\\&(\{b\},\{c\},\{a,d\}),(\{b\},\emptyset,\{a,c,d\}),\\&(\{c\},\{a,d\},\{b\}),(\{c\},\{d\},\{a,b\}),\\&(\{c\},\{a\},\{b,d\}),(\{c\},\emptyset,\{a,b,d\}),\\&(\emptyset,\emptyset,\{a,b,c,d\})\}\end{aligned}$$

We obtain $A_{\mathbb{L}} = \mathit{Args}(\mathbb{L}) = \{a,b,c,d\}$. *Regarding* $R_{\mathbb{L}}$ *we have to consider the singletons* $\{a\}, \{b\}, \{c\}$ *and* $\{d\}$.

S	$S \in \mathbb{L}^I$?	$\bigcup \mathbb{L}^O_{I=S}$	$R_{\mathbb{L}}$
$\{a\}$	*yes*	$\{c\}$	(a,c)
$\{b\}$	*yes*	$\{c\}$	(b,c)
$\{c\}$	*yes*	$\{a,d\}$	(c,a), (c,d)
$\{d\}$	*no*	$\emptyset$	(d,d)

Thus, the AF $F^{cf}_{\mathbb{L}}$ *is given as follows.*

$F^{cf}_{\mathbb{L}}$:


3.4 Representational Freedom

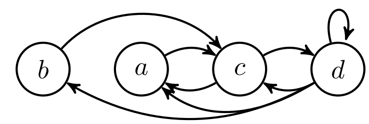
In the former sections we studied whether and how a set $\mathbb{L}$ is realizable. Now, we go a step further. We want to formally describe any possible witnessing AF for $\mathbb{L}$. Are there regularities, similarities and is it possible to easily navigate through the space of options? This question is strongly related to patterns of redundancy and requires no more and no less than a non-semantical characterization of *ordinary equivalence* (Baumann 2018; Dvorák et al. 2019). The latter relation (abbr. $F \equiv^{\sigma} G$) holds for two AFs F and G if $\sigma(F) = \sigma(G)$.

We start with an astonishing result which does not have any counterpart in the realm of extension-based semantics. If starting with a self-loop free AF, then there are no syntactic manipulations preserving ordinary equivalence.

Proposition 7. *Let two AFs* F *and* G *be given. If* G *is self-loop free, then:* $F \equiv^{\mathcal{L}_{cf}} G \Leftrightarrow F = G$.

This means, if the standard AF $F^{cf}_{\mathbb{L}}$ does not possess any self-loop, then $F^{cf}_{\mathbb{L}}$ is the only option to cf-realize $\mathbb{L}$. In other words, in this class the realizing framework is uniquely determined.The next natural question is: Are there representational differences in presence of self-loops?

Example 8 (Example 7 cont.)**.** *Consider the following AF* G. *One may easily verify that* $\mathcal{L}_{cf}(G) = \mathbb{L}$. *Thus,* $F^{cf}_{\mathbb{L}} \equiv^{\mathcal{L}_{cf}} G$.

G :


In case of the above example we observe that the representational freedom is linked to outgoing attacks from self-defeating arguments. The following theorem proves that this observation holds in general. Moreover, there are no other patterns of redundancy. We use the so-called *stable kernel* (Oikarinen and Woltran 2011; Baumann 2016) to formulate the central result in a compact way. For $F = (A,R)$ we have $F^{sk} = (A, R \setminus \{(a,b) \in R \mid a \neq b, (a,a) \in R\})$.

Theorem 8. *Given two AFs* F *and* G, *we have:*

$$F \equiv^{\mathcal{L}_{cf}} G \Leftrightarrow F^{sk} = G^{sk}.$$

3.5 Characterization Logic for Stable Semantics

Strong equivalence is of great interest in non-monotonic formalisms as it guarantees mutually replaceability in arbitrary contexts (Truszczyński 2006). For example, it is known that normal logic programs are strongly equivalent w.r.t. stable models if and only if they are ordinarily equivalent in the logic of here-and-there (Lifschitz, Pearce, and Valverde 2001). This means the logic of here-and-there can be seen as a *characterization logic* as it decides strong equivalence for logic programs (Baumann and Strass 2022). For two AFs F and G *strong equivalence* is given (abbr. $F \equiv^{\sigma}_{s} G$) if for each AF H, $\sigma(F \sqcup H) = \sigma(G \sqcup H)$.[3] The following result shows that conflict-free labellings play a similar role for stable extensions[4] as the logic of here-and-there for logic programs.

Theorem 9. *Given two AFs* F *and* G, *we have:*

$$F \equiv^{\mathcal{L}_{cf}} G \Leftrightarrow F \equiv^{\mathcal{E}_{stb}}_{s} G.$$

4 Conclusion and Related Work

Expressibility is one central issue for knowledge representation formalisms. Regarding AFs only two papers have dealt with labelling-based realizability. The first one considered *realizibility under projection* (Dyrkolbotn 2014). This means, it suffices to come up with an AF F, s.t. its set of labellings restricted to the desired arguments coincide with $\mathbb{L}$. The second work deals with the standard notion of realizability and presents a *propagate-and-guess algorithm* which returns either "No" in case of non-realizability or a witnessing AF (Linsbichler, Pührer, and Strass 2016). The mentioned papers do not consider conflict-free labellings, nor do they provide simple criteria for realizability. For future work we plan to implement and extend our study to more mature semantics such as naive, stage and stable semantics. Moreover, regarding dynamic scenarios the so-called *synthesis problem* seems to be highly significant (Niskanen, Wallner, and Järvisalo 2019). Rather than demanding precise realizability, we are faced with a set of positive labels that need to be realized, and simultaneously, a set of negative labels that must be avoided.

[3] The union $(A,R) \sqcup (B,S)$ is defined as $(A \cup B, R \cup S)$.

[4] A set is stable if it is conflict-free and attacks all other arguments.

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References

Alchourrón, C. E.; Gärdenfors, P.; and Makinson, D. 1985. On the logic of theory change: Partial meet contraction and revision functions. *Journal of Symbolic Logic* 50(2):510–530.

Arieli, O. 2012. Conflict-tolerant semantics for argumentation frameworks. In *Logics in Artificial Intelligence - 13th European Conference, JELIA 2012, Toulouse, France, September 26-28, 2012. Proceedings*, 28–40.

Atkinson, K.; Baroni, P.; Giacomin, M.; Hunter, A.; Prakken, H.; Reed, C.; Simari, G. R.; Thimm, M.; and Villata, S. 2017. Towards artificial argumentation. *AI Magazine* 38(3):25–36.

Baroni, P.; Caminada, M.; and Giacomin, M. 2018. Abstract argumentation frameworks and their semantics. In Baroni, P.; Gabbay, D.; Giacomin, M.; and van der Torre, L., eds., *Handbook of Formal Argumentation*. College Publications. chapter 4.

Baumann, R., and Strass, H. 2022. An abstract, logical approach to characterizing strong equivalence in nonmonotonic knowledge representation formalisms. *Artificial Intelligence* 305:103680.

Baumann, R. 2016. Characterizing equivalence notions for labelling-based semantics. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Fifteenth International Conference, KR 2016, Cape Town, South Africa, April 25-29, 2016*, 22–32.

Baumann, R. 2018. On the nature of argumentation semantics: Existence and uniqueness, expressibility, and replaceability. In Baroni, P.; Gabbay, D.; Giacomin, M.; and van der Torre, L., eds., *Handbook of Formal Argumentation*. College Publications. chapter 14.

Caminada, M. 2011. A labelling approach for ideal and stage semantics. *Argument & Computation* 2(1):1–21.

Delgrande, J. P.; Peppas, P.; and Woltran, S. 2013. Agm-style belief revision of logic programs under answer set semantics. In *Logic Programming and Nonmonotonic Reasoning, 12th International Conference, LPNMR 2013, Corunna, Spain, September 15-19, 2013. Proceedings*, 264–276.

Dung, P. M. 1995. On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games. *Artificial Intelligence* 77(2):321–357.

Dunne, P. E.; Dvořák, W.; Linsbichler, T.; and Woltran, S. 2013. Characteristics of multiple viewpoints in abstract argumentation. In *4th Workshop on Dynamics of Knowledge and Belief*, 16–30.

Dunne, P. E.; Dvořák, W.; Linsbichler, T.; and Woltran, S. 2015. Characteristics of multiple viewpoints in abstract argumentation. *Artificial Intelligence* 228:153–178.

Dvorák, W.; Järvisalo, M.; Linsbichler, T.; Niskanen, A.; and Woltran, S. 2019. Preprocessing argumentation frameworks via replacement patterns. In *Logics in Artificial Intelligence - 16th European Conference, JELIA 2019, Rende, Italy, May 7-11, 2019, Proceedings*, 116–132.

Dyrkolbotn, S. K. 2014. How to argue for anything: Enforcing arbitrary sets of labellings using afs. In *Principles of Knowledge Representation and Reasoning: Proceedings of the Fourteenth International Conference, KR 2014, Vienna, Austria, July 20-24, 2014*.

Eiter, T.; Fink, M.; Pührer, J.; Tompits, H.; and Woltran, S. 2013. Model-based recasting in answer-set programming. *Journal of Applied Non-Classical Logics* 23(1-2):75–104.

Lifschitz, V.; Pearce, D.; and Valverde, A. 2001. Strongly equivalent logic programs. *ACM Transactions on Computational Logic* 2(4):526–541.

Linsbichler, T.; Pührer, J.; and Strass, H. 2016. A uniform account of realizability in abstract argumentation. In *ECAI 2016 - 22nd European Conference on Artificial Intelligence, 29 August-2 September 2016, The Hague, The Netherlands - Including Prestigious Applications of Artificial Intelligence (PAIS 2016)*, 252–260.

Niskanen, A.; Wallner, J.; and Järvisalo, M. 2019. Synthesizing argumentation frameworks from examples. *Journal of Artificial Intelligence Research* 66:503–554.

Oikarinen, E., and Woltran, S. 2011. Characterizing strong equivalence for argumentation frameworks. *Artificial Intelligence* 175:1985–2009.

Truszczyński, M. 2006. Strong and uniform equivalence of nonmonotonic theories – an algebraic approach. *Annals of Mathematics and Artificial Intelligence* 48(3–4):245–265.