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On the Progression of Belief

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Abstract

Based on weighted possible-world semantics, Belle and Lakemeyer recently proposed the logic DS, a probabilistic extension of a modal variant of the situation calculus with a model of belief. The logic has many desirable properties like full introspection and it is able to precisely capture the beliefs of a probabilistic knowledge base in terms of the notion of only-believing. While the proposal is intuitively appealing, it is unclear how to do planning with such logic. The reason behind this is that the logic lacks projection reasoning mechanisms. Projection reasoning, in general, is to decide what holds after actions. Two main solutions to projection exist: regression and progression. Roughly, regression reduces a query about the future to a query about the initial state while progression, on the other hand, changes the initial state according to the effects of actions and then checks whether the formula holds in the updated state. In this paper, we study projection by progression in the logic DS. It is known that the progression of a categorical knowledge base wrt a noise-free action corresponds to what is only-known after that action. We show how to progress a type of probabilistic knowledge base wrt noisy actions by the notion of only-believing after actions. Our notion of only-believing is closely related to Lin and Reiter's notion of progression.

1 Introduction

Rich representation of knowledge and actions has been a goal that many AI researchers pursue. Among all proposals, perhaps, the situation calculus by McCarthy (1963) is the most widely-studied, where actions are treated as logical terms and agent's knowledge is represented by logical formulas. The language has been extended to incorporate many features like time, concurrency, procedures, etc. Later, combining it with probability, Bacchus, Halpern, and Levesque (1999) (BHL) provided a rich account of dealing with degrees of belief and noisy sensing. The main advantage of a logical account like BHL is that it allows partial or incomplete specifications of beliefs depending on what information is actually available in a particular domain.

Alternatively, Belle and Lakemeyer (BL) (2017) proposed a formulation of BHL's ideas based on a modal variant of the situation calculus (Lakemeyer and Levesque 2004), extending earlier work on static probabilistic beliefs (Belle, Lakemeyer, and Levesque 2016). Unlike the axiomatic BHL, BL's logic $\mathcal{DS}$ is based on possible-world semantics with distributions over possible-worlds. More concretely, a distribution is just an assignment of non-negative weights to the possible worlds. An epistemic state is then defined as a set of such distributions and a sentence ϕ is believed with degree r if and only if the normalized sum of the weight of worlds that satisfies ϕ equals r in all distributions of the epistemic state. Later, beliefs after a sequence of actions are defined by the notions of *action likelihood* and *observational-indistinguishability* which captures the idea that the agent might not be able to distinguish between certain actions.

The logic has many interesting properties such as full introspection of beliefs. Besides, it is possible to express all the agent's beliefs of a probabilistic knowledge base (KB) by appealing to a notion of only-believing. Nevertheless, the problem of how to plan with such logic is still open. The reason behind this is the lack of projection reasoning mechanisms. Projection reasoning, in general, is to decide what holds after actions. There are two main solutions to the projection problem: regression and progression. Roughly, regression reduces a query about the future to a query about the initial state while progression, on the other hand, changes the initial state according to the effects of actions and then checks whether the formula holds in the updated state. Compared with regression, progression is more challenging as Lin and Reiter (1997) proved that progression in general requires second-order logic.

Progression has been developed since then, mainly by appealing to the notion of forgetting. Later, Lakemeyer and Levesque (2009) showed that the progression of a categorical knowledge base specified by only-knowing wrt to a noise-free action amounts to what is only known by the agent after that action. In the setting of quantitative beliefs and noisy actions, the progression would correspond to what is only believed after actions. However, the current semantics of the only-believing $\boldsymbol{O}$ in $\mathcal{DS}$ is problematic to reflect this correctly. To see a concrete example, consider a robot moving toward a wall as in Fig. 1. Suppose a fluent h indicates the robot's distance to the wall and the robot is equipped with an accurate sonar (specified by the action model Σ). In Lakemeyer and Levesque's work, the following holds:

$$\models \boldsymbol{O}((h = 1 \vee h = 2) \wedge \Sigma) \supset [sonar(2)]\boldsymbol{O}(h = 2 \wedge \Sigma)$$

In English, only-knowing the distance is 1 or 2 and the ac-

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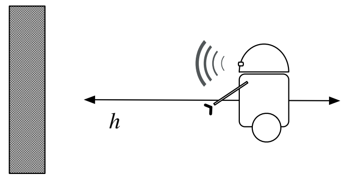

Figure 1: robot moving toward a wall

tion model of sonar entails that after the sonar reads 2 the agent only knows its distance is 2 (and the action model). Likewise, in a stochastic setting, one would expect that:

$$\models \boldsymbol{O}(h = 1\colon 0.5,\ h = 2\colon 0.5,\ \Sigma\colon 1)$$
$$\supset [sonar(2)]\boldsymbol{O}(h = 2 \wedge \Sigma\colon 1)$$

Namely, only-believing h is among $\{1, 2\}$ with equal degree and the action model with degree one entails that after the sensing the agent only believes $h = 2$ (and the action model) with degree one. This does not follow in the logic $\mathcal{DS}$. Because the semantics of only-believing in $\mathcal{DS}$ seems only work for the initial state and it's unclear what an epistemic state that satisfies $[sonar(2)]\boldsymbol{O}(h = 2 \wedge \Sigma\colon 1)$ looks like.

Another issues is that $\mathcal{DS}$ lacks the expressiveness to specify belief distributions. Sentences like $\boldsymbol{O}((\forall u.\boldsymbol{B}(h = u\colon \mathbf{U}_{\{1,2\}}(u)) \wedge \Sigma)\colon 1)$ are unsatisfiable, even if they are intuitively reasonable (here $\mathbf{U}_{\{1,2\}}(u)$ refers to the discrete uniform distribution with points among $\{1, 2\}$).

In this paper, we will address the above issues of the logic $\mathcal{DS}$ by modifying both its language and semantics, which results in a new logic $\mathcal{DS}_p$. More concretely, by special treatment of rigid terms, we are able to express distributions, for example, the above uniform distribution and geometric distributions (with expectation 2) $\forall u.\boldsymbol{B}(h = u\colon \mathcal{G}(\frac{1}{2}, u))$. Besides, only-believing arbitrary formulas is satisfiable including the formula in the robot examples. By virtue of our notion of progressed distribution, we are able to fully reconstruct the results of Lakemeyer and Levesque in the new logic. For a fragment of the logic, we show classical progression is first-order definable. Lastly, we provide our solution for the progression of belief in terms of only-believing after actions.

The rest of the paper is organized as follows. In section 2, we introduce the syntax and semantics of logic $\mathcal{DS}_p$. The semantics of progression is presented in section 3, where we address our solution of progression wrt noisy sensing and stochastic actions. In section 4 and 5, we discuss related work and conclude the paper, respectively.

2 The Logic $\mathcal{DS}_p$

$\mathcal{DS}_p$ is a modal language with equality and sorts of type *object* and *action*. Implicitly, we assume that *number* is a sub-sort of object and refers to the *computable numbers* $\mathbb{C}$.[1] Before presenting the formal definitions, here are the main features:

- *standard names*: The language includes (countably many) standard names $\mathcal{N}$ for both objects $\mathcal{N}_O$ and actions $\mathcal{N}_A$ ($\mathcal{N} = \mathcal{N}_O \cup \mathcal{N}_A$). This can be viewed as a fixed infinite domain closure with the unique name assumption, which further allows first-order (FO) quantification to be understood substitutionally. Moreover, equality can also be treated in a simpler way: every ground term will have a coreferring standard name, and two terms are equal if their coreferring standard names are identical.
- *rigid and fluent functions*: The language contains both fluent and rigid function symbols. For simplicity, all action functions are rigid and we do not include predicate symbols in the language. Fluents vary as the result of actions, yet meaning of rigid functions is fixed.
- *belief and truth*: The language includes modal operators $\boldsymbol{B}$ and $\boldsymbol{O}$ for degrees of belief and only-believing respectively. Such operators allow us to distinguish between sentences that are true and sentences that are believed to be true with positive degrees.
- *observational-indistinguishability*: Finally, unlike deterministic domains, the effects of action could be non-deterministic. This is characterized by stochastic actions. Instead of saying an action might have non-deterministic effects, we say the action is stochastic and has non-deterministic alternatives which are not observationally distinguishable to the agent (as indicated by a special function *oi*).

2.1 The Language

Definition 1. *The symbols of $\mathcal{DS}_p$ are taken from the following vocabulary:*

- *first-order variables: $u, v, x, y \ldots a, a' \ldots$;*
- *second-order (SO) function variables: $F, F' \ldots$;*
- *rigid function symbols of every arity, such as sonar(x), including arithmetical functions like $+, \times$, etc.;*
- *fluent function symbols of every arity, such as distanceTo(x), heightOf(y), including an unary special symbols l and a special binary symbols oi. Roughly, l returns the likelihood of an action and oi describes the* observational-indistinguishability *(alternative choices) among actions* [2]*;*
- *connectives and other symbols: $=, \wedge, \neg, \forall, \boldsymbol{B}, \boldsymbol{O}, [\cdot], \square$, round and square parentheses, period, colon, comma. $\boldsymbol{B}$ and $\boldsymbol{O}$ are called epistemic operators.*

Definition 2. *The **terms** of the language are the least set of expressions such that:*

- *every standard name and FO variable is a term;*
- *if $t_1, \ldots t_k$ are terms, f a k-ary function symbol, then $f(t_1, \ldots t_k)$ is a term;*
- *if $t_1, \ldots t_k$ are terms, F a k-ary SO variable, then $F(t_1, \ldots t_k)$ is a term.*

[1] We use the computable numbers as they are still enumerable and allow representing distributions mentioning real numbers such as Euler's number e (Turing 1937).

[2] We don't include the usual $poss$ function (action precondition), as in stochastic setting, that an action is impossible can always be specified by saying that action has 0 likelihood.

A term is said to be rigid, if and only if it does not contain fluents. *Ground terms* are terms without variables while *SO ground terms* are terms without FO variables. *Primitive terms* are terms of the form $f(n_1, \dots n_k)$, where f is a function symbol and n_i are object standard names. *SO primitive terms* are defined likewise by replacing f with F, a second-order variable. We denote the sets of primitive terms of sort object and action as $\mathcal{P}_O$ and $\mathcal{P}_A$, respectively, and the set of all SO primitive terms as $\mathcal{P}_{SO}$. While object standard names are syntactically like constants, we require that action standard names are all the primitive action terms, i.e. $\mathcal{N}_A = \mathcal{P}_A$. For example, the action $sonar(5)$, where a sonar returns the number 5, is considered a standard action name. Furthermore $\mathcal{Z}$ refers to the set of all finite sequences of action standard names, including the empty sequence $\langle\rangle$. We reserve standard names $\top, \bot$ in $\mathcal{N}_O$ for truth values (to simulate predicates).

Definition 3. *The **well-formed formulas** of the language are the least set of expressions such that:*

- *If t_1, t_2 are terms, then $t_1 = t_2$ is a formula;*
- *If t_a is a term of sort action and α a formula, then $[t_a]\alpha$ is a formula;*
- *If α and β are formulas, v a FO variable, F a SO variable, r, r_i rigid terms, then $\alpha \wedge \beta$, $\neg\alpha$, $\forall v.\alpha$, $\forall F.\alpha$, $\Box\alpha$, $\boldsymbol{B}(\alpha\colon r)$, and $\boldsymbol{O}(\alpha_1 : r_1, \dots, \alpha_k : r_k)$ are also formulas.*

$[t_a]\alpha$ should be read as "α holds after action t_a" and $\Box\alpha$ as "α holds after any sequence of actions." The epistemic expression $\boldsymbol{B}(\alpha\colon r)$ should be read as "α is believed with a degree r,". $\boldsymbol{K}\alpha$ means "α is known" and is an abbreviation for $\boldsymbol{B}(\alpha\colon 1)$. $\boldsymbol{O}(\alpha_1 : r_1, \dots \alpha_k : r_k)$ may be read as "the α_i with a probability r_i are all that is believed". Similarly, $\boldsymbol{O}\alpha$ means "α is only known" and is an abbreviation for $\boldsymbol{O}(\alpha\colon 1)$. For action sequence $z = a_1 \cdots a_k$, we write $[z]\alpha$ to mean $[a_1] \cdots [a_k]\alpha$. α^x_t is the formula obtained by substituting all free occurrences of x in α by t. As usual, we treat $\alpha \vee \beta$, $\alpha \supset \beta$, $\alpha \equiv \beta$, and $\exists v.\alpha$ as abbreviations.

A *sentence* is a formula without free variables. We use TRUE as an abbreviation for $\forall x(x = x)$, and FALSE for its negation. A formula with no $\Box$ is called *bounded*; a formula with no $\Box$ or $[t_a]$ is called *static*; a formula with no $\boldsymbol{B}$ or $\boldsymbol{O}$ is called *objective*; a formula with no fluent, $\Box$ or $[t_a]$ outside $\boldsymbol{B}$ or $\boldsymbol{O}$ is called *subjective*; a formula with no $\boldsymbol{B}$, $\boldsymbol{O}$, $\Box$, $[t_a]$, l, oi is called a *fluent formula*; a fluent formula without fluent functions is called a *rigid formula*.

2.2 The Semantics

The semantics is given in terms of *possible worlds*, which define what is true initially and after any sequence of actions. Compared to non-probabilistic accounts with deterministic actions (Lakemeyer and Levesque 2004), a number of challenges need to be addressed, including how to specify probabilities over *uncountably* many possible worlds, how to deal with multiple probability distributions entertained by the agent, and how to deal with probabilistic action effects, which may be *unobservable* by the agent.

A **world** w is a mapping from the primitive terms ($\mathcal{P}_O \cup \mathcal{P}_A$) and $\mathcal{Z}$ to $\mathcal{N}$ of the right sort, satisfying:

1. **Rigidity:** If t is a rigid primitive term, then for all $(w, z), (w', z')$, $w[t, z] = w'[t, z']$;
2. **Arithmetical Correctness:** If f is an arithmetical expression and val is its value in the usual sense, then for all (w, z), $w[f, z] = val$. For example, $w[1 + 1, z] = 2$.

Let $\mathcal{W}$ be the set of all such worlds. FO free variables are handled substitutionally by using standard names. To interpret free SO variables, we need variable maps. A variable map λ is a mapping from $\mathcal{P}_{SO}$ to $\mathcal{N}_O$. We write $\lambda \sim_F \lambda'$ to mean λ and λ' agree excepts perhaps on SO primitives involving F. We now define the co-referring standard names for SO ground terms (essentially, the denotation of terms). Given a SO ground term t, a world w, and action sequence z, a variable map λ, we define $|t|^z_{w,\lambda}$ (read: the co-referring standard name for t given w, z, λ) recursively by :

1. If $t \in \mathcal{N}$, then $|t|^z_{w,\lambda} = t$;
2. $|f(t_1, \dots t_k)|^z_{w,\lambda} = w[f(|t_1|^z_{w,\lambda}, \dots |t_k|^z_{w,\lambda}), z]$;
3. $|F(t_1, \dots t_k)|^z_{w,\lambda} = \lambda[F(|t_1|^z_{w,\lambda}, \dots |t_k|^z_{w,\lambda})]$.

For a rigid SO ground term t, we use $|t|_\lambda$ instead of $|t|^z_{w,\lambda}$ for its denotation, moreover, if t is first-order, we write $|t|$. We will require that $l(a)$ is of sort number, and $oi(a, a')$ only take values $\top$ or $\bot$, and oi behaves like an equivalence relation (reflexive, symmetric, and transitive).

$oi(a, a')$ means a and a' are observationally indistinguishable actions. In the example of Fig. 1, the robot might perform a stochastic action $fwd(x, y)$, where x is its intended forward distance and y is the actual outcome selected by nature. x is observable to the robot while y is not. Then, $oi(fwd(1, 1.1), fwd(1, 0.9))$ says that nature can non-deterministically select 1.1 or 0.9 as a result for the intended value 1.

By a **distribution** d we mean a mapping from $\mathcal{W}$ to $\mathbb{R}^{\geq 0}$ (non-negative real) and an **epistemic state** e is any set of distributions. By a model, we mean a 4-tuple (e, w, z, λ). In order to prepare for the semantics, we need to extend $l(a), oi(a, a')$ from actions to action sequences:

Definition 4. *We define*

1. $l^*\colon \mathcal{W} \times \mathcal{Z} \rightarrow \mathbb{R}^{\geq 0}$ *as*
   $l^*(w, \langle\rangle) = 1$ *, for every* $w \in \mathcal{W}$;
   $l^*(w, z \cdot a) = l^*(w, z) \times n$ *where* $w[l(a), z] = n$;
2. $z \sim_w z'$ *as*
   $\langle\rangle \sim_w z'$ *iff* $z' = \langle\rangle$;
   $z \cdot a \sim_w z'$ *iff* $z' = z^* \cdot a^*, z \sim_w z^*, w[oi(a, a^*), z] = \top$;

Obviously, $\sim_w$ is an equivalence relation. We require that $w[l(a), z] \leq 1$ and $\sum_{\{a' : a' \sim_w a\}} w[l(a), z] = 1$, for all $w \in \mathcal{W}, z \in \mathcal{Z}$, and $a \in \mathcal{N}_A$. Since $\mathcal{W}$ is uncountable, to obtain a well-defined sum over uncountably many worlds, the following three conditions are used for evaluating beliefs:

Definition 5. *We define* BND, EQ, NORM *for any distribution d and any set $\mathcal{V} = \{(w_1, z_1), (w_2, z_2), \dots\}$ as follows:*

1. BND$(d, \mathcal{V}, n)$ *iff* $\neg\exists k, (w_1, z_1), \dots, (w_k, z_k) \in \mathcal{V}$ *such that* $\sum_{i=1}^{k} d(w_i) \times l^*(w_i, z_i) > n$.
2. EQ$(d, \mathcal{V}, n)$ *iff* BND$(d, \mathcal{V}, n)$ *and there is no* $n' < n$ *such that* BND$(d, \mathcal{V}, n')$ *holds.*

3. *for any* $\mathcal{U} \subseteq \mathcal{V}$, $\textsc{Norm}(d,\mathcal{U},\mathcal{V},n)$ *iff* $\exists b \neq 0$ *such that* $\textsc{Eq}(d,\mathcal{U},b\times n)$ *and* $\textsc{Eq}(d,\mathcal{V},b)$.

Intuitively, given $\textsc{Norm}(d,\mathcal{U},\mathcal{V},n)$, n can be viewed as the *normalized* sum of the weights of worlds in $\mathcal{U}$ wrt d in relation to $\mathcal{V}$. Here $\textsc{Eq}(d,\mathcal{V},b)$ expresses that the weight of the worlds wrt d in $\mathcal{V}$ is b, and finally $\textsc{Bnd}(d,\mathcal{V},b)$ ensures the weights of worlds in $\mathcal{V}$ is bounded by b. In essence, even if $\mathcal{W}$ is uncountable, the condition $\textsc{Eq}$ and $\textsc{Bnd}$ on d admit a well-defined summation of the weights on worlds, i.e. only countably many worlds have non-zero weight wrt d (Belle, Lakemeyer, and Levesque 2016).

The truth of sentences in $\mathcal{DS}_p$ is defined as:

- $e,w,z,\lambda \models t_1 = t_2$ iff $|t_1|^z_{w,\lambda}$ and $|t_2|^z_{w,\lambda}$ are identical;
- $e,w,z,\lambda \models \neg\alpha$ iff $e,w,z,\lambda \not\models \alpha$;
- $e,w,z,\lambda \models \alpha \wedge \beta$ iff $e,w,z,\lambda \models \alpha$ and $e,w,z,\lambda \models \beta$;
- $e,w,z,\lambda \models \forall v.\alpha$ iff $e,w,z,\lambda \models \alpha^v_n$ for every standard name n of the right sort;
- $e,w,z,\lambda \models \forall F.\alpha$ iff $e,w,z,\lambda' \models \alpha$ for all $\lambda' \sim_F \lambda$;
- $e,w,z,\lambda \models [t_a]\alpha$ iff $e,w,z\cdot n,\lambda \models \alpha$ and $n = |t_a|^z_{w,\lambda}$;
- $e,w,z,\lambda \models \Box\alpha$ iff $e,w,z\cdot z',\lambda \models \alpha$ for all $z' \in \mathcal{Z}$.

To prepare for the semantics of epistemic operators, let $\mathcal{W}^{e,z,\lambda}_{\alpha} = \{(w',z') \mid z' \sim_{w'} z, \text{ and } e,w',\langle\rangle,\lambda \models [z']\alpha\}$. If $z = \langle\rangle$, we ignore z and write $\mathcal{W}^{e,\lambda}_{\alpha}$. If α is FO, we ignore λ and write $\mathcal{W}^{e,z}_{\alpha}$. If the context is clear, we write $\mathcal{W}_\alpha$. Intuitively, $\mathcal{W}_\alpha$ is the set of alternatives (world and action sequence pairs) of z that might result in α. A distribution d is **regular** iff $\textsc{Eq}(d,\mathcal{W}^{\{d\}}_{\text{TRUE}},n)$ for some $n \in \mathbb{R}^{>0}$. We denote the set of all regular distributions as $\mathcal{D}$.

Definition 6. *Given* $w \in \mathcal{W}, d \in \mathcal{D}, z \in \mathcal{Z}$, *we define*

- w_z *as a world such that for all primitive term* t *and* $z' \in \mathcal{Z}$, $w_z[t,z'] = w[t,z\cdot z']$;
- d_z *a mapping such that for all* $w \in \mathcal{W}$, $d_z(w) = \sum_{\{w':d(w')>0\}}\sum_{\{z':z'\sim_{w'}z\ ,\ (w')_{z'}=w\}} d(w')\times l^*(w',z')$.

w_z is called the **progressed world** of w wrt z. A remark is that the d_z might not be regular for a regular d. For example, if the likelihood of z's alternatives is all zero in worlds with non-zero weights, then $\textsc{Eq}(d_z,\mathcal{W}^{d_z}_{\text{TRUE}},0)$. Hence we define:

Definition 7. *A distribution* d *is* ***compatible*** *with action sequence* z, $d \sim_{comp} z$ *iff* $d_z \in \mathcal{D}$; *given an epistemic state* e, *the set* $e_z = \{d_z | d \in e\cap\mathcal{D}, d \sim_{comp} z\}$ *is called the* ***progressed epistemic state*** *of* e *wrt* z.

As a consequence, $d \sim_{comp} \langle\rangle$ iff $d \in \mathcal{D}$. Note that the progressed epistemic state of e are only about it's regular subset $e\cap\mathcal{D}$ and $e_z \subseteq \mathcal{D}$, therefore $e \neq e_{\langle\rangle}$ in general.

Proposition 1. *Given* $d \in \mathcal{D}$ *and* $z,z' \in \mathcal{Z}$, $d_{z\cdot z'} = (d_z)_{z'}$.

The truth of $\boldsymbol{B}$ and $\boldsymbol{O}$ are given by:

- $e,w,z,\lambda \models \boldsymbol{B}(\alpha\colon r)$ iff $\forall d \in e_z$, $\textsc{Norm}(d,\mathcal{W}^{\{d\},\lambda}_{\alpha},\mathcal{W}^{\{d\},\lambda}_{\text{TRUE}},n)$ for $n \in \mathbb{R}$ and $n = |r|_\lambda$;
- $e,w,z,\lambda \models \boldsymbol{O}(\alpha_1\colon r_1,\ldots,\alpha_k\colon r_k)$ iff $\forall d$, $d \in e_z$ iff for all $1 \leq i \leq k$, $\textsc{Norm}(d,\mathcal{W}^{\{d\},\lambda}_{\alpha_i},\mathcal{W}^{\{d\},\lambda}_{\text{TRUE}},n_i)$ for $n_i \in \mathbb{R}$, and $n_i = |r_i|_\lambda$;

Intuitively, an epistemic state e only believes something after action sequence z, if and only if the progressed epistemic state e_z is the maximal epistemic state consisting of all distributions that believe it. However, this definition of $\boldsymbol{O}$ is problematic to some extents. To see why we need the following notation.

Let $S(d)$ be the support set of d, i.e. $S(d) = \{w | d(w) > 0\}$, we define a distance function ρ for regular distributions $\mathcal{D}$ as $\rho(d,d') = \sum_{w\in S(d)\cup S(d')} |d(w) - d'(w)|$. Clearly $(\mathcal{D},\rho)$ forms a metric space. Given an infinite sequence of regular distributions $\{d_1,d_2,\ldots d_n,\ldots\}$, we say it converges to a regular distribution d if for any $\epsilon > 0$, there exists a natural number $N \in \mathbb{N}$ s.t. for any $n > N$, $\rho(d_n,d) < \epsilon$. If such a d exists, we write $\lim_{n\to\infty} d_n = d$ and call d the limit of $\{d_1,d_2,\ldots d_n,\ldots\}$.

Definition 8 (Closure). *For all* $e \subseteq \mathcal{D}$, *the closure of* e *is defined as* $cl(e) = \{d | d \in \mathcal{D}, \exists\{d_1,d_2,\ldots d_n,\ldots\}, \forall i \in \mathbb{N}, d_i \in e, \lim_{i\to\infty} d_i = d\}$.

We call an $e \subseteq \mathcal{D}$ closed if and only if $e = cl(e)$.

Proposition 2. *Given regular* d *and* $\{d_1,d_2,\ldots d_n,\ldots\}$ *s.t.* $d = \lim_{n\to\infty} d_n$, *for any* $w \in \mathcal{W}$, *the limit of* $d_n(w)$ *exists and* $d(w) = \lim_{n\to\infty} d_n(w)$

Theorem 1. *Let* e *be an epistemic state,* λ *a variable map,* α *a sentence* α, *and* r *a rigid term, if for all* $d \in e$, $\textsc{Norm}(d,\mathcal{W}^{\{d\},\lambda}_{\alpha},\mathcal{W}_{\text{TRUE}},|r|_\lambda)$, *then for all* $d' \in cl(e)$, $\textsc{Norm}(d',\mathcal{W}^{\{d'\},\lambda}_{\alpha},\mathcal{W}_{\text{TRUE}},|r|_\lambda)$.

The proof require the definition of limit and Prop. 2. The theorem suggests that $\{d | \textsc{Norm}(d,\mathcal{W}^{\{d\},\lambda}_{\alpha},\mathcal{W}_{\text{TRUE}},|r|_\lambda)\}$ is a closed set for any α,λ, and r. Therefore, e_z should be closed to enable the maximal semantics for only-believing. Hence, we have the following definition:

Definition 9 (**Progressed Epistemic State**). *Given* e, $z \in \mathcal{Z}$, *we defined* e_z *as* $e_z = cl(\{d_z \in \mathcal{D} | d \in e\cap\mathcal{D}, d \sim_{comp} z\})$, *i.e.* $e_z = \{d \in \mathcal{D} | \exists\{d_1,d_2,...,d_n,...\}, \forall i \in \mathbb{N}, d_i \in e\cap\mathcal{D}, d_i \sim_{comp} z, d = \lim_{i\to\infty}(d_i)_z\}$

A remark is that Definition 9 ensures that e_z is closed, yet whether the e_z by Definition 7 already ensures e_z is closed remains open. In case the answer is positive, the two definitions are exactly the same. In case negative, the maximal semantics for only-believing only works for Definition 9. Now, the truth of $\boldsymbol{B}$ and $\boldsymbol{O}$ are given exactly the same as before except e_z with the new definition.

For sentence α, we write $e,w \models \alpha$ to mean $e,w,\langle\rangle,\lambda \models \alpha$ for all variable maps λ. When Σ is a set of sentences and α is a sentence, we write $\Sigma \models \alpha$ (read: Σ logically entails α) to mean that for every set of regular distributions e and w, if $e,w \models \alpha'$ for every $\alpha' \in \Sigma$, then $e,w \models \alpha$. We say that α is valid ($\models \alpha$) if $\{\} \models \alpha$. Satisfiability is then defined in the usual way. If α is an objective formula, we write $w \models \alpha$ instead of $e,w \models \alpha$. Similarly, we write $e \models \alpha$ instead of $e,w \models \alpha$ if α is subjective.

2.3 Comparison with $\mathcal{DS}$ and Some Properties

The language of $\mathcal{DS}$ and $\mathcal{DS}_p$ are rather similar except that $\mathcal{DS}$ considers fluent predicates while $\mathcal{DS}_p$ only has fluent functions. While in $\mathcal{DS}$ every closed term is a standard name, we follow (Lakemeyer and Levesque 2011), who use special standard names for objects and actions. The semantic structures in both logics are essentially the same, consisting of worlds, sets of distributions over worlds serving as epistemic states, and action sequences. Our use of rigid mathematical functions, which are not considered in $\mathcal{DS}$, is similar to the $\mathcal{R}$-interpretation in (Belle and Levesque 2018). Among other things, this allows us later to express degrees of belief specified by arbitrary rigid terms.

The main difference lies in the semantics of beliefs and only-believing. To appreciate the difference, it is instructive to review the semantics in $\mathcal{DS}$. While many notations are exactly the same, things diverge from the definition of $\mathcal{W}_\alpha$. First, $\mathcal{DS}$ keeps the traditional special $poss$ predicate to specify action preconditions and a notion of $exec(z)$, defined recursively by $poss$, to express action sequence z is executable. Second, $\mathcal{DS}$ defines a compatibility between worlds wrt oi: $w' \sim_{oi} w$ iff for all a, a', and z, $w'[oi(a,a'),z] = w[oi(a,a'),z]$, that is w' is compatible with w if they agree on oi. Finally, $\mathcal{W}_\alpha$ in $\mathcal{DS}$ is then defined wrt triples e, w, z [3]: $\mathcal{W}_\alpha^{e,w,z} = \{(w',z') | z' \sim_{w'} z, w' \sim_{oi} w$, and $e, w', \langle\rangle \models [z'] \wedge exec(z')\}$. Note that worlds in $\mathcal{W}_\alpha^{e,w,z}$ agree with w wrt observational-indistinguishability. Truth of beliefs ($\boldsymbol{B}'$) and only-believing ($\boldsymbol{O}'$) is given by

- $e, w, z \models \boldsymbol{B}'(\alpha\colon n)$ iff $\forall d \in e$ (not e_z), $\text{NORM}(\mathcal{W}_\alpha^{e,w,z}, \mathcal{W}_{\text{TRUE}}, n)$ for constant n;
- $e, w, z \models \boldsymbol{O}'(\alpha_1\colon n_1, \ldots, \alpha_k\colon n_k)$ iff $\forall d$, $d \in e$ iff $\forall i.1 \leq i \leq k$, $\text{NORM}(d, \mathcal{W}_{\alpha_i}^{e,w,z}, \mathcal{W}_{\text{TRUE}}, n_i)$ for constants n_i.

One consequence of only allowing constants in degrees of belief is that formula like $\boldsymbol{B}(p\colon 0.1 + 0.2)$ is not well-defined. The new logic overcomes this by a special treatment of rigid terms that is terms like $0.1+0.2$ have the same denotations in all worlds, i.e. 0.3. This, among other things, enables us to include formulas like $\forall u.\boldsymbol{B}(h = u\colon \mathcal{G}(\frac{1}{2}, u))$ as well-formed formula, where h is a fluent and $\mathcal{G}(\frac{1}{2}, u)$ stands for geometry distribution with expectation 2.

Another observation is that, while $\mathcal{W}_\alpha$ in $\boldsymbol{B}'$ and $\boldsymbol{O}'$ involves e and w, $\mathcal{W}_\alpha$ in $\boldsymbol{B}$ and $\boldsymbol{O}$, however, only involves individual $d \in e$. Such a change has major impacts on properties of the logic like introspection and meta-beliefs. Discussing them would go beyond the scope of this paper.

We comment that the idea of special treatment of rigid terms and using individual $d \in e$ in $\mathcal{W}_\alpha$ for $\boldsymbol{B}$ and $\boldsymbol{O}$ is from (Liu and Lakemeyer 2021). For the reason of being self-contained, we reiterate it here and it's not the main contribution of this work. Yet, we want to emphasize that:

Theorem 2. *For any sentence α and rigid ground term r, $\boldsymbol{O}(\alpha\colon r)$ is satisfiable.*

For any λ, let $e = \{d | \text{NORM}(\mathcal{W}_\alpha^{\{d\},\lambda}, \mathcal{W}_{\text{TRUE}}, |r|_\lambda)\}$, clearly $e, \lambda \models \boldsymbol{O}(\alpha\colon r)$. This is not the case when replacing $\boldsymbol{O}$ with $\boldsymbol{O}'$, even if r is FO, due to the e in $\mathcal{W}_\alpha^{e,w,z}$ in $\boldsymbol{O}'$.

Theorem 3. *Given an objective sentence α and constant n,*

- $\models \boldsymbol{B}(\alpha\colon n) \equiv \boldsymbol{B}'(\alpha\colon n)$;
- $\models \boldsymbol{O}'(\alpha\colon n) \supset \boldsymbol{O}(\alpha\colon n)$;

[3]Since $\mathcal{DS}$ is first-order, the variable map λ is not required.

Intuitively, in static case, when α is objective, $poss$, e, and $\{d\}$ in $\mathcal{W}_\alpha$ play no role, therefore $\mathcal{W}_\alpha$ coincides in the two logics. Hence $\boldsymbol{B}$ and $\boldsymbol{B}'$ are equivalent. As for only-believing, suppose an $e \subseteq \mathcal{D}$ s.t $e \models \boldsymbol{O}'(\alpha\colon n)$, we have $e = \{d | \text{NORM}(d, \mathcal{W}_\alpha, \mathcal{W}_{\text{TRUE}}, n)\}$. By Theorem 1, e is closed, therefore, $e = e_{\langle\rangle}$. By semantics, $e \models \boldsymbol{O}(\alpha\colon n)$. The converse does not hold, since there might exists an open set e s.t. $e \models \boldsymbol{O}(\alpha\colon n)$, yet $e \neq \{d | \text{NORM}(d, \mathcal{W}_\alpha, \mathcal{W}_{\text{TRUE}}, n)\} = cl(e)$ and therefore $e \not\models \boldsymbol{O}'(\alpha\colon n)$.

Although our notion of only-believing is weaker than its counterpart of $\mathcal{DS}$, we still retain the properties of only-believing:

Proposition 3. *Let α and α_i be arbitrary sentences, we have*

1. $\models \boldsymbol{O}(\alpha_1\colon r_1; \ldots \alpha_k\colon r_k) \supset \bigwedge \boldsymbol{B}(\alpha_i\colon r_i)$;
2. $\models \boldsymbol{O}(\alpha\colon r) \supset \neg\boldsymbol{B}(h(\vec{n}) = m\colon r')$ *for all r, r', and α, where $\vec{n}$ and m are std. names and h is a fluent not in α;*
3. *For any e, $e \models \boldsymbol{O}(\alpha_1\colon r_1, \ldots, \alpha_k\colon r_k)$ iff $cl(e) \models \boldsymbol{O}(\alpha_1\colon r_1, \ldots, \alpha_k\colon r_k)$*

The second part says that the agent has no beliefs about things not mentioned in the KB. Note that this is not true if $\boldsymbol{O}$ is replaced by $\boldsymbol{B}$. In the rest of paper, whenever we write $e \models \boldsymbol{O}(\alpha_1\colon r_1, \ldots, \alpha_k\colon r_k)$, we mean a closed e, unless stated otherwise.

Before we move on, let us turn back to the motivational example in the introduction:

Example 1. *Suppose Σ is the conjunction of the following* [4]
1. $\Box poss(a) = \text{TRUE}$
2. $\Box oi(a,a') = \top \equiv \exists y.a = sonar(y) \wedge a = a'$
3. $\Box l(a) = x \equiv x = 1 \wedge \exists y.h = y \wedge a = sonar(y)$,
then $\not\models \boldsymbol{O}'(h = 1\colon 0.5, h = 2\colon 0.5, \Sigma\colon 1) \supset [sonar(2)]\boldsymbol{O}'(h = 2 \wedge \Sigma\colon 1)$

Suppose the opposite holds and $e, w \models \boldsymbol{O}'(h = 1\colon 0.5, h = 2\colon 0.5, \Sigma\colon 1)$. Consider a distribution d s.t. $\{d\}, w \models \boldsymbol{K}'(h = 1 \wedge \Sigma\colon 1)$, clearly, $d \notin e$ since $e, w \models \boldsymbol{B}(h = 1\colon 0.5)$. In addition, one can check $\{d\}, w \models [sonar(2)]\boldsymbol{K}'(h = 1 \wedge \Sigma\colon 1)$. However, by hypothesis, $e, w \models [sonar(2)]\boldsymbol{O}'(h = 2 \wedge \Sigma\colon 1)$. By semantics of $\boldsymbol{O}'$, $d \in e$, contradiction. The reason is that truth of $\boldsymbol{O}'$ refers to e but not the progressed epistemic state e_z.

Lemma 1. *$(w_z)_{z'} = w_{z \cdot z'}$ and $(e_z)_{z'} = e_{z \cdot z'}$.*

The proof of the first is trivial by definition of w_z while the proof of the later requires not just the definition of e_z but also the triangle property of distance ρ.

Theorem 4. *For any sentence ψ, $e, w, z \models \psi$ iff $e_z, w_z \models \psi$*

The proof is by induction on the structure of ψ. The base case is by definition of w_z while the induction on $\neg, \vee, \exists, \boldsymbol{B}$, and $\boldsymbol{O}$ are simply by semantics. The induction on $[\cdot]$ and $\Box$ requires Lemma 1.

[4]Free variables are implicitly universal quantified outside. The modality $\Box$ has lower syntactic precedence than the connectives, and $[\cdot]$ has the highest priority.

3 The Semantics of Progression

Sometimes, it would be desirable to include usual mathematical functions as logical terms not just $+, \times$ like the uniform distribution in the introduction $\mathbf{U}_{\{1,2\}}$. Specially, we might use summation $\sum$ as a logical term [5]. One way is to assume infinite rigid function symbols, one for each such function, and semantically ensure all worlds in $\mathcal{W}$ interpret them identically, another way is to specify them syntactically by axioms. Here, we take the later approach and call these axioms *definitional axioms* [6] and these functions *definitional functions*, terms constructed by definitional functions *definitional terms*. E.g. the following axiom specifies $\mathbf{U}_{\{1,2\}}$.

$$\forall v.\forall u.\mathbf{U}_{\{1,2\}}(u) = v \equiv (u = 1 \lor u = 2) \land v = 0.5 \\ \lor \neg(u = 1 \lor u = 2) \land v = 0 \tag{1}$$

3.1 Basic Action Theories

BATs were first introduced by Reiter (2001) to encode the effects and preconditions of actions. Given a finite set of fluents $\mathcal{H}$, a BAT Σ over $\mathcal{H}$ consists of the union of the following sets:

- Σ_{ssa}: A set of successor state of axioms, one for each fluent in $\mathcal{H}$, of the form $\square[a]h(\vec{p}) = u \equiv \gamma_h$ to characterize action effects, also providing a solution to the frame problem (Reiter 2001). Here γ_h is a fluent formula with free variables $\vec{p}, u$ and it is functional in u,
- Σ_{oi}: A single axiom of the form $\square oi(a, a') = \top \equiv \psi$, where ψ is a rigid formula, to represent the observational-indistinguishability relation among actions.
- Σ_l: A set of axioms of the form $\square l(a) = \mathcal{L}(a)$ to capture action likelihoods. Here $\mathcal{L}(a)$ is a definitional term with free variable a.

We require Σ_{ssa}, Σ_{oi}, and Σ_l to be first-order. The condition that ψ is rigid ensures that the observational-indistinguishability among actions is fixed. Besides BATs, to infer future state, we need to specify what holds initially. This is achieved by a set of fluent sentences Σ_0 (might be second-order). By **belief distribution**, we mean the joint distribution of a finite set of random variables. Formally, assuming all fluents in $\mathcal{H}$ are **nullary**, [7] $\mathcal{H} = \{h_1, \ldots h_k\}$, a belief distribution $\boldsymbol{B}^f$ of $\mathcal{H}$ is a formula of the form $\forall \vec{u}.\boldsymbol{B}(\vec{h} = \vec{u}\colon f(\vec{u}))$, where $\vec{u}$ is a set of variables, $\vec{h} = \vec{u}$ stands for $\bigwedge h_i = u_i$, f is a definitional function of sort number with free variables $\vec{u}$. Finally, by a knowledge base, we mean a sentence of the form $\boldsymbol{O}(\boldsymbol{B}^f \land \Sigma)$. Although the logic allows KB specified by multiple distributions, for example, $\boldsymbol{O}(\boldsymbol{B}^{f_1} \lor \boldsymbol{B}^{f_2})$, we only consider KB with single distribution in this paper.

Example 2. *The following is a possible BAT Σ for our robot moving example:*

$$\square[a]h = u \equiv \exists x, y.a = fwd(x, y) \land u = \max\{0, h - y\} \\ \lor \forall x, y.a \neq fwd(x, y) \land h = u$$

$$\square oi(a, a') = \top \equiv \exists x, y, z.a = fwd(x, y) \land a' = fwd(x, z) \\ \lor a = sonar(z) \land a' = a$$

$$\square l(a) = \mathcal{L}(a) \quad \textit{with}$$

$$\mathcal{L}(a) = \begin{cases} \theta(x, y, 0.2, 0.6) & \exists x.\exists y.a = fwd(x, y) \\ \theta(z, h, 0.1, 0.8) & \exists z.a = sonar(z) \end{cases} \quad \textit{and}$$

further more, θ is given by

$$\theta(x, y, m, n) = \begin{cases} m & |x - y| = 1 \\ n & x = y \\ 0 & o.w. \end{cases}$$

a possible KB $\boldsymbol{O}(\boldsymbol{B}^f \land \Sigma)$ where f is given by [8].

$$f(u) = \begin{cases} 1/3 & u \in \{1, 2, 3\} \\ 0 & o.w. \end{cases}$$

In English, distance h can only be affected by $fwd(x, y)$ and the value is determined by value y, not the intended value x; the robot cannot get across the wall ($u = \max\{0, h-y\}$); two actions are observationally indistinguishable if and only if they are both forward actions with the same intended value or they are identical sensing action; likelihood of stochastic action $fwd(x, y)$ and noisy sensing $sonar(z)$ is specified by θ. The agent considers a uniform distribution among $\{1, 2, 3\}$ possible initially.

For the objective fragment, our definition of progression is similar to (Claßen 2013):

Definition 10. *Given Σ_0, Σ, and an rigid ground action term t, a set of fluent formula Σ_0' is called the progression of Σ_0 wrt t, Σ iff for all $w' \models \Sigma_0' \cup \Sigma$ iff there exists w s.t. $w \models \Sigma_0 \land \Sigma$ and $w_{|t|} = w'$.*

Theorem 5 (Lin and Reiter's progression)**.** *The following is a progression of Σ_0 wrt Σ, t:*

$$\exists \vec{F}.(\Sigma_0)^{\mathcal{H}}_{\vec{F}} \bigwedge \forall \vec{p}.\forall u.h(\vec{p}) = u \equiv (\gamma_h)^{a,\mathcal{H}}_{t,\vec{F}}$$

, where $\vec{F}$ are SO variables s.t. F_i, h_i are of the same arity.

Since LR progression was proposed, efforts have been made to find fragments where Σ_0' is first-order. For example, if the successor state of axioms is *local-effect* (Liu and Levesque 2005), the progression is first-order definable. Intuitively, local-effect means actions can only affect locally, like the block world example, the action $move(x, y, z)$, i.e. moving object x from y to z, only affects locally on the object x and location y and z but nothing globally. Here we show that the progression of the **nullary fluents** fragment is first-order definable.

[5] Summation is second-order definable see (Belle and Levesque 2018) for details. A problem of summation as a logical term is that summation is not closed under the computable domain as Weihrauch (2012) showed the limit of a infinite summation of rational could be a non-computable number, see also *Specker Sequence* (Specker 1949). Hence, for some terms with infinite summations, we cannot assign decent denotations. We use a special reserved standard name *undefined* for this purpose.

[6] In the rest of paper, whenever we write logical implication $\Sigma \models \alpha$, we implicitly mean $\Sigma \land \Delta \models \alpha$, where Δ is the set of all definitional axioms of functions involved in Σ and α.

[7] As discussed in (Belle and Levesque 2018), allowing fluents with arguments would result in joint distribution over infinitely many random variables, which is generally problematic in probability theory.

[8] Here, "$\in$" should be understood as a finite disjunction. For readability, we write the definition functions in this form, they should be understood as logical formula as Equation 1.

Theorem 6. *Given Σ_0, Σ, and an ground action term t where every fluents in $\mathcal{H}$ are nullary, then the following is a progression of Σ_0 wrt Σ, t:*

$$\exists\vec{v}.(\Sigma_0)^{\vec{h}}_{\vec{v}} \bigwedge \forall u.h = u \equiv (\gamma_h)^{a,\vec{h}}_{t,\vec{v}}$$

The proof is based on the fact that $\exists V.\alpha$ is logically equivalent to $\exists v.(\alpha)^V_v$ if V is nullary. A formal proof is by an induction on the size of α. We omit it here for space reasons. We mark the FO progression given by Theorem 6 as $Pro(\Sigma_0, \Sigma, t)$, when the context is clear we write $Pro(\Sigma_0, t)$. For example, let $\Sigma_0 = \{h = 1 \vee h = 2\}$, then $Pro(\Sigma_0, \Sigma, fwd(1, 2)) = \{\exists v.(v = 1 \vee v = 2) \wedge \forall u.h = u \equiv u = \max\{0, v - 2\}\} = \{h = 0\}$.

A comment is that Belle and Levesque (2020) claims that

> "... because we are assuming a finite set of nullary fluents, any basic action theory can be shown to be local-effect, where progression is first-order definable."

Yet, this is incorrect since we can construct a BAT with only nullary fluents to simulate a two-counter machine as in (Zarrieß and Claßen 2016) whose Σ_{ssa} is not local-effect.

For the subjective fragment, as mentioned in the introduction, the progression of a probabilistic knowledge base should correspond to the agent's only-believing after actions. Formally, we have

Definition 11. *We call a formula $\boldsymbol{O}(\Psi \wedge \Sigma)$ the progression of the knowledge base $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ wrt ground action t, if and only if $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma) \models [t]\boldsymbol{O}(\Psi \wedge \Sigma)$.*

Essentially, the task of progression is to find such a Ψ that follows from $\boldsymbol{B}^f$.

3.2 Progression after Sensing Actions

To begin with, it's necessary to define what are sensing actions. Our view of sensing actions is the same as (Lakemeyer and Levesque 2009), namely these are actions that provide information about the world but do not change it. In a formal way, they are actions that appear in the likelihood axioms but not in the successor state axioms, like the *sonar* in Example 2. Additionally, sensing actions have no alternatives. Intuitively, this means that when the sonar reads some value, the agent knows it reads that value but no others.

Theorem 7. *Given a KB $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ and a sensing action t_{sen} s.t. $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma) \models \boldsymbol{K}(l(t_{sen}) > 0)$, then $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma) \models [t_{sen}]\boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \Sigma)$ where f' is a definitional function in term of f as:*

$$f'(\vec{u}) = \frac{1}{\eta} f(\vec{u}) \times \mathcal{L}(t_{sen})^{\vec{h}}_{\vec{u}},$$

and $\eta = \sum_{\vec{u}' \in (\mathcal{N}_O)^k} f(\vec{u}') \times \mathcal{L}(t_{sen})^{\vec{h}}_{\vec{u}'}$.

Namely, the progression of a KB is another KB with belief distribution $\boldsymbol{B}^{f'}$; the relation between $\boldsymbol{B}^f$ and $\boldsymbol{B}^{f'}$ is such that the new degrees of belief of $(\vec{h} = \vec{u})$ is just the normalized product of the old degrees of belief and likelihoods of the sensing action. While intuitively the result might be straightforward. The proof is non-trivial. Suppose $e \models \boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ and $e' \models \boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \Sigma)$, the central task of the proof is to show that $e_{t_{sen}} = e'$. The direction $e_{t_{sen}} \subseteq e'$ is straightforward, whereas the other one is sophisticated. In fact, for the direction $e_{t_{sen}} \subseteq e'$, we have

Lemma 2. $\boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma \models [t_{sen}]\boldsymbol{B}^{f'}$, *where $\boldsymbol{B}^f, \Sigma, \boldsymbol{B}^f$, and t_{sen} are the same as Theorem 7.*

Definition 12. *Given BAT Σ wrt fluents $\mathcal{H}$, let $\mathcal{P}\bar{\mathcal{H}}$ be the set of all primitive terms of fluents not in $\mathcal{H}$, we define a relation $\simeq_{\mathcal{H}}$ over $\mathcal{W}$ as $w \simeq_{\mathcal{H}} w'$ iff for all $t \in \mathcal{P}_{\bar{\mathcal{H}}}$ and all $z \in \mathcal{Z}$, $w[t, z] = w'[t, z]$.*

Namely, $w \simeq_{\mathcal{H}} w'$ iff w and w' assigns the same denotation for terms without fluent in $\mathcal{H}$. Clearly, $\simeq_{\mathcal{H}}$ is an equivalence relation. We denote the set of all equivalence classes wrt BAT Σ as $\mathcal{W}_{\mathcal{H}}$. Note that $\mathcal{W}_{\mathcal{H}}$ is uncountable.

Proposition 4. *Given BAT Σ and $\mathcal{C} \in \mathcal{W}_{\mathcal{H}}$, for all standard names $\vec{n}$, there is a unique world w s.t. $w \in \mathcal{C}$ and $w \models \vec{h} = \vec{n} \wedge \Sigma$, we mark this world as $w_{\mathcal{C},\vec{n}}$.*

Lemma 3. *Let $\boldsymbol{B}^f, \boldsymbol{B}^{f'}, \Sigma$, and t_{sen} be as in Theorem 7. For all $d' \in \mathcal{D}$ such that $\{d'\} \models \boldsymbol{B}^{f'} \wedge \boldsymbol{K}\Sigma$, there exists a distribution $d \in \mathcal{D}$ s.t. $\{d\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ and $d_{t_{sen}} = d'$.*

The construction of such d is based on three main steps:

1. By virtue of Prop. 4 and the fact that $S(d')$ is countable, there exists a minimal countable set $\mathcal{W}_{cov}(d') \subseteq \mathcal{W}_{\mathcal{H}}$ such that $\forall w \in S(d'), \exists \mathcal{C}' \in \mathcal{W}_{cov}(d'), w \in \mathcal{C}'$, namely $\mathcal{W}_{cov}(d')$ covers all worlds in $S(d')$;
2. For each $\mathcal{C}' \in \mathcal{W}_{cov}(d')$, we select a $\mathcal{C} \in \mathcal{W}_{\mathcal{H}}$ such that for every world $w' \in \mathcal{C}'$, there exists a unique world $w \in \mathcal{C}$ which can progress to w' after t_{sen}. The selected $\mathcal{C}$ forms $\mathcal{W}_{cov}(d)$;
3. The last step is to assign weights to w which is exactly the weight of w' under d' divided by the likelihood of t_{sen}.

Essentially, $f(\vec{n}) = f'(\vec{n})/\mathcal{L}(t_{sen})^{\vec{h}}_{\vec{n}}$ if we ignore the η in Theorem 7. Our construction of d is to reconstruct such relation at the semantical level: If d' assigns some weights to a world w', then d assigns a world w, which progresses to w' after t_{sen}, with the same weights but divided by the likelihood of t_{sen}. Moreover, the feature of sensing actions ensures this semantical property can be reflected correctly at the syntactical level.

Proof of Theorem 7. Suppose two closed e, e' s.t. $e \models \boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ and $e' \models \boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \Sigma)$, by Theorem 4, it suffices to show $e_{t_{sen}} = e'$

"$e_{t_{sen}} \subseteq e'$" : Since $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma) \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$, $e \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ by hypothesis. By Lemma 2 and Theorem 4, $e_{t_{sen}} \models \boldsymbol{B}^{f'}$. By semantics of $\boldsymbol{O}$ and hypothesis, $e_{t_{sen}} \subseteq e'$

"$e' \subseteq e_{t_{sen}}$" : Since $e' \models \boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \boldsymbol{K}\Sigma)$, for all $d' \in e'$, $\{d'\} \models \boldsymbol{B}^{f'} \wedge \boldsymbol{K}\Sigma$. By Lemma 3, $\exists d.\{d\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ and $d_{t_{sen}} = d'$. Since $e \models \boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ by hypothesis, $d \in e$. $d \sim_{comp} t_{sen}$ due to $d_{t_{sen}} = d'$. Therefore $d' \in \{d \in \mathcal{D} | d \in e \wedge d \sim_{comp} t_{sen}\}$, hence $d' \in e_{t_{sen}}$. That is $e' \subseteq e_{t_{sen}}$. $\square$

Example 3. *Given KB $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ as Example 2, then $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma) \models [sonar(2)]\boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \Sigma)$, where f' is a definitional function as:*

$$f'(u) = \frac{1}{\eta} f(u)\mathcal{L}(sonar(2))^h_u = \frac{1}{\eta} f(u)\theta(2, u, 0.1, 0.8)$$

$$= \begin{cases} \frac{1}{\eta}(\frac{1}{3} \times 0.1) & u \in \{1, 3\} \\ \frac{1}{\eta}(\frac{1}{3} \times 0.8) & u = 2 \\ 0 & o.w. \end{cases} = \begin{cases} 0.1 & u \in \{1, 3\} \\ 0.8 & u = 2 \\ 0 & o.w. \end{cases}$$

The second equality is by the specification of $\mathcal{L}(a)$, the third equal is due to $f(u)$ is non-zero only among $\{1, 2, 3\}$. The last one is because $\eta = \frac{1}{3}$.

3.3 Progression after Stochastic Actions

Unlike sensing, stochastic actions have observationally indistinguishable actions as alternatives, sometimes even infinite alternatives. Besides, stochastic actions do affect the real world. This makes the progression wrt stochastic actions more complicated than sensing actions

Theorem 8. *Given a KB* $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ *and a stochastic action* t_{sa}, $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma) \models [t_{sa}]\boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \Sigma)$, *where* f' *is a definitional function in term of* f *as:*
$f'(\vec{u}) = \sum_{\vec{u}' \in (\mathcal{N}_O)^k} \sum_{a \in \mathcal{N}_A} f(\vec{u}') \times \mathcal{L}(a)^{\vec{h}}_{\vec{u}'} \times \mathbb{I}(\vec{u}, \vec{u}', a, t_{sa})$,
where $\mathbb{I}$ *is a definitional function given by*

$$\mathbb{I}(\vec{u}, \vec{u}', a, t_{sa}) = \begin{cases} 1 & Pro(\vec{h} = \vec{u}', a)^{\vec{h}}_{\vec{u}} \wedge (\psi)^{a'}_{t_{sa}} \\ 0 & o.w. \end{cases}$$

Like sensing actions, the central task of the proof is to show $e_{t_{sa}} = e'$ and the direction $e_{t_{sa}} \subseteq e'$ is straightforward. Formally, we have

Lemma 4. $\boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma \models [t_{sa}]\boldsymbol{B}^{f'}$, *where* $\boldsymbol{B}^f$, $\boldsymbol{B}^{f'}$, *and* t_{sa} *are the same as Theorem 8.*

The proof is rather similar to Lemma 2 but additionally requires the fact that for all world w and stochastic action a, $\sum_{\{a':a' \sim_w a\}} l(a) = 1$. Unfortunately, the techniques for sensing actions to construct a distribution as Lemma 3 does not apply to stochastic actions. This is because: 1) there is not an explicit inverse of f in terms of f'; and 2) the correspondence between $\mathcal{C}$ and $\mathcal{C}'$ breaks. More concretely, assuming $\{a_1, a_2, \ldots a_m\}$ are mutual alternatives, given $w' \in \mathcal{C}'$, there might be a set of world $\{w_1, w_2 \ldots\}$ s.t. $w' = (w_i)_{a_i}$ and $w_i \in \mathcal{C}_i$ for different $\mathcal{C}_i$. Conversely, given a world $w \in \mathcal{C}$, w_{a_i} might belong to different $\mathcal{C}'_i$.

To solve this problem, we first consider the case where action alternatives are finite.

Lemma 5 (Finite Action Alternatives)**.** *Let* $\boldsymbol{B}^f, \boldsymbol{B}^{f'}$, *and* Σ *be as in Theorem 8,* t_{sa} *a stochastic actions with finitely many alternatives under* Σ*. For all* $d' \in \mathcal{D}$ *such that* $\{d'\} \models \boldsymbol{B}^{f'} \wedge \boldsymbol{K}\Sigma$, *there exists a distribution* $d \in \mathcal{D}$ *s.t.* $\{d\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ *and* $d_{t_{sa}} = d'$.

In the following, we only consider the case that $\text{EQ}(d', \mathcal{W}_{\text{TRUE}}, 1)$. If $\text{EQ}(d', \mathcal{W}_{\text{TRUE}}, c)$ for $c \neq 1$, a distribution can be constructed in the same way except that the weight of worlds is proportionally increased by c.

We first observe that given a world w s.t. $w \models \vec{h} = \vec{n} \wedge \Sigma$ for some $\vec{n}$, due to the finite alternatives hypothesis, there are only finite alternatives $\{a_{\vec{n},1}, a_{\vec{n},2}, \ldots a_{\vec{n},m}\}$ whose likelihoods are positive. Moreover, w might progress to m worlds $w_{a_{\vec{n},1}}, w_{a_{\vec{n},2}}, \ldots w_{a_{\vec{n},m}}$ with equivalence class $\mathcal{C}'_{a_{\vec{n},i}}, \mathcal{C}'_{a_{\vec{n},2}} \ldots \mathcal{C}'_{a_{\vec{n},m}}$. Conversely, given m equivalence class $\mathcal{C}'_{a_{\vec{n},1}}, \mathcal{C}'_{a_{\vec{n},2}} \ldots \mathcal{C}'_{a_{\vec{n},m}}$, there exists a world w s.t. $w \models \vec{h} = \vec{n} \wedge \Sigma$ and $w_{a_{\vec{n},i}} \in \mathcal{C}'_{a_{\vec{n},i}}$ for all $1 \leq i \leq m$. In fact, there are infinitely many such worlds: the condition $w_{a_{\vec{n},i}} \in \mathcal{C}'_{a_{\vec{n},i}}$ only restricts how w interprets terms $t \in \mathcal{P}_{\vec{\mathcal{H}}}$ after actions

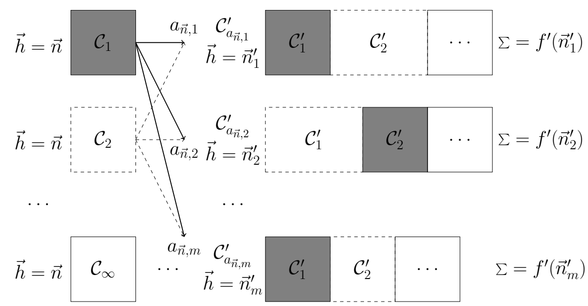

Figure 2: The selection of $S(d)$ for stochastic actions with finite alternatives.

$a_{\vec{n},i}$ for $1 \leq i \leq m$, worlds might interpret t differently when $z \neq a_{\vec{n},i}$. We only need to select one such world and denote the selected world as $w_{\vec{n}, \mathcal{C}'_{a_{\vec{n},1}}, \mathcal{C}'_{a_{\vec{n},2}} \ldots \mathcal{C}'_{a_{\vec{n},m}}}$. Clearly, we only care about equivalence classes in $\mathcal{W}_{cov}(d')$, namely, $\mathcal{C}'_{a_{\vec{n},i}} \in \mathcal{W}_{cov}(d')$. Intuitively, the selected worlds form the support of d, i.e. $S(d)$. Since m is bounded and $\mathcal{C}'_{a_{\vec{n},i}}$ has countably many possible values, $S(d)$ is countable.

Figure 2 illustrates the selection procedure. Each rectangle box stands for a world and the text over the box indicates the equivalence class it belongs to. The size of the box indicates that relative weight of the world. Boxes on the LHS are worlds that satisfy $\vec{h} = \vec{n}$ and Σ, their possible progressed worlds are presented on the RHS. The selected world $w_{\vec{n}, \mathcal{C}_1}$ corresponds to the combination $\mathcal{C}'_{a_{\vec{n},2}} = \mathcal{C}'_2$ and $\mathcal{C}'_{a_{\vec{n},i}} = \mathcal{C}'_1$ for all $i \neq 2$ (boxes filled by gray), while $w_{\vec{n}, \mathcal{C}_2}$ corresponds to the combination $\mathcal{C}'_{a_{\vec{n},2}} = \mathcal{C}'_1$ and $\mathcal{C}'_{a_{\vec{n},i}} = \mathcal{C}'_2$ for all $i \neq 2$ (boxes with dashed boarder). Note that our selection automatically guarantees that different combinations will select different worlds due to the restriction $w_{a_{\vec{n},i}} \in \mathcal{C}'_{a_{\vec{n},i}}$.

Now consider a distribution d as follows:

$$d(w) = \begin{cases} f(\vec{n}) \displaystyle\prod_{i=1}^{m} \frac{d'(w_{a_{\vec{n},i}})}{f'(\vec{n}'_i)} & \begin{array}{l} w = w_{\vec{n}, \mathcal{C}'_{a_{\vec{n},1}}, \mathcal{C}'_{a_{\vec{n},2}} \ldots \mathcal{C}'_{a_{\vec{n},m}}} \\ \text{for some } \vec{n}, \mathcal{C}'_{a_{\vec{n},1}}, \ldots \mathcal{C}'_{a_{\vec{n},m}} \\ \text{and } w \models \bigwedge_i [a_{\vec{n},i}]\vec{h} = \vec{n}'_i \\ \text{for some } \vec{n}'_1, \vec{n}'_2, \ldots \vec{n}'_m \\ \text{and } d'(w_{a_{\vec{n},i}}) > 0 \text{ for all } i \end{array} \\ 0 & o.w. \end{cases}$$

The construction is based on the observation that for every selected world $w_{\vec{n}, \mathcal{C}'_{a_{\vec{n},1}}, \mathcal{C}'_{a_{\vec{n},2}} \ldots \mathcal{C}'_{a_{\vec{n},m}}}$, the proportion of its weight in d to the summed weights of all worlds which satisfies $\vec{h} = \vec{n} \wedge \Sigma$ in d, i.e. $f(\vec{n})$, equals the products of proportions of individual progressed world's weight in d' to and the summed weights in d' of all worlds which assigns the same values to fluents in $\mathcal{H}$, i.e. $f'(\vec{n}'_i)$. The constructed distribution d satisfies $\{d\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ and $d_{t_{sa}} = d'$.

While the above construction shows that Lemma 5 indeed holds for the finite alternatives case, it cannot be generalized to the infinite alternatives case. A direct reason is that infinite $\mathcal{C}'_{a_{\vec{n},i}}$ (i is not bounded) need to be considered which results in combinations of infinite dimensions where each dimension has countably infinite candidates. Consequently, $S(d)$ is uncountable. Nevertheless, a weaker lemma exists and is sufficient to prove the progression theorem.

Lemma 6. *Let $\boldsymbol{B}^f, \boldsymbol{B}^{f'}$, and Σ be as in Theorem 8, t_{sa} a stochastic actions. For all $d' \in \mathcal{D}$ such that $\{d'\} \models \boldsymbol{B}^{f'} \wedge \boldsymbol{K}\Sigma$ and any $\epsilon > 0$, there exists a distribution $d \in \mathcal{D}$ s.t. $\{d\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ and $\rho(d_{t_{sa}}, d') < \epsilon$.*

The idea is to only consider a finite subset of t_{sa}'s alternatives and construct a distribution d using the above procedure wrt the finite set of alternatives. It can be shown that d satisfies $\boldsymbol{B}^{f_i} \wedge \boldsymbol{K}\Sigma$. Additionally, by increasing the size of the finite set of alternatives, the distance $\rho(d_{t_{sen}}, d')$ decreases accordingly and is eventually less than ϵ.

Proof of Theorem 8. Suppose $e \models \boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma)$ and $e' \models \boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \Sigma)$, by Theorem 4, it suffices to show $e_{t_{sen}} = e'$

"$e_{t_{sen}} \subseteq e'$" : the proof is exactly the same as its counterpart of noisy sensing.

"$e' \subseteq e_{t_{sen}}$" : Since $e' \models \boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \boldsymbol{K}\Sigma)$, for all $d' \in e'$, $\{d'\} \models \boldsymbol{B}^{f'} \wedge \boldsymbol{K}\Sigma$. By Lemma 6, $\exists d.\{d\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ and $\rho(d_{t_{sen}}, d') < \epsilon$ for any ϵ. Therefore, $\exists\{d_1, \ldots d_n \ldots\}$ s.t. $\{d_n\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ and there exists N for $n > N$, $\rho((d_n)_{t_{sen}}, d') < \epsilon$ for any ϵ. That is, $\exists\{d_1, \ldots d_n \ldots\}$ s.t. $\{d_n\} \models \boldsymbol{B}^f \wedge \boldsymbol{K}\Sigma$ and $\lim_{n\to\infty}(d_n)_{t_{sa}} = d'$. By definition of $e_{t_{sen}}$, $d' \in e_{t_{sen}}$. □

Example 4. *Let $\boldsymbol{B}^f$ and Σ) be as Example 2, then $\boldsymbol{O}(\boldsymbol{B}^f \wedge \Sigma) \models [fwd(2,2)]\boldsymbol{O}(\boldsymbol{B}^{f'} \wedge \Sigma)$ where f' is given by and*

$$f'(u) = \begin{cases} 1/15 & u = 2 \\ 4/15 & u = 1 \\ 1/3 & u = 0 \\ 0 & o.w. \end{cases}$$

By definition, we have:

$$\mathbb{I}(u, u', a, fwd(2,2)) = \begin{cases} 1 & Pro(h = u', a)^h_u \wedge \psi^{a'}_{fwd(2,2)} \\ 0 & o.w. \end{cases}$$

$$= \begin{cases} 1 & \begin{array}{c}(\exists x, y.a = fwd(x,y) \wedge u = \max\{0, u' - y\} \\ \vee \forall x, y.a \neq fwf(x,y) \wedge u = u') \\ \wedge \exists y.a = fwd(2,y)\end{array} \\ 0 & o.w. \end{cases}$$

$$= \begin{cases} 1 & \exists y.a = fwd(2,y) \wedge u = \max\{0, u' - y\} \\ 0 & o.w. \end{cases}$$

and $\mathcal{L}(a)^h_{u'} = \begin{cases} \theta(x, y, 0.2, 0.6) & \exists x, y.a = fwd(x,y) \\ \theta(z, u', 0.1, 0.8) & \exists z.a = sonar(z) \end{cases}$,

therefore

$$f'(u) = \sum_{u'} \sum_a f(u') \mathcal{L}(a)^h_{u'} \mathbb{I}(u, u', a, fwd(2,2))$$

$$= \sum_{u'} \sum_a f(u') \begin{cases} \theta(2, y, 0.2, 0.6) & \begin{array}{c}\exists y.a = fwd(2,y) \wedge \\ u = \max\{0, u' - y\}\end{array} \\ 0 & o.w. \end{cases}$$

$$= \begin{cases} \frac{1}{3} \times 0.2 & u = 2 \\ \frac{1}{3} \times 0.2 + \frac{1}{3} \times 0.6 & u = 1 \\ \left(\begin{array}{c} \frac{1}{3} \times 0.2 + \frac{1}{3} \times 0.6 + \frac{1}{3} \times 0.2 \\ + \frac{1}{3} \times 0.2 + \frac{1}{3} \times 0.6 + \frac{1}{3} \times 0.2 \end{array}\right) & u = 0 \\ 0 & o.w. \end{cases}$$

The second line is by $\mathcal{L}(a)^h_u \times \mathbb{I}(u, u', a, fwd(2,2))$. The third is because f and θ are zero when u' and y are not among $\{1, 2, 3\}$, respectively, therefore f' is non-zero only ony if $u \in \{0, 1, 2\}$ and the degrees of belief for each value of u equals to the sum of products between $f(u')$ and $\theta(2, y, 0.2, 0.6)$ of all combinations of u' and y that result that value (according to $u = \max\{0, u' - y\}$).

4 Related Work

We revisit related work from two aspects: knowledge representation and projection by progression.

In terms of knowledge representation, our logic builds on the logic $\mathcal{DS}$, a probabilistic extension of a modal variant of the situation calculus with a model of belief and only-believing. What distinguishes us is that our proposed logic has richer expressiveness that allows us to express a probabilistic knowledge base with arbitrary belief distributions and the utility of our notion of only-believing does not constrain to the static case. The logic $\mathcal{DS}$ is based on the first-order logic $\mathcal{OBL}$ (Belle, Lakemeyer, and Levesque 2016), a probabilistic logic of only-knowing. It is shown that $\mathcal{OBL}$ fully captures the features of the logic $\mathcal{OL}$, the pioneering work on only-knowing by Levesque (1990). In a game theory context, Halpern and Pass (2009) have considered a (propositional) version of only knowing with probability distributions.

For the aspect of degrees of belief, $\mathcal{DS}_p$ is inspired by the work BHL (Bacchus, Halpern, and Levesque 1999), an axiomatic proposal with a conceptually attractive definition of belief in a first-order dynamic setting. In a less restricted setting, reasoning about knowledge and probability was studied prior to BHL, (Nilsson 1986; Fagin and Halpern 1994; Monderer and Samet 1989). Notably, the work of Fagin and Halpern (1994) can be seen to be at the heart of BHL.

On progression, Lin and Reiter (1997) proposed the most general account of progression and showed that progression is second-order definable. Restricted forms of LR-progression, which are first-order definable, are discussed there as well and later in (Liu and Levesque 2005; Claßen et al. 2007; Vassos and Levesque 2007). Based on the notion of progressed worlds, Lakemeyer and Levesque (2009) show that the progression of categorical knowledge against noise-free actions amounts to only-knowing after actions. For a limited type of theory, the progression of discrete degrees of belief wrt context-completeness is considered in (Belle and Lakemeyer 2011). Belle and Levesque (2020) studied the progression of continuous degrees of belief for the so called *invertible* BATs which exclude our BATs in Example 2. As a result, our work fills the gap of a general account of progression in discrete degrees of belief.

5 Conclusion

In this work, we lift the expressiveness of the logic $\mathcal{DS}$. As a result, we are able to express a probabilistic knowledge base with arbitrary belief distribution. For an interesting fragment, we show that classical progression is first-order definable. Besides, based on our notion of progressed distributions, we show how the progression of discrete degrees of belief is related to only-believing after actions. In terms of future work, it would be interesting to see how the idea of progression can be used in verification of belief programs (Belle and Levesque 2015).

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