Learning Generalized Policies without Supervision Using GNNs

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Abstract
We consider the problem of learning generalized policies for classical planning domains using graph neural networks from small instances represented in lifted STRIPS. The problem has been considered before but the proposed neural architectures are complex and the results are often mixed. In this work, we use a simple and general GNN architecture and aim at obtaining crisp experimental results and a deeper understanding: either the policy greedy in the learned value function achieves close to 100% generalization over instances larger than those used in training, or the failure must be understood, logically. For this, we exploit the relation established between the expressive power of GNNs and the C2 fragment of first-order logic (namely, FOL with 2 variables and counting quantifiers). We find for example that domains with general policies that require more expressive features can be solved with GNNs once the states are extended with suitable "derived atoms" encoding role compositions and transitive closures that do not fit into C2. The work follows an existing approach based on GNNs for learning optimal general policies in a supervised fashion, but the learned policies are no longer required to be optimal (which expands the scope, as many planning domains do not have general optimal policies) and are learned without supervision. Interestingly, value-based reinforcement learning methods that aim to produce optimal policies, do not always yield policies that generalize, as the goals of optimality and generality are in conflict in domains where optimal planning is NP-hard.

1 Introduction
Generalized planning is concerned with the computation of general policies for families of planning instances over the same domain that span different state spaces. For example, a general policy for solving Blocks problems can place all blocks on the table and stack them the desired towers, bottom up, one at at time. The formulation and the computation of general policies is particularly interesting as it involves ideas from planning, knowledge representation, and learning. Indeed, the language for representing the general policies is key, in particular in domains where the set of ground actions change from instance to instance (Bonet and Geffner 2018). Also learning policies from examples has been found to be simpler than synthesizing them from specifications (Khardon 1999; Srivastava, Immerman, and Zilberstein 2008; Bonet, Palacios, and Geffner 2009; Hu and De Giacomo 2011; Belle and Levesque 2016; Segovia, Jiménez, and Jonsson 2016). In planning, it is common to approach the problem assuming that domain predicates are known, while some deep learning and deep reinforcement learning approaches address the problem with no domain knowledge. representing the states, for example, as 2D images (Chevalier-Boisvert et al. 2019; Campero et al. 2021; Cobbe et al. 2020).

In this paper, we consider the problem of learning generalized policies for classical planning domains using graph neural networks (Scarselli et al. 2008; Hamilton 2020) from small instances represented in lifted STRIPS. The problem has been considered before but using neural architectures that are more complex and with results that are often less crisp, involving in certain cases heuristic information or search (Toyer et al. 2020; Garg, Bajpai, and Mausam 2020; Rivlin, Hazan, and Karpas 2020; Karia and Srivastava 2021; Shen, Trevizan, and Thiébaux 2020). We use a simple and general GNN architecture and aim at obtaining crisp experimental results and a deeper understanding: either the policy greedy in the learned value function achieves close to 100% generalization over instances larger than those used in training, or the failure must be understood and, possibly fixed, using logical methods. For this, we exploit the relation between the expressive power of GNNs and the two-variable fragment of first-order logic with counting, that includes the standard description logics (Barceló et al. 2020; Grohe 2020). Description logic features have been used indeed for expressing general policies and general value functions (Martín and Geffner 2004; Fern, Yoon, and Givan 2006; Bonet, Francès, and Geffner 2019; Francès et al. 2019; Francès, Bonet, and Geffner 2021). We find for example that domains with general policies that require more expressive features can be solved with GNNs once the states are extended with suitable "derived atoms" for encoding role compositions and transitive closures that do not fit into C2.

The work follows the GNN approach for learning optimal general policies in a supervised fashion (Ståhlberg, Bonet, and Geffner 2022) but the learned policies are no longer required to be optimal, which expands the scope of the approach, as many planning domains do not admit general optimal policies, and are learned without supervision. The learning problem becomes the problem of learning a value function \( V \) that can be applied to the states \( s \) of any domain instance, such that the greedy policy in \( V \)
solves the training instances. Versions of this idea have been used in combinatorial settings (Francès et al. 2019; Francès, Bonet, and Geffner 2021). Interestingly, value-based reinforcement learning methods that aim to produce optimal value functions \( V = V^* \) are shown not to generalize as well in domains that admit (non-optimal) general policies but where optimal planning is NP-hard.

The rest of the paper is organized as follows. First we discuss related research, then cover the background (classical planning, general policies and value functions, and GNNs) and the actual GNN architecture and loss functions used for learning. This is followed by the experimental section, analyses, and a summary.

2 Related Work

Some related research threads are the following.

Generalized planning (GP). Formulations of generalized planning differ in the way in which general policies are represented: most often, as logic programs, finite-state controllers, or programs with loops (Khardon 1999; Srivastava, Immerman, and Zilberstein 2008; Bonet, Palacios, and Geffner 2009; Hu and De Giacomo 2011; Belle and Levesque 2016; Segovia, Jiménez, and Jonsson 2016). In all cases, the most compact policies that manage to solve a family of examples are sought, and the key question is how the space of possible programs or controllers is defined.

GP with logical features. An alternative approach is to define the general policies as collection of rules over a set of logical features (Bonet and Geffner 2018), often derived from the domain predicates using a description logic grammar (Martín and Geffner 2004; Fern, Yoon, and Givan 2006). Recent methods learn such policies from pools of such features (Bonet, Francès, and Geffner 2019; Francès, Bonet, and Geffner 2021); in some cases, by learning value functions (Francès et al. 2019). The Boolean and numerical features are closely related to the variables used in qualitative numerical planning models (Srivastava et al. 2011; Bonet and Geffner 2020b).

Generalized policies using deep learning. Deep learning and deep reinforcement learning methods have been used to compute general policies from sampled problems without having to redefine the space of possible features. In some cases, the planning representation of the domains is used (Toyer et al. 2020; Garg, Bajpai, and Mauzam 2020; Rivlin, Hazan, and Karpar 2020); in other cases, it is not (Groshev et al. 2018; Chevalier-Boisvert et al. 2019; Campero et al. 2021; Cobbe et al. 2020). Also in some cases, the learning is supervised; in others, it is based on reinforcement learning (Bertsekas 1995; Sutton and Barto 2018; François-Lavet et al. 2018). The neural networks learn to map states into a feature representation that is mapped into the value or policy associated to the state.

GNNs and logic. A graph neural network learns to map vertices of a graph into feature representations that can be aggregated and fed into a feedforward neural network for classifying graphs, and more generally, for computing functions over graphs independently of their size (Scarselli et al. 2008; Hamilton 2020). Since the computational model is based on message passing, GNNs cannot distinguish all pairs of graphs that are not isomorphic but can distinguish those that are distinguished by the WL coloring procedure (Morris et al. 2019; Xu et al. 2019). These correspond in turn to those that can be distinguished by formulas in the two-variable fragment of first-order logic with counting quantifiers, \( \mathbb{C}_2 \), which includes the standard description logics (Barceló et al. 2020; Grohe 2020).

GNNs and optimal general policies. Stähler, Bonet, and Geffner (2022) use GNNs to learn optimal general policies in a supervised fashion from targets \( V^*(s) \) and sampled states \( s \), taking advantage of a GNN architecture introduced for learning to solve Max-CSPs (Toenshoff et al. 2021), extended to the more general relational structures underlying planning states where objects define the universe, predicates define the relations, and atoms define their denotations. In this work, we build on these results to learn general policies that are not necessarily optimal (and which hence cover more domains) without supervision and without having to define a pool of features (Francès et al. 2019).

3 Classical Planning

A classical planning problem is a pair \( P = \langle D, I \rangle \) where \( D \) is a first-order domain and \( I \) contains information about the instance (Geffner and Bonet 2013; Ghallab, Nau, and Traverso 2016; Haslum et al. 2019a). The domain \( D \) contains a set of predicate symbols \( p \) and a set of action schemas with preconditions and effects given by atoms \( p(x_1, \ldots, x_k) \) where each \( x_i \) is an argument of the schema. An instance is a tuple \( I = \langle O, Init, Goal \rangle \) where \( O \) is a set of object names \( c_i \), and \( Init \) and \( Goal \) are sets of ground atoms \( p(c_1, \ldots, c_k) \).

A classical problem \( P = \langle D, I \rangle \) encodes a state model \( S(P) = \langle S, s_0, S_G, Act, A, f \rangle \) in compact form where the states \( s \in S \) are sets of ground atoms from \( P \), \( s_0 \) is the initial state \( I \), \( S_G \) is the set of goal states \( s \) such that \( S_G \subseteq s \), \( Act \) is the set of ground actions in \( P \), and \( A(s) \) is the set of ground actions whose preconditions are (true) in \( s \), and \( f \) is the transition function so that \( f(a, s) \) for \( a \in A(s) \) represents the state \( s' \) that follows action \( a \) in the state \( s \). An action sequence \( a_0, \ldots, a_n \) is applicable in \( P \) if \( a_i \in A(s_i) \) and \( s_{i+1} = f(a_i, s_i) \), for \( i = 1, \ldots, n \), and it is a plan if \( s_{n+1} \in S_G \). The cost of a plan is assumed to be given by its length and a plan is optimal if there is no shorter plan.

The representation of planning problems \( P \) in two parts \( D \) and \( I \), one that is general, and the other that is specific, is essential for defining and computing general policies, as the instances are assumed to come all from the same domain. Recent work has addressed the problem of learning the action schemas and predicates (Cresswell, McCluskey, and West 2013; Asai 2019; Bonet and Geffner 2020a; Rodriguez et al. 2021).

4 General Policies and Value Functions

One approach for expressing general policies is as rules \( C \rightarrow E \) where the condition \( C \) and the effect \( E \) are defined in terms of state features (Bonet and Geffner 2018). State features or simply, features, refer to functions \( \phi \) over
the state, and Boolean and numerical features refer to state functions that return Boolean and numerical values. For example, a general policy for clearing a block \( x \) can be expressed in terms of the two features \( \Phi = \{ H, n \} \), where \( H \) is a true in a state if a block is being held, and \( n \) represents the number of blocks above \( x \). The policy rules are

\[
-H, n > 0 \rightarrow H, n\downarrow, \quad H \rightarrow -H
\]

that say that, when the gripper is empty and there are blocks above \( x \), any action that decreases \( n \) and makes \( H \) true should be selected, and that when the gripper is not empty, any action that makes \( H \) false and does not affect \( n \) should be selected. General policies of this form can be learned without supervision by solving a combinatorial optimization problem \( T(S, F) \) where \( S \) is a set of sampled state transitions and \( F \) is a large but finite pool of description logic features obtained from the domain predicates (Bonet, Francès, and Geffner 2019; Francès, Bonet, and Geffner 2021).

Another way to represent (general) policies is by means of (general) value functions. In dynamic programming and RL (Bellman 1957; Sutton and Barto 2018; Bertsekas 1995), a value function \( V \) defines a (non-deterministic) greedy policy \( \pi_V \) that selects in a state \( s \) any possible successor state \( s' \) with minimum \( V(s') \) value under the assumption that actions are deterministic and have the same cost. A policy \( \pi \) solves an instance \( P \) if the state transitions compatible with \( \pi \), starting with the initial state, eventually end up in a goal state. If \( V \) is optimal, i.e., \( V = V^* \), the greedy policy \( \pi_V \) is optimal too, selecting state transitions along optimal paths.

General value functions for a class of problems are defined in terms of features \( \phi_i \) that have well-defined values over all states of such problems as:

\[
V(s) = F(\phi_1(s), \ldots, \phi_k(s)).
\]

Linear value functions have the form

\[
V(s) = \sum_{1 \leq i \leq k} w_i \phi_i(s)
\]

where the coefficients \( w_i \) are constants that do not depend on the states. For example, a general, linear value function for clearing block \( x \) while having an empty gripper is \( V = 2n + H \), where the states are left implicit, and the Boolean feature \( H \) is assumed to have value 1 when true, and 0 otherwise.

Linear value functions using description logic features (Bonet, Francès, and Geffner 2019), called generalized potential heuristics, can be learned from small instances via a mixed integer programming formulation, leading to an alternative representation of general policies that solve many standard planning domains (Francès et al. 2019).

5 Features

Logical features derived from the domain predicates using a description logic grammar have been used to define and learn policies of the form (1) and value functions of the form (3).

\footnote{These logical features have also been used to encode “sketches”, a generalization of policies that split problems into (polynomial) subproblems of bounded width (Drexler, Seipp, and Geffner 2021). Policies are a special type of sketches where the subproblems can be solved in one step (Bonet and Geffner 2021).} The complexity of such features is defined in terms of the number of grammar rules required to derive them, and the pool of features used is obtained by placing a bound on the complexity of the features. An important limitation of these methods is that the pool of features grows exponentially with the complexity bound, and that some domains require complex features. For example, Francès et al. (2019) cannot learn general value functions for Logistics and Blocks because they appear to require features of complexity 22 and 49, respectively. Interestingly, the features required to express the policy rules for some of these domains is much smaller (Francès, Bonet, and Geffner 2021).

For learning general policies without using a precomputed pool of features, it turns out to be simpler and more direct to learn general value functions, and then define greedy policies from them. A first step in this direction was taken by Stählberg, Bonet, and Geffner (2022) where the value function \( V \) was learned in a supervised fashion using graph neural networks from optimal targets \( V^* \). Graph neural networks have also been used in other approaches to generalized planning using deep nets (Toyer et al. 2020; Garg, Bajpai, and Mausam 2020; Rivlin, Hazan, and Karpas 2020), but in combination with other techniques and without drawing on the relation between the features that can be learned by GNNs and those that are actually needed.

6 Graph Neural Networks

The GNN architecture for learning value functions follows the one used by Stählberg, Bonet, and Geffner (2022): it accepts states \( s \) over arbitrary instances of a given planning domain, and outputs the scalar value \( V(s) \). For this, the form of the general value function \( V(s) \) in (2) is reformulated as:

\[
V(s) = F(\phi(o_1), \ldots, \phi(o_n))
\]

where \( o_1, \ldots, o_n \) represent the objects in the instance where the state \( s \) is drawn from, \( \phi(o) \) is a vector of feature values associated with object \( o \) in state \( s \) (dependence on \( s \) omitted), represented as a vector of real numbers, and \( F \) is a function that aggregates these feature vectors and produces the scalar output \( V(s) \). The vectors \( \phi(o) \) are usually called object embeddings and the function \( F \), the readout. Before revising the details of the architecture, it is worth discussing the meaning and the implication of the transition from the fully general value function form expressed in (2) to the specific form expressed in (4).

6.1 From State Features to Object Embeddings

We are moving from state features to object features \( \phi(o) \) that depend not just on the state \( s \) but on the objects \( o \). In addition, the same feature function \( \phi \) is applied to all the objects, and the same aggregation function \( F \) is applied to the states \( s \) of any of the domain instances so that the number of feature vectors \( \phi(o) \) expands or contracts according to the number of objects in the instance. This is key for having a well-defined value function over the whole collection of domain instances that involve a different numbers of objects, not necessarily bounded.

The reasons for why the restricted value function form (4) is rich enough for capturing the value functions needed...
for generalized planning can be understood by comparing (4) with the linear value functions (3) used by Francès et al. (2019) in combination with description logic features. These Boolean and numerical features $b_q(s)$ and $n_q(s)$ are defined in terms of derived unary predicates $q$, where $b_q(s) = 1$ (true) if there is an object $o$ such that $q(o)$ is true in $s$, otherwise 0; and $n_q(s) = m_o$ is the number of objects $o$ for which $q(o)$ is true in $s$. Clearly, if the feature vectors $\phi_i$ in (4) contain a bit encoding whether $q(o)$ is true in $s$, then the readout function $F$ would just need to take the max and the sum of the bits $q(o)$ as

$$\begin{align*}
b_q(s) &= \max_o q(o), \\
n_q(s) &= \sum_o q(o),
\end{align*}$$

in order to capture such features, where the objects $o$ range over all the objects $o$ in the instance. In other words, the object-embedding form (4) is no less expressible than the linear form that uses description logic features, provided that the feature vectors $\phi_i$ are expressive enough to represent the bits $q_i(o)$ for unary predicates $q_i$ derived from the domain predicates using the description logic grammar. This in turn is known to be within the capabilities of standard, message passing GNNs, that can capture the properties that can be expressed in the guarded fragment of the variable logic with counting $C_2$, which includes the standard description logics (Barceló et al. 2020).

Below we follow the terminology of graph neural networks and refer to graphs and not states, and to vertex embeddings $f(v)$ and not object embeddings $\phi(o)$. After considering standard GNNs for undirected graphs, we introduce the generalization needed for dealing with the relational structures represented by planning states.

### 6.2 GNNs on Graphs

GNNs represent trainable, parametric, and generalizable functions over graphs (Scarselli et al. 2008; Hamilton 2020) specified by means of aggregate and combination functions $agg_i$ and $comb_i$, and a readout function $F$. For each vertex $v$ of the input graph $G$, the GNN maintains a state (vector) $f_i(v) \in \mathbb{R}^k$, the vertex embedding, $i = 0, \ldots, L$, where $L$ is the number of iterations or layers. The vertex embeddings $f_0(v)$ are fixed and the embeddings $f_{i+1}$ for all $v$ are computed from the $f_i$ embeddings as:

$$f_{i+1}(v) := \text{comb}_i(f_i(v), agg_i(\{f_i(w)|w \in \mathcal{N}_G(v)\}))$$

where $\mathcal{N}_G(v)$ is the set of neighbors for vertex $v$ in $G$, and $\{\ldots\}$ denotes a multiset. In words, the embeddings $f_{i+1}(v)$ at iteration $i + 1$ are obtained by combining the aggregation of neighbors’ embeddings $f_i(w)$ at iteration $i$ with $v$’s own embeddings $f_i(v)$. This process is usually seen as an exchange of messages among neighbor nodes in the graph. The aggregation functions $agg_i$ map arbitrary collections of real vectors of dimension $k$ into a single $\mathbb{R}^k$ vector. Common aggregation functions are sum, max, and smooth-max (a smooth approximation of the max function). The combination functions $\text{comb}_i$ map pairs of $\mathbb{R}^k$ vectors into a single $\mathbb{R}^k$ vector. The embeddings $f_L(v)$ in the last layer are aggregated and mapped into the output of the GNN by means of a readout function $F$. In our setting, the output will be a scalar $V$, and the aggregation and combination functions $agg_i$ and $\text{comb}_i$ will be homogeneous and not depend on the layer index $i$. All the functions are parametrized with weights that are adjusted by minimizing a suitable loss function. By design, the function computed by a GNN is invariant with respect to graph isomorphisms, and once a GNN is trained, its output is well defined for any graph $G$ regardless size.

### 6.3 GNNs for Planning States

States $s$ in planning do not represent graphs but more general relational structures that are defined by the set objects, the set of domain predicates, and the atoms $p(o_1, \ldots, o_m)$ that are true in the state: the objects define the universe, the domain predicates, the relations, and the atoms, their denotations. The set of predicate symbols $p$ and their arities are fixed by the domain, but the sets of objects $o_i$ may change from instance to instance. The adaptation of the basic GNN architecture for dealing with planning states $s$ follows (Stählberg, Bonet, and Geffner 2022), which is an elaboration of the architecture for learning to solve Max-CSP problems over a fixed class of binary relations introduced by Toenishoff et al. (2021). The new GNN still maintains just the object embeddings $f_i(o)$ for each of the objects $o$ in the input state $s$, $i = 0, \ldots, L$, but now rather than messages flowing from “neighbor” objects to objects as in (7), the messages flow from objects $o_i$ to the true atoms $q$ in $s$ that include $o_i$, $q = p(o_1, \ldots, o_m)$, $1 \leq i \leq m$, and from such atoms $q$ to all the objects $o_j$ involved in $q$ as:

$$f_{i+1}(o) := \text{comb}_i(f_i(o), agg_i(\{m_{q,o}|o \in q, q \in s\}))$$

where $m_{q,o}$ for $q = p(o_1, \ldots, o_m)$ and $o = o_j$ is:

$$m_{q,o} := |\text{comb}_p(f_i(o_1), \ldots, f_i(o_m))|.$$  

In these updates, the combination function $\text{comb}_i$ takes the concatenation of two real vectors of size $k$ and outputs a vector of size $k$, while the combination function $\text{comb}_p$, that depends on the predicate symbol $p$, takes the concatenation of $m$ vectors of size $k$, where $m$ is the arity of $p$, and outputs $m$ vectors of size $k$ as well, one for each object involved in
the \( p \)-atom. The expression \([\ldots]_j \) in (9) selects the \( j \)-th such vector in the output.

The resulting trainable function that maps states \( s \) into their values \( V(s) \) is shown in Algorithm 1 with all the combination functions replaced by the multilayer perceptrons (MLPs) that implement them. During the iterations \( i = 0, \ldots, L \), a single \( \text{MLP}_i \) is used for updating the object embeddings following (7), and a single \( \text{MLP}_p \) per predicate is used to collect the messages from atoms to objects as in (9). The readout function, the last line in Algorithm 1, uses two MLPs and a sum aggregator. Finally, for the aggregator in line 6, we use the differentiable smooth max function \( \text{softmax}(x_1, \ldots, x_n) \) defined as

\[
x^* + \alpha^{-1} \log \left( \sum_{1 \leq j \leq n} \exp(\alpha (x_j - x^*)) \right) \tag{10}
\]

where \( x^* = \max\{x_1, \ldots, x_n\} \) and \( \alpha = 8 \).

All MLPs consists of a dense layer with a ReLU activation function, followed by a dense layer with a linear activation function. The hyperparameter in the networks are the embedding dimension \( k \) and the number of layers \( L \). The initial embeddings \( f_0(o) \) are obtained by concatenating a zero vector with a random vector, each of dimension \( k/2 \), to break symmetries. Random initialization increase expressive power for instances of fixed size (Abboud et al. 2021), however, we aim to learn policies for arbitrary sizes. Key for the GNN to apply to any state over the domain is the use of a single MLP per each predicate symbol \( p \) in the domain.

7 Learning the GNN Parameters

The parameters of the network displayed in Algorithm 1 are learned by stochastic gradient descent by minimizing a loss function. In the work of Ståhlberg, Bonet, and Geffner (2022), the training data \( D \) is a collection of pairs \( \langle s, V^*(s) \rangle \) for sampled states \( s \) from selected instances, and \( V^*(s) \) is the optimal cost for reaching the goal from \( s \) (min. number of steps). The loss is the average sum of the differences

\[
L(s) = |V(s) - V^*(s)| \tag{11}
\]

over the states \( s \) in the training set. The computation of the optimal targets \( V^*(s) \) is not a problem because we are computing them over small instances. The real problem is that by forcing the value function to be optimal over the training instances, domains such as Blocks or Miconic, where optimal planning is NP-hard (Gupta and Nau 1992; Helmert 2001), are excluded (except when the goals are restricted to be single atoms).

Interestingly, as discussed in the next section, this limitation pops up also in unsupervised, reinforcement learning approaches where the optimal target values \( V^*(s) \) are not given but are sought by minimizing the Bellman error:

\[
L'_0(s) = |V(s) - (1 + \min_{s' \in N(s)} V(s'))| \tag{12}
\]

for non-goal states \( s \), where \( N(s) \) are the states reachable from \( s \) in one step (possible successor states). For goal states, \( L'_0(s) = |V(s)| \). The optimal function \( V^* \) is the unique value function that minimizes the resulting loss, provided that actions costs are all 1 and the goal is reachable from all states. In this work, rather than penalizing departures from the Bellman optimality equation

\[
V(s) = 1 + \min_{s' \in N(s)} V(s'), \tag{13}
\]

departures from the inequality \( V(s) \geq 1 + \min_{s' \in N(s)} V(s') \) are penalized with a loss for non-goal states \( s \) defined as

\[
L'_1(s) = \max\{0, 1 + \min_{s' \in N(s)} V(s') - V(s)\} \tag{14}
\]

Furthermore, this loss is extended with two regularization terms that penalize large departures from \( V^* \); namely, as done by Francès, Bonet, and Geffner (2021), we want a value function \( V \) that also satisfies \( V^* \leq V \leq \delta V^* \), and thus settle for the minimization of the loss:

\[
L_1(s) = L'_1(s) + \max\{0, V^*(s) - V(s)\} + \max\{0, V(s) - \delta V^*(s)\}, \tag{15}
\]

where \( \delta = 2 \). The loss over a set \( S \) of states is the sum of the average of \( L_1(s) \) for non-goal states \( s \in S \) and the average of \( |V(s)| \) for goal states \( s \in S \). For comparison purposes, the \( L_0' \) loss is extend into the regularized \( L_0 \) loss as well as:

\[
L_0(s) = L'_0(s) + \max\{0, V^*(s) - V(s)\} + \max\{0, V(s) - \delta V^*(s)\}, \tag{16}
\]

If all the states in a small instance are in \( S \) and the overall loss is close to zero, the loss function \( L_1 \) results in value functions that lead greedily to the goal (by picking the min-V successors), while the loss \( L_0 \) results in value functions that lead greedily and optimally to the goal. For simplicity, it is assumed that the domains considered do not have dead-ends, i.e. states from which the goal is not reachable and where \( V^*(s) \) is not well-defined. Learning to plan in such domains requires an slight extension, with extra inputs, for labeling states as dead-ends in the training data, and extra outputs, for predicting if a state is a dead-end (Ståhlberg, Francès, and Seipp 2021). This extension is implemented and tested, but it will be skipped over in the presentation.

8 Experiments

The experiments are aimed to test the generalization, coverage, and quality of the plans obtained by the policy \( \pi_{V} \) greedy in the learned value function \( V \), using the unsupervised losses \( L_0 \) and \( L_1 \). We describe the training and testing data used, and the results. A key difference with prior work (Ståhlberg, Bonet, and Geffner 2022) is that the test instances are standard IPC planning problems from standard planning domains, several of which are intractable for optimal planning. We seek crisp experimental results, which means close to 100% generalization, or alternatively, crisp explanations of why this is not possible, with logical fixes that restore generalization in certain cases.

Data. The states in the training and validation sets are obtained by fully expanding selected instances from the initial state through a breadth-first search. For each reachable state, the length of the shortest path to a goal state is computed. For instances with large state spaces we keep up to 40,000 sampled reachable states to avoid large instances from dominating the training set. The actual size of the instances used
in training, validation, and testing are shown in Table 1, measured by the number of objects involved. In almost all cases, the testing instances are IPC (International Planning Competition) instances. The exception is the domain Spanner*, which is a slight variant of the Spanner domain that does not give rise to dead-end states by allowing the agent to move not just forward but also backward.

**Domains.** The domains are those used by Francies, Bonet, and Geffner (2021) with the addition of Logistics, and the above modification of Spanner. Briefly, Blocks is the standard blocks world. Delivery is the problem of picking up objects in an empty grid and delivering them one by one to a target cell. Gripper is about moving balls from one room to another with a moving robot that can have more than one gripper. Logistics involves trucks and airplanes that move within city locations and across cities, where packages have to be moved from one location to another location, possibly in a different city. Miconic is about controlling an elevator to pick up passengers in different floors to their destination floors. Rewards is about reaching certain cells in a grid while avoiding others. Spanner is about collecting spanners spread in a one dimensional grid, each one to be used to tighten up a single nut at the other end. Visitall is about visiting all or some cells in an empty grid.

**Setup.** The hyperparameters $k$ and $L$ in Algorithm 1 are set to 64 and 30, respectively: $k$ is the number of “features” per object; i.e., the size of the real object embedding vectors; and $L$ the number of layers in the GNN (fixed for training and testing). Both hyperparameters affect training speed, memory, and generalization. Hyperparameter $L$ affects how far messages can propagate in the graph, and indeed, the GNN cannot capture shortest paths between two objects if longer than $L$, even if the existence of paths up to length $2L$ can be determined. The architecture is implemented in PyTorch (Paszke and et. al. 2019) and the optimizer Adam (Kingma and Ba 2015) is used with a learning rate of 0.0002. The networks are trained with NVIDIA A100 GPUs for up to 12 hours. Five models for each domain are trained to ensure that the optimizer did not get stuck in “bad” local minima, and the final model used is the one with the best validation loss (i.e., loss measured on the validation set). The quality of the plans obtained by following the greedy policy $\pi_V$ for the learned value function $V$ are evaluated in comparison with optimal plans that are computed with the Fast Downward (FD) planner (Helmert 2006) using the seq-opt-merge-and-shrink configuration with time and memory outs set to 10 minutes and 64 GB, respectively, on a Ryzen 9 5900X CPU.

### 8.1 Testing the Greedy Policy $\pi_V$: Two Modes

The greedy policy $\pi_V$ selects the action applicable in a non-goal state $s$ that leads to the child state $s'$ with minimum $V(s')$ value (action costs are all assumed to be 1). It is common to add “noise” in this selection process by either breaking ties randomly or by choosing the action leading to the best child probabilistically, by soft-ranking the children values $V(s')$ into probabilities that add up to 1. The addition of “noise” in action selection has the benefit that it helps to avoid cycles in the execution, but at the same time, it blurs the results. Instead, Table 2 shows on the right the results of the executions that follow the deterministic greedy policy $\pi_V$, which always chooses the action leading to the child $s'$ with lowest $V(s')$ value, breaking ties for the first such action encountered. Since the learned value function is not perfect, we show on the left the execution of the greedy policy but with cycle avoidance; namely, executions keep track of the visited states and deterministically select the first action leading to the best unvisited child (min-$V$ value). When there are no such children, the execution fails. Executions are also terminated when the goal is not reached within 1,000 steps.

### 8.2 Results: $L_1$ Loss

Table 2 shows the results for various experiments: learning using the $L_1$ loss (top), learning using the $L_0$ loss (middle), and learning using states augmented with derived atoms in domains that benefit from $C_2$ features (explained below). Furthermore, the three subtables are divided horizontally in two, according to the way in which the greedy policy $\pi_V$ for the learned value function $V$ is used: with cycle avoidance, on the left, and without cycle avoidance, on the right. We focus now on the top part of the table.

**Coverage.** The first thing to notice is that in 4 out of the 8 domains considered, Blocks, Delivery, Gripper, and Miconic, the deterministic greedy policy $\pi_V$ for the learned value function $V$ solves all the test instances. This is pretty remarkable as the resulting plans are often long. In Blocks, the average plan length is $790/20 = 39.5$ steps, while in Miconic, it is $7,331/120 = 61.09$. As we will see, while the plans are not optimal, they are very good, and moreover, in none of these cases, the deterministic greedy policy generates an execution where a state is revisited. Indeed, if revisits are explicitly excluded by executing the greedy policy while avoiding cycles (left), a fifth domain is solved in full: Visitall. The other three domains are not solved in full in either mode: Logistics, Reward, and Spanner. In the case of Logistics, the reason, as we will see, is purely logical:

<table>
<thead>
<tr>
<th>Domain</th>
<th>Train</th>
<th>Validation</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blocks</td>
<td>[4, 7]</td>
<td>[8, 8]</td>
<td>[9, 17]</td>
</tr>
<tr>
<td>Delivery</td>
<td>[12, 20]</td>
<td>[28, 28]</td>
<td>[29, 85]</td>
</tr>
<tr>
<td>Gripper</td>
<td>[8, 12]</td>
<td>[14, 14]</td>
<td>[16, 46]</td>
</tr>
<tr>
<td>Logistics</td>
<td>[5, 18]</td>
<td>[13, 16]</td>
<td>[15, 37]</td>
</tr>
<tr>
<td>Miconic</td>
<td>[3, 18]</td>
<td>[18, 18]</td>
<td>[21, 90]</td>
</tr>
<tr>
<td>Reward</td>
<td>[9, 100]</td>
<td>[100, 100]</td>
<td>[225, 625]</td>
</tr>
<tr>
<td>Spanner*</td>
<td>[6, 33]</td>
<td>[27, 30]</td>
<td>[22, 320]</td>
</tr>
<tr>
<td>Visitall</td>
<td>[4, 16]</td>
<td>[16, 16]</td>
<td>[25, 121]</td>
</tr>
</tbody>
</table>

Table 1: Instance sizes used training, validation, and testing datasets, as measured by the number of objects involved. E.g., the training set for Blocks consists of IPC instances with a number of blocks between 4 and 7. There is no instance that is in more than 1 set (same number of objects, initial state and goal description).
given the domain representation of Logistics, the feature expressing that a package is in a location or in a city, while possibly within a vehicle, involves the composition of two or three binary relations, requiring three variables, which is not possible in $C_2$. We address this expressive limitation of GNNs below by adding suitable “derived” atoms to the state that bypass the need for such compositions. The limitations observed in Reward and Spanner are not logical: these two domains, as others in the list, require the computation of distances to determine in which direction to move (e.g., to the nearest reward or right exit). Yet GNNs cannot compute distances that exceed their number of layers $L$. Actually, there are other domains solved in full that require the computation of distances, but the magnitude of the distances needed in the test set does not defy these bounds. Indeed, even a simple problem such a clearing a block $x$ may be found to be unsolvable by the learned policy if the number of blocks above $x$ is much larger than $L$. Interestingly, this limitation has an easy logical “fix” in some of the domains, where derived atoms capturing the transitive closure of some binary predicates manage to decouple the computation of distances from the number of layers in the GNN. In the domains where these expressive limitations arise, the greedy policy with cycle avoidance does better than the pure greedy policy, as the latter is more likely to be trapped in cycles.

**Quality.** Somewhat surprisingly, the quality of the executions delivered by the models trained with the $L_1$ loss is very close to optimal, as measured with respect to the optimal plans computed by FD. The only exception is the Logistics domain where plans are up to 10 times longer than optimal, on average. These results are surprising not just because the $L_1$ loss does not force the value function $V$ to be optimal, but because optimal planning in several of these domains, certainly Blocks, Miconic, and Logistics, and possibly in Re-
ward and Visitall as well, is NP-hard (Gupta and Nau 1992; Helmert 2001). For example, FD with the given time and memory bounds computes optimal solutions for 35 instances in Miconic comprising a total of 1,164 actions, while the sum of execution lengths for the learned, greedy policy \( \pi_V \) with or without cycle avoidance on the same 35 instances is 1,170. Indeed, the execution lengths that follow from the learned value function do not exceed the optimal plan lengths in more than 12% with the exception of Logistics.

### 8.3 \( L_0 \) Loss: General Policies and RL

The differences between the \( L_1 \) loss (15) and the \( L_0 \) loss (16) are small but significant. Zero loss for \( L_0 \) arises just when the learned \( V \) function has zero Bellman error over the training set; i.e., \( \text{when } V(s) = 1 + \min_{s' \in N(s)} V(s') \) for the possible children \( s' \) of \( s \), and thus when \( V \) is the optimal cost function \( V^* \). Zero loss for \( L_1 \), on the other hand, arises just when the learned \( V \) function is such that \( V(s) \geq 1 + \min_{s' \in N(s)} V(s') \). Thus, zero \( L_0 \) loss implies zero \( L_1 \) loss, but not the other way around, as the \( L_1 \) loss captures just one half of Bellman’s optimality equation. Provided that only the goal states have zero value and that non-goal states have positive values, one can use a value function \( V \) with zero \( L_0 \) loss to solve problems greedily by always moving to the best child (min \( V \)). On the other hand, a value function \( V \) with zero \( L_0 \) loss can be used in the same manner to solve problems greedily and optimally. The difference between solving a class of problems optimally or suboptimally is crucial in domains where optimal planning is NP-hard. Such domains, like Blocks, often admit general policies but no general policies that are optimal.

So the question arises as to whether the minimization of the \( L_0 \) loss leads to greedy policies \( \pi_V \) that are as good as, or better than those obtained by minimization of the \( L_1 \) loss. The question is particularly relevant because the standard methods for learning policies without supervision are usually based on reinforcement learning, which in their value-based variant (as opposed to the policy gradient version) are based on the minimization of Bellman error (Sutton and Barto 2018). The expectation is that the minimization of \( L_0 \) loss will not be as good. Indeed, the value functions \( V \) that yield greedy policies \( \pi_V \) that generalize correctly over domains that are intractable for optimal planning are unlikely to yield zero \( L_0 \) loss.

The middle part of Table 2 shows the results of the greedy policies \( \pi_V \) for value functions \( V \) learned by minimizing \( L_0 \) loss instead of \( L_1 \). The \( L_0 \)-based policies are observed to perform worse than the \( L_1 \)-based policies. The extreme case is precisely in Blocks where coverage drops from 100% to 0% when using the greedy policy with cycle avoidance and also without. A big difference also surfaces in Logistics where coverage drops from 60% to 3% with cycle avoidance (otherwise no instances are solved). For the other domains, the drops are not as drastic, yet the greedy policy with no cycle avoidance based on \( L_1 \) solves four domains fully (100% coverage) while the same policy based on \( L_0 \) does not solve fully any single domain. The \( L_0 \)-policies, however, do slightly better in two of the domains where the \( L_1 \)-policy is not good: Reward and Visitall where coverage increases from 20% and 78% to 46% and 86%. As expected, the lower coverage of \( L_0 \)-policies goes along with executions whose lengths are better overall. With cycle avoidance, the performance resulting from the two loss functions is closer, with the aforementioned exceptions. In general, the ability of the learned value functions \( V \) to yield greedy policies that generalize can be predicted from the corresponding loss on the validation set. In both Blocks and Logistics, the validation loss after \( L_1 \) training is close to zero, but significantly higher than zero after \( L_0 \) training.

### 8.4 Derived Atoms: Beyond \( C_2 \)

The failure of the learned policies to generalize fully when using the \( L_1 \) loss function in domains such as Logistics, Reward, and Spanner* can be traced to two limitations. Logistics requires features that cannot be expressed in \( C_2 \) and which therefore are not captured by GNNs (Barceló et al. 2020; Grohe 2020). Spanner*, like Reward and other domains, involves the computation of distances in the test instances that exceed the number of layers used in the GNN. The bottom part of Table 2 shows the results that are obtained in Logistics and Spanner* when these limitations are addressed logically by extending the states (in training, validation, and testing) with suitable derived atoms and predicates, a facility provided by PDDL (Thiébaut, Hoffmann, and Nebel 2005; Haslum et al. 2019b). For example, one can extend the states in Blocks with the derived predicate above that corresponds to the transitive closure of the domain predicate on, so that every state \( s \) contains additional atoms above(x, y) when block \( x \) is above block \( y \) in \( s \).

In Logistics, four derived predicates are added, following the four role compositions used by Francês et al. (2019) to obtain a general value function. These role compositions go beyond the expressive capabilities of \( C_2 \) and GNNs. In Logistics, there are binary predicates (roles) to express that a package or truck is at some location (‘at’), to express that a package is inside a truck or airplane (‘in’), and to express that a location is in a city (‘in-city’). Additionally, as done in previous works, “goal versions” of these predicates (indeed, of all predicates denoted by ‘at@’, ‘in@’ and ‘in-city@’ whose denotation is provided by the goal descriptions are added to the domain. The Logistics domain is extended with the following role compositions from Francês et al. (2019):

- ‘at\circ\text{in-city}’ and ‘at@\circ\text{in-city}’ that tells the city where a package is located, either in the current or goal state,
- ‘in\circ\text{at}’ that tells the location of a package that is inside a truck, and
- ‘in\circ\text{at}\circ\text{in-city}’ that tells the city where a package that is inside a truck is located.

In Spanner*, a single derived predicate is added which is the transitive closure of the ‘link’ predicate. Provided with the new \( \text{link}^+ \) predicate, the required distances in Spanner* are not restricted by the number of layers \( L \) in the GNN and can be computed in a single layer, as the distance to the exit location equals the number of locations to the right of the current location \( c \); i.e., dist2exit = \(|\{x|\text{link}^+(c, x)\}|\).

The results obtained by learning from states with these derived predicates in Logistics and Spanner* are shown at
the bottom of Table 2. In Logistics, the simple addition of the atoms makes the coverage jump from from 0% to 14% for the greedy policy alone, and from 60% to 100% for the greedy policy with cycle avoidance. For Spanner*, three rows are shown: the first is for the domain without derived atoms but with two modifications that preclude comparison with the Spanner* results reported previously in the same table. The first is that the test instances involving more than 100 locations have been replaced by smaller instances with up to 45 and 50 locations. The second is that the number of layers \( L \) in the GNN are reduced from 30 to 10. These modifications provide a more convenient baseline for evaluating the impact of derived atoms: with 100 locations, there are 10,000 = 100\(^2\) extra derived atoms in the states, that make training and testing much slower (this is a weakness of adding derived atoms). It is because of these modifications, and in particular from the reduction in the value of \( L \) from 30 to 10, that the coverage of the learned policies in the modified Spanner* setting is reduced to 33% and 22% percent (first of the last three rows in the table). This number however increases to 86% and 77% when the derived atoms are included, even if the number of GNN layers is reduced from 10 to 5 (second of the last three rows in table). Moreover, coverage increases further to 100% when the derived atoms are included and the number of GNN layers is reduced further to just 2 (last row in the table). This additional increase in coverage is likely due by reduced overfitting as the number of layers \( L \) is reduced from 5 to 2.

9 Conclusions

We have considered the problem of learning generalized policies for classical planning domains from small instances represented in lifted STRIPS. Unlike previous work that makes use of a predefined pool of features based on description logic and combinatorial solvers, we have followed the GNN approach for learning general policies advanced by Ståhlberg, Bonet, and Geffner (2022) that exploits the relation between \( C_2 \) features and those that can be computed by GNNs. However, instead of learning optimal value functions in a supervised manner, we learn non-optimal value functions without supervision. For this, the change is technically small, as it affects the loss function and not the GNN architecture, but the consequences are interesting as the new method can be applied to domains that have general policies but no general policies that are optimal. We have shown that 100% generalization is achieved in many such domains, and have discussed and addressed two important additional issues: the limitations of value-based RL methods for computing general policies over domains where optimal planning is intractable, and the limitations of GNNs for capturing general value functions that require non-C\(_2\) features. We have addressed the first limitation by using a novel loss function \((L_1)\) different than the more natural loss function \(L_0\) associated with value-based RL methods, and the second limitation, by extending planning states with derived atoms. In the future, we would like to make the point about the limitations of RL methods for learning generalized plans, sharper, and to consider the use of recent GNN architectures that compute features beyond \( C_2 \) (Bevilaquca et al. 2021). At the same time, we are interested in “domesticating” the use of deep learning engines in the context of planning and representation learning for planning, so that they can be used as alternatives to ASP and Weighted Max-SAT solvers, for avoiding scalability issues and for opening up new possibilities. This requires understanding what can be computed with them in a clean way and how. This work is also a step in that direction.

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References


