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Relating Abstract and Structured Accounts of Argumentation Dynamics: the Case of Expansions

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Abstract

This paper proposes a structured variant in *ASPIC*$^+$ of the notion of expansions of abstract argumentation frameworks. The purpose of this is threefold: studying what it takes to instantiate the abstract notion of expansions with a structured account of argumentation, studying to which extent assumptions implicitly made at the abstract level hold for structured instantiations and studying which potentially interesting distinctions can be made at the structured level that cannot be expressed at the abstract level.

1 Introduction

There is much abstract work on argumentation dynamics, such as work on modifying abstract (Dung 1995) or bipolar (Cayrol and Lagasquie-Schiex 2009) argumentation frameworks by adding or deleting (sets of) arguments, attacks or supports. See Doutre and Mailly (2018) for an overview. Much of this work is motivated by the aim to study preservation and enforcement properties. Preservation is about the extent to which the current status of arguments is preserved under change, while enforcement concerns the extent to which desirable outcomes can or will be obtained by changing a framework. While this body of work is very interesting, a limitation is that it disregards the structure of arguments and the nature of their relations. In consequence, it cannot recognise that some arguments are not attackable (such as arguments without assumptions in assumption-based argumentation) or that some attacks cannot be deleted (for example, undercutting attacks in *ASPIC*$^+$), or that the deletion of one argument implies the deletion of other arguments (for example, when the deleted argument is part of another argument), or that the deletion or addition of one attack implies the deletion or addition of other attacks (for example, attacking an argument implies that all arguments of which it is part are also attacked). For these reasons, formal results established at the abstract level may depend on assumptions that do not hold in general.

Accordingly, there is a need for studying argumentation dynamics in structured accounts of argumentation (Hunter 2014) and for relating such studies to abstract accounts. This paper aims to do so for the notion of expansions of abstract argumentation frameworks proposed by Baumann and Brewka (2010), which will be instantiated with *ASPIC*$^+$ (Modgil and Prakken 2018). The purpose of this is threefold: studying what it takes to instantiate the abstract notion of expansions, studying to which extent assumptions implicitly made at the abstract level hold for structured instantiations and studying which distinctions can be made at the structured level that cannot be expressed at the abstract level. The choice for *ASPIC*$^+$ is motivated by the facts that it is well-studied and often applied while variants of assumption-based (Toni 2014) and classical (Gorogiannis and Hunter 2011) argumentation can be reconstructed as special cases of *ASPIC*$^+$ (Modgil and Prakken 2018).

In our paper we will abstract from the particular ways to use expansions. We will also abstract from the procedural context in which argumentation takes place in that we will disregard the question whether an expansion is allowed according to the rules of debate (for example, whether arguments of particular types are admissible in a legal sense). Instead, we will only take structural and logical constraints on expansions into account as induced by the underlying structured account of argumentation expressed by *ASPIC*$^+$.

The rest of this paper is organised as follows. After presenting the formal preliminaries in Section 2, we will in Section 3 recall Prakken's (2022) refinement at the abstract level of Baumann and Brewka's (2010) notions of expansions. These refined notions will allow us to make explicit some implicit assumptions underlying a central result of Baumann and Brewka (2010). We will then instantiate the refined notions of expansions in terms of *ASPIC*$^+$ in Section 4 and explore some formal properties in Section 5. We discuss related work in Section 6, after which we conclude.

2 Formal Preliminaries

In this section we summarise the theory of abstract argumentation frameworks and their expansions, and *ASPIC*$^+$.

2.1 Abstract Argumentation Frameworks and their Expansions

An *abstract argumentation framework* (AF) is a pair $(\mathcal{A}, \mathcal{D})$, where $\mathcal{A}$ is a set of arguments and $\mathcal{D} \subseteq \mathcal{A} \times \mathcal{A}$ is a relation of defeat.[1] The theory of AFs (Dung 1995) identifies sets of arguments (called *extensions*) which are internally coherent and defend themselves against defeat. An

[1] Dung used the term 'attack' but since we will interpret it as the *ASPIC*$^+$ defeat relation, we will use 'defeat'.

argument $A \in \mathcal{A}$ is *defended* by a set by $S \subseteq \mathcal{A}$ if for all $B \in \mathcal{A}$: if B defeats A, then some $C \in S$ defeats B. Then relative to a given AF, $E \subseteq \mathcal{A}$ is *admissible* if E is conflict-free and defends all its members; E is a *complete extension* if E is admissible and $A \in E$ iff A is defended by E; E is a *preferred extension* if E is a $\subseteq$-maximal admissible set; E is a *stable extension* if E is admissible and attacks all arguments outside it; and $E \subseteq \mathcal{A}$ is the *grounded extension* if E is the least fixpoint of operator F, where $F(S)$ returns all arguments defended by S.

It holds that any preferred, stable or grounded extension is a complete extension. For $T \in$ {complete, preferred, grounded, stable}[2], X is *skeptically* or *credulously* justified under the T semantics if X belongs to all, respectively at least one, T extension.

Baumann and Brewka (2010) define various kinds of expansions of AFs as follows.

Definition 1. [**Expansions**] An abstract argumentation framework AF' is an *expansion* of an abstract argumentation framework $AF = (A, \mathcal{D})$ iff $AF' = (\mathcal{A} \cup \mathcal{A}', \mathcal{D} \cup \mathcal{D}')$ for some nonempty $\mathcal{A}'$ disjoint from $\mathcal{A}$. An expansion is

1. *normal* iff for all A, B: if $(A, B) \in \mathcal{D}'$ then $A \in \mathcal{A}'$ or $B \in \mathcal{A}'$,
2. *strong* iff it is normal and for all A, B: if $(A, B) \in \mathcal{D}'$ then it is not the case that $A \in \mathcal{A}$ and $B \in \mathcal{A}'$,
3. *weak* iff it is normal and for all A, B: if $(A, B) \in \mathcal{D}'$ then it is not the case that $A \in \mathcal{A}'$ and $B \in \mathcal{A}$.

In this paper we will mainly focus on normal expansions, since when instantiated with *ASPIC*$^+$, new defeats involving old arguments will be possible.

2.2 The *ASPIC*$^+$ Framework

The *ASPIC*$^+$ framework defines abstract argumentation systems as structures consisting of a logical language $\mathcal{L}$ and two sets $\mathcal{R}_s$ and $\mathcal{R}_d$ of strict and defeasible inference rules defined over $\mathcal{L}$. In this paper we for simplicity assume that $\mathcal{L}$ contains ordinary negation $\neg$ but all new definitions proposed in this paper can be easily adapted to versions of *ASPIC*$^+$ with asymmetric negation. Arguments are constructed from a knowledge base (a subset of $\mathcal{L}$) by chaining inferences over $\mathcal{L}$ into acyclic graphs (which are trees if no premise is used more than once). Formally,

Definition 2. [**Argumentation System**] an *argumentation system* (AS) is a triple $AS = (\mathcal{L}, \mathcal{R}, n)$ where:

- $\mathcal{L}$ is a logical language with a negation symbol $\neg$;
- $\mathcal{R} = \mathcal{R}_s \cup \mathcal{R}_d$ is a finite set of strict ($\mathcal{R}_s$) and defeasible ($\mathcal{R}_d$) inference rules of the form $\varphi_1, \ldots, \varphi_n \to \varphi$ and $\{\varphi_1, \ldots, \varphi_n\} \Rightarrow \varphi$ respectively (where φ_i, φ are meta-variables ranging over wff in $\mathcal{L}$), such that $\mathcal{R}_s \cap \mathcal{R}_d = \emptyset$. Here, $\varphi_1, \ldots, \varphi_n$ are called the *antecedents* and φ the *consequent* of the rule.
- n is a partial function such that $n : \mathcal{R}_d \longrightarrow \mathcal{L}$.

[2]In later papers new semantics have been introduced, see Baroni, Caminada, and Giacomin (2011), but we only discuss these semantics of Dung (1995).

Informally, $n(r)$ is a well-formed formula (wff) in $\mathcal{L}$ which says that the defeasible rule $r \in \mathcal{R}$ is applicable, so that an argument claiming $\neg n(r)$ attacks an inference step in the argument using r. We write $\psi = -\varphi$ just in case $\psi = \neg\varphi$ or $\varphi = \neg\psi$. We use $\leadsto$ as a variable ranging over $\{\to, \Rightarrow\}$. Since the order of antecedents of a rule does not matter, we sometimes write $S \leadsto \varphi$ where S is the set of all antecedents of the rule.

Definition 3. [**Knowledge bases**] A *knowledge base* in an $AS = (\mathcal{L}, \mathcal{R}, n)$ is a set $\mathcal{K} \subseteq \mathcal{L}$ consisting of two disjoint subsets $\mathcal{K}_n$ (the *axioms*) and $\mathcal{K}_p$ (the *ordinary premises*).

Definition 4. [**Argumentation theories**] An *argumentation theory* is a pair $(AS, \mathcal{K})$ where AS is an argumentation system and $\mathcal{K}$ a knowledge base in AS.

Definition 5. [**Arguments**] A *argument* A on the basis of an argumentation theory AT is a structure obtainable by applying one or more of the following steps finitely many times:

1. φ if $\varphi \in \mathcal{K}$ with: $\texttt{Prem}(A) = \{\varphi\}$; $\texttt{Conc}(A) = \varphi$; $\texttt{Prop}(A) = \{\varphi\}$,
   $\texttt{Sub}(A) = \{\varphi\}$; $\texttt{Rules}(A) = \emptyset$; $\texttt{DefRules}(A) = \emptyset$; $\texttt{TopRule}(A) =$ undefined.
2. $A_1, \ldots, A_n \leadsto \psi$ if $A_1, \ldots, A_n$ are arguments such that $\psi \notin \texttt{Conc}(\{A_1, \ldots, A_n\})$ and $\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \leadsto \psi \in \mathcal{R}$ with:
   $\texttt{Prem}(A) = \texttt{Prem}(A_1) \cup \ldots \cup \texttt{Prem}(A_n)$;
   $\texttt{Conc}(A) = \psi$;
   $\texttt{Prop}(A) = \texttt{Prop}(A_1) \cup \ldots \cup \texttt{Prop}(A_n) \cup \{\psi\}$,
   $\texttt{Sub}(A) = \texttt{Sub}(A_1) \cup \ldots \cup \texttt{Sub}(A_n) \cup \{A\}$;
   $\texttt{Rules}(A) = \texttt{Rules}(A_1) \cup \ldots \cup \texttt{Rules}(A_n) \cup \{\texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \leadsto \psi\}$;
   $\texttt{DefRules}(A) = \texttt{Rules}(A) \cap \mathcal{R}_d$;
   $\texttt{TopRule}(A) = \texttt{Conc}(A_1), \ldots, \texttt{Conc}(A_n) \leadsto \psi$.

$\texttt{Prem}_n(A) = \texttt{Prem}(A) \cap \mathcal{K}_n$ and $\texttt{Prem}_p(A) = \texttt{Prem}(A) \cap \mathcal{K}_p$. Furthermore, argument A is *strict* if $\texttt{DefRules}(A) = \emptyset$ and *defeasible* otherwise, and A is *firm* if $\texttt{Prem}_p(A) = \emptyset$, otherwise A is *plausible*. The set of all arguments on the basis of AT is denoted by $\mathcal{A}_{AT}$.

Each of the functions $\texttt{Func}$ in this definition is also defined on sets of arguments $S = \{A_1, \ldots, A_n\}$ as follows: $\texttt{Func}(S) = \texttt{Func}(A_1) \cup \ldots \cup \texttt{Func}(A_n)$. Note that the $\to$ and $\Rightarrow$ symbols are overloaded to denote both inference rules and arguments. In this paper we do not discuss variants of *ASPIC*$^+$ in which the premises of an argument must be consistent (see Modgil and Prakken (2018)). All new definitions proposed in this paper directly apply to these versions.

Definition 6. [**Attack**] Argument A *attacks* argument B iff A *undercuts* or *rebuts* or undermines B, where:

- A *undercuts* B (on B') iff $\texttt{Conc}(A) = -n(r)$ and $B' \in \texttt{Sub}(B)$ such that B''s top rule r is defeasible.
- A *rebuts* B (on B') iff $\texttt{Conc}(A) = -\varphi$ for some $B' \in \texttt{Sub}(B)$ of the form $B''_1, \ldots, B''_n \Rightarrow \varphi$.
- A *undermines* B (on φ) iff $\texttt{Conc}(A) = -\varphi$ for some $\varphi \in \texttt{Prem}(B) \cap \mathcal{K}_p$.

Definition 7. [**Structured Argumentation Frameworks**] A *structured argumentation framework (SAF)* defined by an argumentation theory AT is a triple $(\mathcal{A}, \mathcal{C}, \preceq)$ where $\mathcal{A}$ is the set of all arguments on the basis of AT, $\preceq$ is an ordering on $\mathcal{A}$ and $(X, Y) \in \mathcal{C}$ iff X attacks Y.

The notion of *defeat* is now defined as follows. Undercutting attacks succeed as *defeats* independently of preferences over arguments, since they express exceptions to defeasible inference rules. Rebutting and undermining attacks succeed only if the attacked argument is not stronger than the attacking argument, where $A \prec B$ is defined as usual as $A \preceq B$ and $B \npreceq A$ and $A \approx B$ as $A \preceq B$ and $B \preceq A$. Below we assume that for no arguments A and B both $A \prec B$ and $B \prec A$ hold, while, moreover, if A is strict and firm, then $A \prec B$ does not hold.

Definition 8. [**Defeat**] Argument A *defeats* argument B iff either A undercuts B; or A rebuts or undermines B on B' and $A \nprec B'$.

Abstract argumentation frameworks are then generated from $SAFs$ as follows:

Definition 9. [**Argumentation frameworks**] An *abstract argumentation framework (AF) corresponding to a* $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ is a pair $(\mathcal{A}, \mathcal{D})$ such that $\mathcal{D}$ is the defeat relation on $\mathcal{A}$ determined by SAF.

3 An Abstract Framework for Specifying Expansions

In this section we recall Prakken's (2022) refined versions of Baumann and Brewka's (2010) notions of an expansion. The main refinements are that expansions are now relative to a given background *universal argumentation framework* $UAF = (\mathcal{A}^u, \mathcal{D}^u)$ and that expansions can be allowed or not allowed. Prakken (2022) made these refinements part of an abstract account of dialectical argument strength. For present purposes they will turn out to be useful for avoiding implicit assumptions at the abstract level that are not always satisfied by instantiations. An interesting question is how a UAF can be sensibly fixed in applications. In the following sections we will make several observations on how this can be done on logical, dialogical or knowledge-based grounds.

Definition 10. [**Argumentation frameworks in a universal *AF***] Given a universal argumentation framework $UAF = (\mathcal{A}^u, \mathcal{D}^u)$, an *argumentation framework in* UAF is any $AF = (\mathcal{A}, \mathcal{D})$ such that $\mathcal{A} \subseteq \mathcal{A}^u$ and $\mathcal{D} \subseteq \mathcal{D}^u_{|\mathcal{A} \times \mathcal{A}}$.

The fact that $\mathcal{D}$ is not required to equal $\mathcal{D}^u_{|\mathcal{A} \times \mathcal{A}}$ is to allow for instantiations with systems like *ASPIC*$^+$ that use preferences to resolve attacks.

We must also distinguish between allowed and not allowed expansions. One reason is that the dialogical protocol may impose constraints, such as admissibility of particular types of evidence or arguments. The problem context may also impose restrictions. For example, in investigation procedures in which information gathering is interchanged with argument construction from the gathered information, there may be requirements that all and only relevant arguments constructible from the gathered information are included. Finally, and the most relevant for the present paper, underlying structured accounts of argumentation may impose such constraints, as we will see in Section 4 for *ASPIC*$^+$.

Definition 11. [**Expansions given a universal argumentation framework**] Let $AF = (\mathcal{A}, \mathcal{D})$ and AF' be two abstract argumentation frameworks in UAF. Then AF' is an *expansion* of AF given UAF if $AF' = (\mathcal{A} \cup \mathcal{A}', \mathcal{D} \cup \mathcal{D}')$ for some nonempty $\mathcal{A}'$ disjoint from $\mathcal{A}$. The notions of a normal, weak and strong expansion in UAF are defined as the corresponding notions in Definition 1.

Let $X_{UAF}(AF)$ be the set of all expansions of AF given UAF. Then the set of *allowed expansions* of AF given UAF is some designated subset of $X_{UAF}(AF)$.

We can now explain how a central result of Baumann and Brewka (2010) depends on some implicit assumptions. Their Theorem 4 says that for $T \in$ {complete, preferred, grounded, stable}[3], for any $AF = (\mathcal{A}, \mathcal{D})$ and for any conflict-free $C \subset \mathcal{A}$ unequal to a T-extension of AF, there exists a strong expansion AF' of AF such that $C \subset E$ for some T-extension E of AF', where the expansion can be chosen such that E is the unique T-extension of AF'. The proof of this result shows how a single argument can be added that defeats all arguments in AF outside C. This construction depends on the assumptions that this expansion is available given UAF and is allowed, which, as explained in Section 4 may not be the case. However, this assumption is not implied by our Definitions 10 and 11 and in Section 4 we will see that this is for good reasons. For now we give a simple abstract example (also given by Baumann and Brewka), with an AF consisting of two arguments A_2 and A_1 where A_2 defeats A_1. According to Baumann and Brewka the AF can be expanded by adding some A_3 defeating A_2 but if A_2 is unattackable, (for instance, since it is a strict-and-firm *ASPIC*$^+$ argument or an argument without assumptions in assumption-based argumentation), then no UAF respecting the underlying structured account will enable or allow such an expansion. This illustrates one benefit of our Definitions 10 and 11, namely, that they enable the explicitation of assumptions that are implicit in Definition 1.

4 Instantiating the Abstract Framework for *ASPIC*$^+$

In this section we instantiate Definitions 10 and 11 for *ASPIC*$^+$. This requires a specification of how the UAF can be generated by a universal *structured* argumentation framework to which it corresponds. Since a *SAF* is in *ASPIC*$^+$ determined by an argumentation theory, we must also specify the notion of a universal argumentation theory.

4.1 Universal Structured Argumentation Frameworks

A UAF is now defined as corresponding to a universal structured argumentation framework, which is in turn defined by a universal argumentation theory. Together, they define the space of possible knowledge bases, possible sets of inference rules and possible argument orderings and thus define the space of possible argumentation frameworks.

[3] Baumann and Brewka (2010) also include ideal semantics.

Definition 12. [**Universal Argumentation theories and universal structured *AFs***] A *universal argumentation theory* is a tuple $UAT = ((\mathcal{L}^u, \mathcal{R}_s^u \cup \mathcal{R}_d^u, n^u), \mathcal{K}_n^u \cup \mathcal{K}_p^u)$ where all elements are defined as for *ASPIC*$^+$ argumentation theories except that $\mathcal{K}_n^u$ and $\mathcal{K}_p^u$ do not have to be disjoint. Then a *universal structured argumentation framework* defined by UAT is a tuple $USAF = (\mathcal{A}^u, \mathcal{C}^u, \preceq^u)$ defined according to Definition 7, where $\preceq^u$ is an empty preference ordering on $\mathcal{A}^u$. A $UAF = (\mathcal{A}^u, \mathcal{D}^u)$ that is the abstract argumentation framework corresponding to some given $USAF$ is denoted by $sUAF$.

Note that the sets $\mathcal{R}_s^u$ and $\mathcal{R}_d^u$ of a UAT are not required to contain all well-formed strict, respectively, defeasible rules over $\mathcal{L}^u$. This is to allow for instantiations where the strict rules are defined by a logical interpretation of $\mathcal{L}^u$ and/or the defeasible rules correspond to some recognised set of argument schemes. The limiting case where $\mathcal{R}_s^u$ and $\mathcal{R}_d^u$ do contain all well-formed rules over $\mathcal{L}$ is suitable for applications where the choice of strict and/or defeasible rules is fully free, as, for instance, in online debate settings. For similar reasons $\mathcal{K}_p^u$ and K_p^u are not required but are allowed to equal $\mathcal{L}^u$. The reason why $\mathcal{K}_n^u$ and K_p^u can overlap is to allow that the type of a premise is unspecified until determined when constructing an AT in UAT. Accordingly, to keep the notion of an argument on the basis of a UAT well-defined, we now assume that in Definition 5(1) it is explicitly indicated whether a premise is taken from $\mathcal{K}_n^u$ or from $\mathcal{K}_p^u$. Finally, the idea behind the choice of $\preceq^u$ as the empty ordering is that a universal SAF does not commit to any way to resolve preference-dependent conflicts. Note that the empty ordering induces the greatest set of defeat relations in that every attack succeeds as defeat. Commitments on how conflicts should be resolved can be expressed in the specification of a SAF in a $USAF$, by adopting any nonempty argument ordering. At the abstract level this was captured in Definition 10 in the use of $\subseteq$ instead of $=$ in the requirement that $\mathcal{D} \subseteq \mathcal{D}^u_{|\mathcal{A}\times\mathcal{A}}$. The structural counterpart of this definition looks as follows.

Definition 13. [**Argumentation theories and structured *AFs* in a universal *AT***] An *argumentation theory in* a given UAT is an *ASPIC*$^+$ argumentation theory $AT = ((\mathcal{L}, \mathcal{R}_s \cup \mathcal{R}_d, n), \mathcal{K}_n \cup \mathcal{K}_p)$ where

- $\mathcal{L} \subseteq \mathcal{L}^u$;
- $\mathcal{R} \subseteq \{S \rightsquigarrow \varphi \in \mathcal{R}^u \mid S \subseteq \mathcal{L}$ and $\varphi \in \mathcal{L}\}$;
- $\mathcal{K}_n \subseteq \mathcal{K}_n^u$;
- $\mathcal{K}_p \subseteq \mathcal{K}_p^u$;
- $n = n^u \cap \{(r, \varphi) \mid r \in \mathcal{R}_d\}$.

An argumentation theory in UAT is *objective* iff $\mathcal{K}_n = \mathcal{K}_n^u \cap \mathcal{L}$, it is *logic-based* iff $\mathcal{R}_s = \{S \to \varphi \in \mathcal{R}_s^u \mid S \subseteq \mathcal{L}$ and $\varphi \in \mathcal{L}\}$, and it is *strongly logic-based* iff $\mathcal{R} = \{S \rightsquigarrow \varphi \in \mathcal{R}^u \mid S \subseteq \mathcal{L}$ and $\varphi \in \mathcal{L}\}$.

A *structured argumentation framework in* UAT is a structured argumentation framework $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ defined by an AT in UAT for some ordering $\preceq$ on $\mathcal{A}$.

Objective ATs are called objective since they accept all necessary premisses from UAT that can be expressed in their language. Objective ATs may be suitable for knowledge-based systems (such as for medical diagnosis or crime investigation), in which the general knowledge is fixed but investigations must be done to gather specific observations (such as medical tests on a person who is ill, or searching for evidence predicted by a crime scenario). (Strongly) logical ATs are called thus since they accept all (defeasible and) strict inference rules from UAT that can be expressed over their language. Consider, for example, a UAT with $\mathcal{L}$ a propositional language and $\mathcal{R}_s = \{S \to \varphi \mid S \subseteq \mathcal{L}$ and S is finite and $\varphi \in \mathcal{L}$ and $S \vdash \varphi\}$ where $\vdash$ denotes propositional-logical consequence. Then all logic-based AT's in UAT allow for deductive reasoning with the full power of propositional logic over their language. Non-logic-based and non-objective ATs make sense for the formal reconstruction of natural-language debates, in which often anything can be said and challenged. So then the universal sets of inference rules correspond to any argument that can be constructed, whether 'valid' in some sense or not, and the universal knowledge base consists of any premise that can be used, whether it corresponds to genuine knowledge or not.

The following proposition captures that Definitions 12 and 13 indeed instantiate Definitions 10 and 11 since it implies that every AF that can be generated from a universal argumentation theory is an AF in the same universal AF as required by Definition 10 (which is used in Definition 11).

Proposition 1. Given a $USAF = (\mathcal{A}^u, \mathcal{C}^u, \preceq^u)$ in a $UAT = ((\mathcal{L}^u, \mathcal{R}_s^u \cup \mathcal{R}_d^u, n^u), \mathcal{K}_n^u \cup \mathcal{K}_p^u)$, an $AF = (\mathcal{A}, \mathcal{D})$ corresponding to a $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ in UAT is an AF in $sUAF = (\mathcal{A}^u, \mathcal{D}^u)$, where $sUAF$ corresponds to $USAF$.

Proof. It holds that $\mathcal{A} \subseteq \mathcal{A}^u$ by definition of a SAF in UAT. Furthermore, it holds that $\mathcal{D} \subseteq \mathcal{D}'$ since $\preceq^u = \emptyset$ and therefore makes every attack succeed as defeat, so no additional defeat relations are possible. □

4.2 Allowed Expansions

So far all we have done is instantiating the notion of an AF in a UAF for *ASPIC*$^+$(as captured by Proposition 1). The next step is to define the *allowed expansions* of an AF that corresponds to a SAF in a universal argumentation theory. The main task is to ensure that the result of such an expansion still corresponds to a structured AF in the universal argumentation theory, in order to respect the structural constraints imposed by *ASPIC*$^+$. Since the idea of expansions as originally proposed by Baumann and Brewka (2010) is that information is only added and not deleted, a natural way to achieve this is to require that expansions correspond to a SAF that expand (in a sense to be defined) the SAF to which the expanded AF corresponds. This is directly stated by the following definition. It assumes that the argument ordering $\preceq$ of a SAF comes with a definition of its type, as, for example, the definitions of a basic, weakest- or last link ordering (Modgil and Prakken 2018).

Definition 14. [**Allowed expansions**] Consider any AF in a given $sUAF$ that corresponds to a $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ in UAT defined by $AT = ((\mathcal{L}, \mathcal{R}, n), \mathcal{K})$, and consider any AF' in $sUAF$ that expands AF. Then AF' is an *allowed expansion* of AF given UAF iff AF' corresponds to a $SAF' = (\mathcal{A}', \mathcal{C}', \preceq')$ in UAT defined by $AT' = ((\mathcal{L}', \mathcal{R}', n'), \mathcal{K}')$ such that:

1. $\mathcal{L} \subseteq \mathcal{L}'$;
2. $\mathcal{R} \subseteq \mathcal{R}'$;
3. $\mathcal{K}_n \subseteq \mathcal{K}'_n$ and $\mathcal{K}_p \subseteq \mathcal{K}'_p$;
4. for $\preceq'$ it holds that
   (a) $\preceq'$ is of the same type as $\preceq$;
   (b) $\preceq \subseteq \preceq'$;
   (c) $A \prec' B$ if $A \prec B$;
5. if AT is (strongly) logic-based then AT' is (strongly) logic-based;
6. if AT is objective then AT' is objective.

Strictly speaking there is no need to define additional constraints on $\preceq'$ since the defeat relation of an expansion is by definition contained in the defeat relation of the expanded AF. Nevertheless, the specified constraints agree with the idea that only information is added. Note that the constraints on $\preceq'$ together make that SAF' *extends* SAF in the sense of Modgil and Prakken (2012). In this case we also say that $\preceq'$ extends $\preceq$. Adding preferences to resolve attacks (in the sense of Modgil and Prakken (2012)) between arguments makes sense in the present setup, since it is analogous to adding new rules or premises to an AT.

Although at the structured level expansions can be generated by simply expanding an argumentation theory, they can also be induced by sets of new arguments, for instance, put forward in a debate. Therefore, it is useful to define the notion of an AT extended by a set of arguments.

Definition 15. [**Argumentation theories extended by sets of arguments**] Let $AT = ((\mathcal{L}, \mathcal{R}_s \cup \mathcal{R}_d, n), \mathcal{K}_n \cup \mathcal{K}_d)$ be an argumentation theory in a given UAT and let $S \subseteq \mathcal{A}^u$. Then $AT + S$ is defined as $((\mathcal{L}', \mathcal{R}'_s \cup \mathcal{R}'_d, n'), \mathcal{K}'_n \cup \mathcal{K}'_d)$ such that:

1. $\mathcal{L}' = \mathcal{L} \cup \texttt{Prop}(S)$;
2. $\mathcal{R}'_s = \mathcal{R}_s \cup \texttt{Rules}(S) \cap \mathcal{R}^u_s$ if AT is not logic-based, otherwise $\mathcal{R}'_s = \{S \to \varphi \in \mathcal{R}^u_s \mid S \subseteq \mathcal{L}' \text{ and } \varphi \in \mathcal{L}'\}$;
3. $\mathcal{R}'_d = \mathcal{R}_d \cup \texttt{Rules}(S) \cap \mathcal{R}^u_d$ if AT is not strongly logic-based, otherwise $\mathcal{R}'_d = \{S \Rightarrow \varphi \in \mathcal{R}^u_d \mid S \subseteq \mathcal{L}' \text{ and } \varphi \in \mathcal{L}'\}$;
4. $n' = n \cup n^u \cap \{(d, \varphi) \mid d \in \mathcal{R}^u_d \text{ and } \varphi = n^u(d)\}$;
5. $\mathcal{K}'_n = \mathcal{K}_n \cup \{\varphi \mid \varphi \in \texttt{Prem}_n(S) \cap \mathcal{K}^u_n \text{ and } \varphi \notin \mathcal{K}_p\}$ if AT is not objective, otherwise $\mathcal{K}'_n = \mathcal{K}^u_n \cap \mathcal{L}'$;
6. $\mathcal{K}'_p = \mathcal{K}_p \cup \{\varphi \mid \varphi \in \texttt{Prem}_p(S) \cap \mathcal{K}^u_p \text{ and } \varphi \notin \mathcal{K}_n\}$.

One idea here is that $AT + S$ adds all language elements, rules and premises of any argument in S to the corresponding elements of AT. A complication is that if an argument in S uses a premise that is in both $\mathcal{K}^u_n$ and $\mathcal{K}^u_p$, then $AT + S$ has to respect the choice of the type of the premise made in AT. A second idea is that if AT is (strongly) logic-based, then $AT + S$ also adds all strict (and defeasible) rules over the extended language that are not in either AT nor used in any argument in S. This ensures that 'implied' arguments that are neither constructible on the basis of AT nor a member of S are constructible on the basis of AT'. Finally, note that arguments in S cannot change the type of elements in $\mathcal{K}$. If S fully respects the type of elements of $\mathcal{K}$, we say that S *respects* $\mathcal{K}$ of AT. (Similar concerns do not arise for the set of rules, since there can be strict and defeasible rules with the same antecedents and consequent.)

Proposition 2. The following observations hold:

1. $AT + S$ is an argumentation theory in UAF.
2. If AT is (strongly) logic-based then $AT + S$ is (strongly) logic-based and $\mathcal{R}_s \subseteq \mathcal{R}'_s$ (and $\mathcal{R}_d \subseteq \mathcal{R}'_d$).
3. Let $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ and $SAF' = (\mathcal{A}', \mathcal{C}', \preceq')$ be structured argumentation frameworks defined by, respectively, AT and $AT + S$ and let $\preceq$ and $\preceq'$ satisfy condition (4) of Definition 14. Let AF correspond to SAF and AF' correspond to SAF. Then AF' is an allowed expansion of AF.

Item (3) of this proposition implies that allowed expansions at the abstract level can be generated from a given SAF by extending the argumentation theory defining the SAF in a way that satisfies according to Definition 15 and extending the argument ordering of the SAF in a way that satisfies condition (4) of Definition 14.

5 Properties

In this section we investigate some properties of the formal account of the previous section. To start with, since an expansion that is allowed according to Definition 14 corresponds to a SAF, it by definition satisfies closure under argument construction, under the subargument relation and under the constraints that *ASPIC*$^+$ imposes on the defeat relation. For example, it satisfies the constraint that if A defeats B and B is a subargument of C, then A defeats C (in the literature on bipolar argumentation frameworks (Cohen et al. 2018) called *closure under secondary attacks*).

Next it holds that each allowed expansion adds at least one rule or one premise, otherwise it contains no new arguments. Furthermore, if all argumentation theories in UAT are strongly logic-based, then each allowed expansion adds at least one new premise, since otherwise such expansions cannot add new rules and so cannot give rise to new arguments. The same does not hold if all argumentation theories in UAT are logic-based but not strongly so, since then an allowed expansion can add a new defeasible rule.

We next identify a set of assumptions under which the structured counterpart of Theorem 4 of Baumann and Brewka (2010) holds. We first prove necessary-and-sufficient conditions for *credulous enforcement* in complete and preferred semantics in that a set of arguments can be made part of a preferred or complete extension of an expansion. We will focus on sets of arguments that are not in any extension. Note that this is a special case of sets of arguments that are unequal to any T-extension, which are the focus of Theorem 4 of Baumann and Brewka (2010). Yet another option would be to consider sets of arguments that are not a subset of any extension. One reason for deviating from Baumann and Brewka (2010) is that if S is a proper subset of some extension, then there is no need to consider expansions to make S part of an extension (note that all expansions add at least one argument, so this case cannot be included as a special case of enforcement).

Theorem 1. *Let $T \in \{complete, preferred\}$, let $AF = (\mathcal{A}, \mathcal{D})$ be an abstract argumentation framework in a $sUAF$ that corresponds to a $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ in a UAT, and $S \subseteq \mathcal{A}$ any nonempty conflict-free set of arguments that are not a member of any T-extension of AF. Then there exists an allowed normal expansion AF' of AF in $sUAF$ corresponding to a $SAF' = (\mathcal{A}', \mathcal{C}', \preceq')$ such that $S \subset E'$ for a T-extension of AF' iff there exists a nonempty set $S' \in \mathcal{A}^u \setminus \mathcal{A}$ respecting $\mathcal{K}$ of the AT defining SAF and some $\preceq'$ extending $\preceq$ while preserving its type, such that according to $\preceq'$:*

1. *$S \cup S'$ is conflict-free;*
2. *every defeater of $S \cup S'$ in $\mathcal{A}_{AT+S'}$ is defeated by $S \cup S'$.*

Proof. The if-part is immediate from the definition of an admissible set and the facts that any admissible set is included in a preferred extension and that any preferred extension is complete, while the only-if part follows from the fact that every preferred or complete extension is an admissible set (Dung 1995). □

For stable semantics there are counterexamples. Consider $AF = (\{A, B\}, \{(A, A)\})$: then $\{B\}$ cannot be credulously enforced if the UAF contains no defeater of A. If S is instead chosen to be any conflict-free set not included in any T-extension then the if-part still holds but there are counterexamples to the only-if part. For an abstract counterexample see Figure 1 and let $S = \{A, C\}$ and let $USAF$ contain an undefeated defeater of B but not of C. Then condition (2) of Theorem 1 is not satisfied but expanding AF with the defeater of B makes S part of every preferred and every complete extension, since C is defended by E. A structured instantiation of this example can be easily defined.

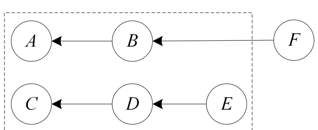

Figure 1: Counterexample to modified Theorem (only-if part).

Admittedly the conditions under which Theorem 1 holds are quite high-level but useful examples can be given about more concrete conditions under which these conditions are or are not satisfied. Before discussing such examples, we first identify a set of sufficient conditions for *skeptical enforcement* in complete, preferred and grounded semantics (i.e., for when a set of arguments can be made part of all extensions of these kinds for some expansion).

Theorem 2. *Let $T \in \{complete, preferred, grounded\}$, let $AF = (\mathcal{A}, \mathcal{D})$ be an abstract argumentation framework in a $sUAF$ that corresponds to a $SAF = (\mathcal{A}, \mathcal{C}, \preceq)$ in a UAT, and $S \subseteq \mathcal{A}$ any nonempty conflict-free set of arguments that are not a member of any T-extension of AF. Then there exists an allowed normal expansion AF' of AF in $sUAF$ corresponding to a $SAF' = (\mathcal{A}', \mathcal{C}', \preceq')$ such that $S \subset E'$ for all T-extensions of AF' if there exists a nonempty set $S' \in \mathcal{A}^u \setminus \mathcal{A}$ respecting $\mathcal{K}$ of the AT defining SAF and and some $\preceq'$ extending $\preceq$ while preserving its type such that according to $\preceq'$*

1. *$\mathcal{A}_{AT+S'}$ does not defeat S';*
2. *every defeater of S in $\mathcal{A}_{AT+S'}$ is defeated by S'.*

Proof. Consider without loss of generality a minimal set S' satisfying assumptions (1-2), and let AF' correspond to the $SAF' = (\mathcal{A}', \mathcal{C}', \preceq')$ determined by $AT + S'$ with $\preceq'$ some extension of $\preceq$ that preserves its type. With this choice of SAF', AF' corresponds to a SAF' that satisfies the conditions of Definition 14 so it is an allowed expansion of AF. We can choose S' in such a way that it equals $\mathcal{A}_{AT+S'} \setminus \mathcal{A}$. Let us do so.

By assumption (1) we have that S' is conflict-free and all its members are undefeated in AF'. Then S' is included in the grounded extension of AF' since it is undefeated. But then assumption 2 gives that S' is defended by the grounded extension of AF' so S' is included in it. Finally, since a grounded extension is included in every preferred and every complete extension (Dung 1995), S is included in every preferred and every complete extension of AF'.

□

We next prove a result on skeptical enforcement of single arguments given a particular kind of UAT by adapting the notion of strong unacceptability from Rapberger and Ulbricht (2022) to our setting. For any AF in a given UAF, let an argument A be *strongly unacceptable* in AF iff AF contains a defeater B such that any argument C in UAF that defeats B also defeats A. Then the following holds.

Theorem 3. *Let $T \in \{complete, preferred, stable, grounded\}$, let $AF = (\mathcal{A}, \mathcal{D})$ be an abstract argumentation framework in a $sUAF$ that corresponds to a SAF in a UAT where $\mathcal{R}_s^{UAT}$ contains rules $\rightarrow \varphi$ for every $\varphi \in \mathcal{L}^{UAT}$. Let $A \in \mathcal{A}$ be any argument that is not skeptically T-acceptable in AF. Then there exists an allowed expansion AF' of AF given UAF in which A is skeptically T-acceptable iff A is not strongly unacceptable in AF.*

Proof. Suppose A is not skeptically T-acceptable in AF. If A is strongly unacceptable in AF, then A has a defeater B in AF for which any defeater in $sUAF$ also defeats A in $sUAF$. This in particular holds for all defeaters C of the from $\rightarrow -\varphi$ for a rebuttable conclusion, ordinary premise or name of a defeasible rule of B of the form φ, which defeaters exist by assumption on $\mathcal{R}_s^{UAT}$. Since by assumption on $\preceq$ for no such C it holds that $C \prec B$, these defeat relations also hold in any expansion of AF with C. Then no expansion can make A T-acceptable.

If A is not strongly unacceptable in AF, then any defeater B of A in AF has defeaters in $sUAF$. Any such defeater has a conclusion $-\varphi$ for a wff φ that is either a rebuttable conclusion or an ordinary premise or a name of a defeasible rule of B. But then UAT contains a rule $\rightarrow -\varphi$ which as an argument C is an undefeatable defeater of B in both $sUAF$ and (by the assumption on $\prec$ for strict-and-firm arguments) any extension of AF with C. Then extending AF with C results in an AF' for which A is in the grounded extension.

And since that extension is included in all complete, stable and preferred extensions (Dung 1995), A is also in all those extensions of AF'. □

The proof of the only-if part implies that it also holds for any case where the argument ordering is simple ($A \preceq B$ iff A is defeasible or plausible and B is strict and firm), regardless of the content of $\mathcal{R}_s^{UAT}$. Moreover, for $T \in$ {complete, preferred, grounded} the credulous version of Theorem 3 (where A is not credulously acceptable in AF but can be made so in an allowed expansion) is provable in exactly the same way. For stable semantics the proof of the if-part must be extended to guarantee the existence of stable extensions.

We now discuss more concrete conditions under which the conditions of Theorems 1 and 2 are not satisfied.

Not all arguments are attackable In Section 3 we already observed that Theorem 4 of Baumann and Brewka (2010) depends on the assumption that all arguments are attackable.

No conflict-free set of defenders Let $\mathcal{K}_n^u = \emptyset$ and consider an AF consisting of A: $\Rightarrow q$ defeated by both B: $p \rightarrow \neg q$ and C: $\neg p \rightarrow \neg q$ where $A \prec B$ and $A \prec C$ and where the strict version of the argument ordering $\prec$ is asymmetric. Assume, furthermore, that on the basis of UAT only one defeater D: $r \rightarrow \neg p$ of B and one defeater E: $\neg r \rightarrow p$ of C can be constructed and that $\mathcal{K}_n$ contains r and $\neg r$. Then there is no conflict-free expansion that satisfies assumption (2) of Theorem 1, since for any $\preceq$ it holds that D defeats E or E defeats D.

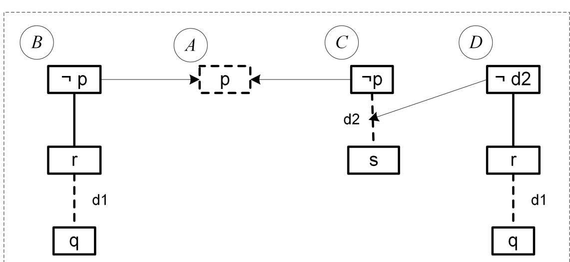

Figure 2: An assumption underlying Theorem 4 of Baumann and Brewka (2010).

All defenders imply a defeater Consider the example in Figure 2 based on an AT with $\mathcal{K}_n = \{q, s\}$, $\mathcal{K}_p = \{p\}$, $\mathcal{R}_s = \{s \rightarrow \neg p; r \rightarrow \neg p; r \rightarrow \neg d_2\}$, $\mathcal{R}_d = \{q \Rightarrow_{d1} r; s \Rightarrow_{d2} \neg p\}$ where the subscripts of $\Rightarrow$ denote the rule names. The AF contains A, B, C, D and all their subarguments, where both B and C defeat A and D defeats C. Assume that $A \prec B$ and $A \prec C$. Note that A is not in any T-extension for T = grounded or complete or preferred, since it is defeated by B which is undefeated. The question is whether A can be made part of all $T-$extensions of some allowed expansion of AF. All such expansions must add a defeater E of B's subargument for r. Assume that on the basis of UAT a single undefeated argument E exists that defeats B but no defeater of C other than D exists. For instance, UAT could differ from AT only in that it also contains a strict rule $s \rightarrow \neg q$. Then any expansion defeating B contains E so also D is strictly defeated (on its subargument for r). But then C is defended and prevents A from being in any T-extension of the expansion. Hence no expansion exists in which A is in any T-extension. Dung (1995) calls arguments like E, which both defend and indirectly defeat an argument, *controversial arguments*. This example illustrates another assumption underlying Theorem 4 of Baumann and Brewka (2010), namely, that a defeat from a new to an old argument has no side effects in that the new argument also defeats other old arguments that are relevant to the status of an argument in the set that should be in an extension of the expansion. In other words, it is not the case in general that a set S' can be found such that $S \cup S'$ is admissible.

Further implicit assumptions in Baumann and Brewka (2010) are visualised in Figure 3, where the dotted boxes contain AFs while the entire graphs are $UAFs$. For the

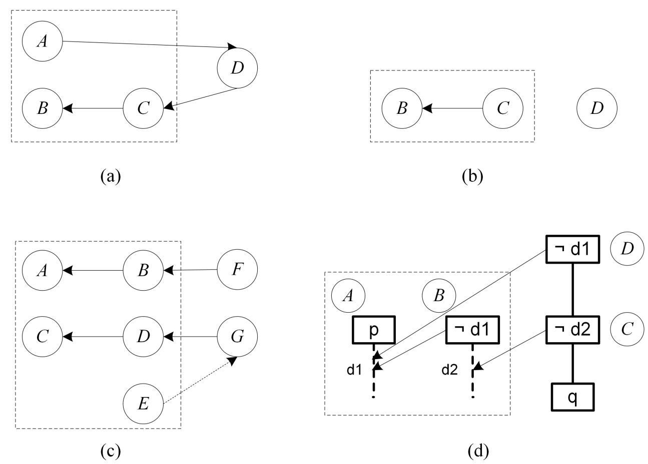

Figure 3: Further assumptions underlying Theorem 4 of Baumann and Brewka (2010).

three abstract examples we leave it to the reader to verify that instantiations for *ASPIC*$^+$ exist.

No undefeated defenders Figure 3(a) refutes the assumption that always an undefeated expansion can be found with an AF with $\mathcal{A} = \{A, B, C\}$, where C defeats B and $USAF$ contains just one argument that defeats C, namely, D but which is defeated by A. Then there is no expansion that makes $\{B\}$ included in any extension.

No defenders Figure 3(b) refutes the assumption that always a defender of any argument in S exists in UAF.

No allowed way to extend the argument ordering For refuting the assumption that $\preceq$ can always be extended in a way that preserves its type and satisfies the other conditions of Theorems 1 and 2 consider a definition that says '$E \prec G$ iff $F \notin \mathcal{A}$, else $G \prec E$' and consider the AF in Figure 3(c) corresponding to a SAF with a $\preceq$ according to which argument F from $USAF$ strictly defeats B and argument G from $USAF$ strictly defeats G. If $USAF$ contains no other arguments, then the only way to make $\{A, C\}$ included in a T-extension is to expand AF with F and G. But then E strictly defeats G according to $\preceq'$ so C is not in any T-extension for any T. Note that $\preceq'$ does not extend $\preceq$ since $E \prec G$ while $G \prec' E$. Similar examples can be constructed for the weakest- or last link argument ordering (Modgil and Prakken 2018) along the lines of Example 7 of Modgil and Prakken (2012). That example illustrates that properties of the argument ordering, such as transitivity, may make that adding explicit preferences to resolve a conflict in a desired way implies the addition of implicit preferences that prevent resolving another conflict in the desired way.

Effects of implied arguments Finally, Figure 3(d) illustrates the possible effects of implied arguments. Consider a logic-based AT with $\mathcal{K}_n = \mathcal{K}_p = \emptyset$, $\mathcal{R}_s = \{\rightarrow \neg d_1; q \rightarrow \neg d_2\}$, $\mathcal{R}_d = \{\Rightarrow_{d1} p; \ \ \Rightarrow_{d2} \neg d_1\}$ and where UAT has $q \in \mathcal{K}_p^u$ and $q \rightarrow \neg d_2 \in \mathcal{R}_s^u$. Consider then the AF in Figure 3(d) and assume that $sUAF$ further only contains q, C and D. No expansion can make $\{A\}$ included in a T-extension for any T, since adding C (the only defender of A against B) also adds D to the expansion, which defeats A.

6 Related Research

As noted in the introduction, most formal work on argumentation dynamics does not take the structure of arguments and the nature of their relations into account. Nevertheless, there is some recent relevant work. Wallner (2020) studies constraints for dynamic operations on abstract or dialectical (Brewka and Woltran 2010) argumentation frameworks, and discusses applications to enforcement in structured accounts of argumentation. His ideas are motivated by similar considerations as ours, namely, that abstract approaches can make implicit assumptions that are not satisfied by all structured instantiations. Wallner distinguishes three kinds of constraints on operators: semantic ones (e.g. constraints on the semantic status of arguments in expansions), structural ones (e.g. that all extensions of expansions are closed under subarguments) and syntactic ones (e.g. that all expansions are an AF). For AFs Wallner illustrates his approach with an instantiation with assumption-based argumentation (ABA), with as structural constraint on expansions that they should contain all arguments implied by the knowledge base (cf. Figure 3(d) above). Wallner does, unlike us, not discuss to which extent enforcement results proved at the abstract level depend on structural assumptions but instead studies complexity and implementation issues. Wallner's notion of constraints on operations is similar to our notion of allowed expansions. However, unlike us he does not consider universal background information. Moreover, in our approach, structural constraints are not *imposed* but are instead *implied* by our definition of allowed expansions as AFs that correspond to a SAF defined by some AT. It would be interesting to study whether explicitly distinguishing between structural and syntactic constraints has additional benefits for our approach.

Early structured work on dynamics in *ASPIC*$^+$ is Modgil and Prakken (2012), who instantiate abstract resolution semantics (Baroni, Dunne, and Giacomin 2011), which studies the effect of deletions of defeat relations on the possible statuses of arguments. This work has a similar aim as ours, namely, to investigate which assumptions are implicitly made by abstract work on resolution semantics. If the condition of expansions that they add at east one new argument is dropped, then Modgil and Prakken's approach can be seen as a special case of our structured approach.

More recently, Odekerken et al. (2020; 2022) have in the context of *ASPIC*$^+$ without preferences studied to which extent argument and conclusion statuses are 'stable' or can change under expansions of the knowledge base. This work is motivated by criminal-investigation applications in which it is useful to check whether searching for further information makes sense. In this work the set of *future argumentation theories* is defined as the set of all argumentation theories that extend the knowledge base of a given argumentation theory $AT = ((\mathcal{L}, \mathcal{R}, n), \mathcal{K})$ with a subset of a set $\mathcal{Q} \subset \mathcal{L}$ of *queryables*. This approach can be reconstructed as an instance of our approach by letting *UAT* be $((\mathcal{L}, \mathcal{R}, n), \mathcal{K} \cup \mathcal{Q})$ and by imposing the further constraint on Definition 14 that an expansion can only add elements to $\mathcal{K}$ and can only take these elements from $\mathcal{Q}$. Formally this makes any (future) argumentation theory strongly logic-based but this is only since the rules capture domain-specific knowledge; no logic is encoded in the rules. Since the knowledge base equates $\mathcal{K}_n$ and can grow, the ATs are not objective. The work of Odekerken et al. (2020) was abstracted by Mailly and Rossit (2020) with *incomplete argumentation frameworks* (Baumeister et al. 2021), which divide an AF in a certain and an uncertain part. Incomplete AFs can be 'specified' by making uncertain arguments or attacks certain.

Borg and Bex (2021) develop a structured account of enforcement in Borg and Strasser's (2018) 'general argumentation setting', in which, among other things, a special case of *ASPIC*$^+$ with no ordinary premises and no preferences was translated. Within this setting Borg and Bex (2021) define several notions of expansions and enforcement. Unlike in our case, these notions of expansions are not formally related to abstract accounts of expansions. Instead, their main focus is on enforcement results. Most of their results assume that the setting is *contrapositable*, which is very similar to closure in *ASPIC*$^+$ of strict consequence under contraposition (capturing what can be derived with only strict-rule application). However, unlike in *ASPIC*$^+$, contraposition is not restricted to the strict part of the logic. Thus most of Borg and Bex's results only apply to special cases of *ASPIC*$^+$ with no defeasible rules (and no preferences and ordinary premises). It would be interesting to investigate how their enforcement results relate to our Theorems 1 and 2 for the special cases of *ASPIC*$^+$ to which their results apply.

Finally, Rapberger and Ulbricht (2022) study enforcement in ABA. Like us, they observe that results for abstract argumentation frameworks do not automatically apply to structured instantiations. They then introduce ABA counterparts of the abstract notion of enforcement and relate them to a generalisation of abstract AFs called *cvAFs*, in which 'instantiated' arguments x are defined as pairs $(cl(x), vul(x))$ where $cl(x)$ is the argument's *conclusion* while $vul(x)$ is its set of *vulnerabilities*. A *cvAF* is *well-formed* iff for every $x, y \in \mathcal{A}$ it holds that x attacks[4] y iff the conclusion of x equals a vulnerability of y. The authors then prove complexity results and necessary-and-sufficient conditions for enforceability of single arguments in well-formed *cvAFs*. Rapberger and Ulbricht then instantiate *cvAFs* with ABA, where an argument's vulnerabilities are the contraries of its assumptions. This by definition results in well-formed *cvAFs*. They observe that, like for AFs, results for *cvAFs* do not automatically apply to ABA and they separately prove complexity results for enforceability of single ABA arguments.

Let us now see how *ASPIC*$^+$ could generate *cvAFs*. For conclusions this is obvious, while the vulnerabilities are the contradictories of all ordinary premises plus the contradictions of all conclusions of any subargument with a defeasible top rule plus the contradictories of all names of defeasible rules used in the argument. Defining attack is then straightforward, namely, A attacks B iff $cl(A) = -v$ for some $v \in vul(B)$. Defining defeat is less straightforward, since the proper application of preferences for determining defeat depends on the structure of arguments, which is lost in a *cvAF* encoding (contrary to Definition 9, which puts the original *ASPIC*$^+$ arguments in an AF). The most sensible way is to record which original *ASPIC*$^+$ argument gave rise to the *cvAF* argument and then define defeat as between the original arguments. Note that the thus generated *cvAFs* are guaranteed to be well-formed with respect to the attack relation but for the defeat relation this is only guaranteed if the argument ordering is empty or simple (the simple ordering says that that $A \preceq B$ iff B is strict-and-firm while A is defeasible or plausible). This is one reason why the results of Rapberger and Ulbricht (2022) do not in general apply to the present setting. Another reason is that they (like Borg and Bex (2021)) do not explicitly work with notions like universal (structured) argumentation frameworks.

Having said so, Rapberger and Ulbricht still seem to implicitly make assumptions about available background information. For example, they implicitly assume a fixed logical language and it seems to us that their Theorem 5.9 (an argument in a *cvAF* is credulously enforceable according to stable, preferred and complete semantics if and only if it is not strongly unacceptable) relies on implicit assumptions on the availability of suitable arguments for expansions. This is since otherwise the following example would be a counterexample, in which *cvAF* consists of the arguments $A = (p, \{v_1\})$, $B = (v_1, \{v_2\})$ and $C = (v_3, \emptyset)$. Note that B attacks A. Suppose that outside *cvAF* only argument $D = (v_2, \{v_3\})$ exists. Then A cannot be made acceptable by expanding *cvAF* with D since C attacks D. One assumption that invalidates this counterexample is that for every vulnerability v of an argument in *cvAF* there exists an argument $(v, \emptyset)$ that can be used in an expansion. (Note the similarity with a condition of our Theorem 3.) Then AF can be expanded with $D' = (v_2, \emptyset)$ to make A acceptable. This illustrates the importance of including an explicit notion of background information in a theory of expansions. It would be interesting to explore how the present approach and the one of Rapberger and Ulbricht can be formally related.

[4]Rapberger and Ulbricht (2022) use 'attack' instead of 'defeat'.

7 Conclusion

In this paper we have proposed a structured variant in *ASPIC*$^+$ of the notion of expansions of abstract argumentation frameworks, with a threefold purpose. First, we wanted to study what it takes to instantiate the abstract notion of expansions with a structured account of argumentation. It turned out that the structured account is more complicated than its abstract counterpart. However, we believe that this is not a problem of the present account; instead our findings illustrate that the simplicity of Bauman and Brewka's (2010) abstract account in is deceptive. This was in particular revealed by the fact that one of their key results turned out to depend on a number of implicit assumptions. We believe it is a merit of our approach that it allows making these assumptions explicit, for which the loss of conceptual simplicity is inevitable. This contribution is not only relevant for *ASPIC*$^+$. For example, because of translation results of Prakken (2010) and Dung and Thang (2014) for many of our examples variants exist in assumption-based argumentation.

A final aim was to study which potentially interesting distinctions can be made at the structured level that cannot be expressed at the abstract level. We defined notions of objective and (strongly) logic-based argumentation theories and proved simple properties of expansions that preserve these types. However, more research on this can be done. More generally, an important ingredient of our approach was the explicit inclusion of universal (abstract or structured) argumentation frameworks from which expansions are constructed. Any structured (and arguably also any abstract) study of expansions needs to fix a universal background, otherwise there is no way to identify possible expansions. Therefore, it is worthwhile to make notions of background information explicit in order to develop a theory about them. In the previous section we discussed how some other work in fact assumes such a background. Moreover, in the course of our paper we made several observations on how in applications a sensible background framework can be determined on logical, dialogical or knowledge-based grounds.

As for other future research, our approach can also be applied to other abstract accounts of argumentation dynamics. For instance, Coste-Marquis et al. (2015) propose a variant of expansions in which the set of attack relations can change, and a proposition similar to Theorem 4 of Baumann and Brewka (2010) is proven. This proposition may make similar implicit assumptions. More generally, this kind of future research is important for any abstract account of argumentation dynamics, such as the approaches reviewed in Section 3.1 of Doutre and Mailly (2018) that study the consequences of adding or deleting a single argument or attack, the work on incomplete argumentation frameworks, the work on control argumentation frameworks, which generalises the notion of enforcement (Dimopoulos, Mailly, and Moraitis 2018) and, finally, the work on argumentation-based belief revision (Baroni et al. 2022).

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