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Choosing What to Believe - New Results in Selective Revision

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Abstract

Selective Revision was proposed by Fermé and Hansson as a belief revision operation in which it is possible to accept only a part of the input information. In this paper, we extend Selective Revision to belief bases and also to logics not closed under negation.

1 Introduction

Knowledge Representation is one of the key problems in Artificial Intelligence. Since knowledge is not constant, we need to be apt to deal with its dynamics. Belief Revision is the study of knowledge dynamics. In the standard model of Belief Revision, known as the AGM paradigm, given a set of beliefs, there are three possible changes towards a new belief: expansion, contraction and revision. Expansion occurs when the base simply absorbs the information without any loss. A contraction consists in retracting beliefs from the base so that the specified information is not derivable from it anymore. Finally, revision happens when the new belief is added in a consistent way. In this work, we are going to focus on a variant of this last operation.

In AGM revision, the new information is always accepted. This is an unrealistic feature, since agents, when confronted with information that contradicts previous beliefs, often reject it altogether or accept only part of it.

Selective Revision (Fermé and Hansson 1999) is a constructive model, based in AGM revision, in which it is possible to accept only a part of the input information.
For example, suppose we believe that birds fly, that birds are warm blooded and that Tweety is a bird. Then, we get the information that Tweety does not fly nor is warm blooded. As we are sure that all birds are warm blooded, we refuse this information and only accept the part about Tweety not flying. This example could be easily expressed using a Description Logic (DL) syntax:

Initial beliefs: $Bird \sqsubseteq FlyingAnimal$, $Bird \sqsubseteq WarmAnimal$, $Bird(Tweety)$. **Input:** $(\neg FlyingAnimal \sqcap \neg WarmAnimal)(Tweety)$.

Selective Revision was initially defined for belief sets (closed under logical consequence) and propositional logic.

In this paper, we propose to extend Selective Revision to belief bases (sets of sentences not necessarily closed under logical consequences) and to logics without negation.

This paper proceeds as follows. Section 2 provides the necessary background. Selective revision for bases is presented in Section 3 while its extension to logics without negation is given in Section 4. Conclusions and future work come in Section 5.

2 Background

2.1 Formal Preliminaries

We will assume a propositional language $\mathcal{L}$ that contains the usual truth functional connectives: $\neg$ (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication) and $\leftrightarrow$ (equivalence). In Section 4 we assume that the logic is not closed under negation. We assume a consequence operation Cn that takes sets of sentences to sets of sentences and which satisfies the standard Tarskian properties, namely *inclusion, monotony* and *iteration*. Furthermore we will assume that Cn satisfies *supraclassicality, compactness* and the *deduction theorem*. We will sometimes use $Cn(\alpha)$ for $Cn(\{\alpha\})$, $A \vdash \alpha$ for $\alpha \in Cn(A)$, $\vdash \alpha$ for $\alpha \in Cn(\emptyset)$, $A \not\vdash \alpha$ for $\alpha \notin Cn(A)$, $\not\vdash \alpha$ for $\alpha \notin Cn(\emptyset)$. The letters $\alpha, \beta, \ldots$ (except for γ and σ) will be used to denote sentences of $\mathcal{L}$. $A, B, \ldots$ shall denote sets of sentences of $\mathcal{L}$. K is reserved to represent a *belief set* (*i.e.* $K = Cn(K)$). We will use $\bot$ for the falsity constant and the symbols $\star$, $*$, $\odot$ and $\circledast$ to denote AGM belief set revision, belief base revision, selective belief set revision and selective belief base revision operators, respectively.

2.2 AGM Revision

A *belief set* is a set of formulas closed under logical consequences. The operation of revision of a belief set by a sentence consists in obtaining a new belief set which contains that sentence, is a similar as possible to the original one and is, whenever possible, consistent. Because of that, in a revision process, some previous beliefs may be retracted. The following six postulates, which were originally presented in (Gärdenfors 1988), are commonly known as the *basic AGM postulates for revision*:

(closure) $K \star \alpha$ is a belief set.
(success) $K \star \alpha \vdash \alpha$.
(inclusion) $K \star \alpha \subseteq Cn(K \cup \{\alpha\})$.
(consistency) If $\not\vdash \neg\alpha$ then $K \star \alpha \not\vdash \bot$.
(vacuity) If $K \not\vdash \neg\alpha$, then $Cn(K \cup \{\alpha\}) \subseteq K \star \alpha$.

(extensionality) If $\vdash \alpha \leftrightarrow \beta$, then $K \star \alpha = K \star \beta$.

A revision operator that satisfies these six postulates is designated by *basic AGM revision*.

2.3 Selective Revision

Fermé and Hansson (1999) proposed a new operator that allows the acceptance of only part of the new information and the non-acceptance of the rest of it. They called this operator *selective revision*. In this subsection we recall from (Fermé and Hansson 1999) the definition of this kind of operator.

From the six basic AGM postulates, four are also valid for selective revision: *closure*, *inclusion*, *consistency* and *extensionality*. Three new postulates were proposed:
(Proxy success) There is a sentence β, such that $K \odot \alpha \vdash \beta$, $\vdash \alpha \rightarrow \beta$ and $K \odot \alpha = K \odot \beta$.
(Weak proxy success) There is a sentence β, such that $K \odot \alpha \vdash \beta$ and $K \odot \alpha = K \odot \beta$.
(Consistent expansion) If $K \not\subseteq K \odot \alpha$, then $K \cup (K \odot \alpha) \vdash \bot$.

An operator of selective revision is constructed from a basic AGM revision and a function f from $\mathcal{L}$ to $\mathcal{L}$:

Definition 1 ((Fermé and Hansson 1999))**.** *Let K be a belief set, $\star$ a basic AGM revision operator for K and f a function from $\mathcal{L}$ to $\mathcal{L}$. The selective revision $\odot$, based on $\star$ and f, is the operation such that for all sentences α:*

$$K \odot \alpha = K \star f(\alpha).$$

f is called the transformation function on which $\odot$ is based.

Intuitively, the transformation function f selects the credible part of every sentence. A natural restriction is that $f(\alpha)$ should not contain more information that the one that is contained in α (*i.e.*, $\vdash \alpha \rightarrow f(\alpha)$). However, Fermé and Hansson proposed the operation in a very general way, without imposing this restriction. The following are some of the proposed properties for transformation functions presented in (Fermé and Hansson 1999):

$\vdash \alpha \rightarrow f(\alpha)$	(Implication)
If $K \nvdash \neg\alpha$, then $\vdash \alpha \rightarrow f(\alpha)$	(Weak implication)
$\vdash f(f(\alpha)) \leftrightarrow f(\alpha)$	(Idempotence)
If $\vdash \alpha \leftrightarrow \beta$, then $\vdash f(\alpha) \leftrightarrow f(\beta)$	(Extensionality)
If $\not\vdash \neg\alpha$, then $\not\vdash \neg f(\alpha)$	(Consistency Preservation)
If $K \not\vdash \neg\alpha$, then $\vdash f(\alpha) \leftrightarrow \alpha$	(Weak Maximality)

2.4 Base Revision

One of the issues encountered when trying to use AGM revision in practice is the need to work with logically closed (and potentially infinite) belief sets. *Belief base revision*, on the other hand, deals with sets that are not necessarily closed. In this subsection, we introduce the belief base revision operators which we will use.

On belief bases, revising a set by α can be done by first contracting that set by $\neg\alpha$ and subsequently adding α to its outcome. This is expressed by the *Levi Identity* adapted to the belief base context: $B * \alpha = (B - \neg\alpha) \cup \{\alpha\}$.

An operator defined by the above equality is called an operator of *internal revision*.[1]
On belief bases, it is also possible to define a revision operator in the reverse order, *i.e.*, first adding the formula α and then contracting the result by $\neg\alpha$. This is expressed by the *reverse Levi identity* (Hansson 1993): $B * \alpha = (B \cup \{\alpha\}) - \neg\alpha$.
In general, internal and external revision do not coincide (Hansson 1993). In this paper we will consider internal and external partial meet and kernel revisions. These revision operators can be defined from the equalities presented above, where the operator $-$ is either a partial meet contraction (Alchourrón, Gärdenfors, and Makinson 1985) or a kernel contraction operator (Hansson 1994).

Given a set B, a partial meet contraction of B by α is based on the intersection of some of the maximal subsets of A that fail to imply α (if there is any such a set, otherwise the set to be contracted is left unchanged). The choice of sets is made by a *selection function*, usually denoted by γ.
Hansson introduced another construction for contraction operators, called *kernel contraction*, which is a generalization of safe contraction (Alchourrón and Makinson 1985). Kernel contractions are based on the removal from the belief base B of at least one element of each minimal subset of B that implies α (if there is any such a set, otherwise the set to be contracted is left unchanged). The choice of the elements to be removed is performed by means of an *incision function*, which is usually denoted by σ.

The following postulates were proposed for belief base revision:
(success) $\alpha \in B * \alpha$.
(inclusion) $B * \alpha \subseteq B \cup \{\alpha\}$.
(consistency) If $\alpha \nvdash \bot$ then $B * \alpha \nvdash \bot$.
(non-contradiction) If $\alpha \nvdash \bot$, then $B * \alpha \nvdash \neg\alpha$.
(uniformity) If for all $B' \subseteq B$, $B' \cup \alpha \vdash \bot$ iff $B' \cup \beta \vdash \bot$, then $B \cap (B * \alpha) = B \cap (B * \beta)$.
(relevance) If $\beta \in B$ and $\beta \notin B * \alpha$, then there is some B' such that $B * \alpha \subseteq B' \subseteq B \cup \{\alpha\}$, $B' \nvdash \bot$ but $B' \cup \{\beta\} \vdash \bot$.
(core-retainment) If $\beta \in B$ and $\beta \notin B * \alpha$, then there is some B' such that $B' \subseteq B \cup \{\alpha\}$, $B' \nvdash \bot$ but $B' \cup \{\beta\} \vdash \bot$.

The next theorem is a characterization of partial meet revision on belief bases:

Theorem 1. *(Hansson 1999) The operator $*$ is an operator of partial meet revision for a belief base B iff it satisfies success, inclusion, consistency, relevance and uniformity.*

In order do characterize external partial meet revisions, we need two extra postulates:
(pre-expansion) $(B \cup \{\alpha\}) * \alpha = B * \alpha$.
(weak uniformity) If α and β are elements of B and it holds for all $B' \subseteq B$ that $\neg\alpha \in Cn(B')$ iff $\neg\beta \in Cn(B')$, then $B \cap (B * \alpha) = B \cap (B * \beta)$.

[1] In this article, the term revision will be used to refer to internal revisions.

Theorem 2. *(Hansson 1993) An operator $*$ is an operator of external partial meet revision on B iff it satisfies* non-contradiction*,* inclusion*,* success*,* relevance*,* pre-expansion *and* weak uniformity*.*

In the following two theorems we recall representation theorems for internal and external kernel revisions.

Theorem 3. *(Hansson 1993; Hansson and Wassermann 2002) An operator $*$ is an operator of internal kernel revision iff it satisfies non-contradiction, inclusion, core-retainment, success and uniformity.*[2]

Theorem 4. *(Hansson 1993; Hansson and Wassermann 2002) An operator $*$ is an operator of external kernel revision iff it satisfies* non-contradiction*,* inclusion*,* success*,* core-retainment*,* pre-expansion *and* weak uniformity*.*[2]

2.5 Negation Free Revision

Although in (Hansson 1999; Hansson and Wassermann 2002) there are definitions and constructions for (internal and external) partial meet and kernel base revisions, they use contraction as an intermediate step, which makes the respective representation theorems dependent on negation of sentences. Nevertheless, many interesting logics, such as most of the DLs, are not closed under negation of formulas, which means that the mentioned constructions are not directly applicable to this context. Aiming at solving this issue, Ribeiro and Wassermann (2009) proposed and axiomatically characterized some base revision operators without assuming that the underlying logic is closed under negation of sentences. They worked only with external revision, *i.e.*, all the constructions adopted the same method: first, the expansion of the belief base by the input α and, then, a (partial meet or kernel) contraction of the resulting base by $\bot$. Within this approach, the dependence on negation is warded off by establishing conditions on the selection or incision functions.

As defined in (Ribeiro and Wassermann 2009), a *selection function γ that protects the input* works this way: if $\alpha \nvdash \bot$, then γ selects among the maximal consistent subsets of $B \cup \{\alpha\}$ containing α. Otherwise, γ selects $B \cup \{\alpha\}$.

Note that this definition of selection function demands two arguments. One of them is the formula to be preserved, so that the function is able to select the suitable maximal consistent subsets. The usage of this kind of selection function implies that *success* is a strong requirement for the operation, while *consistency* is weak (since the input will be absorbed anyway). The same happens with the incision function defined after the theorem below. In (Ribeiro and Wassermann 2009) the names of the operators constructed in this way end with the expression *with success*.

Theorem 5. *(Ribeiro and Wassermann 2009) The operator $*$ is a negation free external partial meet revision for a belief base A iff it satisfies success, inclusion, consistency, relevance and pre-expansion.*

The same idea is used in (Ribeiro and Wassermann 2009) for incision functions. An *incision function σ that protects the input* is a function that, if $\alpha \nvdash \bot$, chooses at least a sentence from every minimal inconsistent subset of $B \cup \{\alpha\}$ as long as the subset is not empty and α is not chosen.

Theorem 6. *(Ribeiro and Wassermann 2009) The operator $*$ is a negation free external kernel revision for a belief base A iff it satisfies success, inclusion, consistency, core-retainment and pre-expansion.*

For the negation free (internal) revisions of belief bases, one can easily adapt a definition originally proposed in the context of belief sets. A *negation free remainder set $B \downarrow \alpha$* is defined as the set of all maximal subsets of B that are inconsistent with the input α. Formally:

Definition 2. *(Ribeiro and Wassermann 2014)[negation free remainder set] $X \in B \downarrow \alpha$ iff: (i) $X \subseteq B$; (ii) $X \cup \{\alpha\} \nvdash \bot$; (iii) If $X \subset X' \subseteq B$, then $X' \cup \{\alpha\} \vdash \bot$.*

A *selection function γ* selects a non-empty subset of $B \downarrow \alpha$ whenever $B \downarrow \alpha \neq \emptyset$. Otherwise it returns $\{B\}$.

Any selection function induces the following revision operation called *negation free internal partial meet revision*:

$$B *_\gamma \alpha = \bigcap \gamma(B \downarrow \alpha) \cup \{\alpha\}$$

Theorem 7. *An operator $*$ is an operator of negation free internal partial meet revision iff it satisfies consistency, inclusion, success, relevance and uniformity.*

The proof can be easily adapted from (Hansson and Wassermann 2002) by using $B \downarrow \alpha$ instead of $B \perp \neg\alpha$.

The same strategy can be used for kernel revision:

Definition 3. *(Ribeiro and Wassermann 2014)[negation free kernel set] $X \in B \downarrow\downarrow \alpha$ iff: (i) $X \subseteq B$; (ii) $X \cup \{\alpha\} \vdash \bot$; (iii) If $X' \subset X$, then $X' \cup \{\alpha\} \nvdash \bot$.*

Theorem 8. *An operator $*$ is an operator of negation free internal kernel revision iff it satisfies consistency, inclusion, success, core-retainment and uniformity.*

3 Selective Base Revision

In this section, we are going to show how to define and characterize axiomatically selective revisions for belief bases.

3.1 Postulates

The next set of postulates are weaker versions of *success*:
(Proxy success) There is a sentence β, such that $\beta \in A \circledast \alpha$, $\vdash \alpha \rightarrow \beta$ and $A \circledast \alpha = A \circledast \beta$.
(Weak proxy success) There is a sentence $\beta \in A \circledast \alpha$ and $A \circledast \alpha = A \circledast \beta$.
(Stability) If $\alpha \in A$, then $\alpha \in A \circledast \alpha$.
(Uniform success) If for all subsets $A' \subseteq A$, $A' \cup \{\alpha\} \vdash \bot$ iff $A' \cup \{\beta\} \vdash \bot$, then $\alpha \in A \circledast \alpha$ iff $\beta \in A \circledast \beta$.

Proxy success states that the selective revision should accept and fully incorporate some part of the input information. *Weak proxy success* is a weaker version of *proxy success*. *Stability* states that any explicit belief of an agent (*i.e. that is in the agent's belief base*) should be kept when a selective revision by that belief is made.. *Uniform success* states that if two beliefs are inconsistent with exactly the same subsets of A, then one of them should be incorporated

[2]The postulate of *non-contradiction* could be replaced by *consistency*

in the outcome of the selective revision by it iff the same thing happens regarding the other one.

The following postulates are weaker versions of the postulates for base revision presented in Subsection 2.4:

(Weak inclusion) If $\alpha \in A \circledast \alpha$, then $A \circledast \alpha \subseteq A \cup \{\alpha\}$.

(Conditional uniformity) If $\alpha \in A \circledast \alpha$ and for all subsets A' of A it holds that $A' \cup \{\alpha\} \vdash \perp$ iff $A' \cup \{\beta\} \vdash \perp$, then $A \cap (A \circledast \alpha) = A \cap (A \circledast \beta)$.

(Weak relevance) If $\alpha \in A \circledast \alpha$, $\beta \in A$ and $\beta \notin A \circledast \alpha$, then there is some A' such that $A \circledast \alpha \subseteq A' \subseteq A \cup \{\alpha\}$, $A' \not\vdash \perp$ but $A' \cup \{\beta\} \vdash \perp$.

(Weak core-retainment) If $\alpha \in A \circledast \alpha$, $\beta \in A$ and $\beta \notin A \circledast \alpha$, then there is some $A' \subseteq A$ such that $A' \not\vdash \neg\alpha$ and $A' \cup \{\beta\} \vdash \neg\alpha$.

The weakening resides in the fact that they are preconditioned by $\alpha \in A \circledast \alpha$. Informally, this means that, if a sentence is in the outcome of a selective revision by it, then that outcome behaves as one coming from a standard revision.

3.2 Constructing the Operation

Definition 4. *Let A be a belief base, $*$ be a base revision operator on A and f a function from $\mathcal{L}$ to $\mathcal{L}$. The selective revision $\circledast$, based on $*$ and f, is the operation such that for all sentences α: $A \circledast \alpha = A * f(\alpha)$.*
f is the transformation function on which $\circledast$ is based.

We now present a list of properties that the transformation function may be expected to satisfy:

$\vdash \alpha \rightarrow f(\alpha)$ (Implication)
$f(f(\alpha)) = f(\alpha)$ (Idempotence)
If $\not\vdash \neg\alpha$, then $\not\vdash \neg f(\alpha)$ (Consistency preservation)
If $A \not\vdash \neg\alpha$, then $f(\alpha) = \alpha$ (Weak Maximality)
If $\alpha \in A$, then $f(\alpha) = \alpha$ (Lower boundary)
If for all $A' \subseteq A, A' \cup \{\alpha\} \vdash \perp$ iff $A' \cup \{\beta\} \vdash \perp$, then $f(\alpha) = \alpha$ iff $f(\beta) = \beta$. (Uniform identity)

The first four properties were already recalled in Subsection 2.3 or result from those by adapting the namesake property to the belief base context. We note that if a transformation function satisfies *implication*, then it also satisfies *consistency preservation*. *Lower boundary* states that an agent's explicit belief should be in the outcome of the selective revision by it. *Uniform identity* states that if two sentences are inconsistent with exactly the same subsets of A, then one of them should be fully accepted and incorporated iff the same thing happens regarding the other one.

The next observation shows how properties of the transformation function (eventually) combined with postulates of base revision give rise to postulates of selective revision.

Observation 1. *Let A be a belief base, $*$ be a revision operator on A that satisfies success, inclusion and consistency and f a transformation function. Let $\circledast$ be the selective revision operator on A based on $*$ and f. If f satisfies:*[3]
1. lower boundary, then $\circledast$ satisfies stability and weak inclusion.
2. consistency preservation, then $\circledast$ satisfies consistency.
3. idempotence, then $\circledast$ satisfies weak proxy success.
4. idempotence and implication, then $\circledast$ satisfies proxy success.
5. lower boundary and $*$ satisfy relevance, then $\circledast$ satisfy weak relevance.
6. lower boundary and $*$ satisfy core-retainment, then $\circledast$ satisfy weak core-retainment.
7. uniform identity and lower boundary, then $\circledast$ satisfies uniform success.
8. uniform identity and lower boundary and $*$ satisfies uniformity, then $\circledast$ satisfies conditional uniformity.

[3]Due to space limitations, the proofs of the results reported herein will be omitted.

3.3 Representation Results

Now we present axiomatic characterizations for four different classes of selective base revision. More precisely, we give representation theorems based on partial meet revisions and kernel revisions, with two variants for each.

Theorem 9. *Let A be a belief base and $\circledast$ be an operator on A. Then the following pair of conditions are equivalent:*

(a) $\circledast$ satisfies weak inclusion, consistency, conditional uniformity, proxy success, stability, uniform success and weak relevance (respectively, weak core-retainment).

(b) There exists a partial meet base revision (respectively, kernel) operator $*$ for A and a transformation function f that satisfies lower boundary, idempotence, implication, uniform identity and such that $A \circledast \alpha = A * f(\alpha)$, for all α.

Theorem 10. *Let A be a belief base and $\circledast$ be an operator on A. Then the following pair of conditions are equivalent:*

(a) $\circledast$ satisfies weak inclusion, consistency, conditional uniformity, weak proxy success, stability, uniform success and weak relevance (respectively, weak core-retainment).

(b) There exists a partial meet base (respectively, kernel) revision operator $*$ for A and a transformation function f that satisfies lower boundary, consistency preservation, idempotence and uniform identity and such that $A \circledast \alpha = A * f(\alpha)$, for all α.

4 Negation Free Selective Base Revision

Keeping in mind the importance of providing a theory suitable for logics not closed under negation of sentences, as explained in Section 2.5, in this section we are going to show how Selective Revision can be extended taking this restriction into account.

4.1 Postulates

Most of the postulates proposed in Section 3 for bases stay the same. The formulation of weak core-retainment needs to be adapted in order to avoid the use of negations.

(Weak core-retainment) If $\alpha \in A \circledast \alpha$, $\beta \in A$ and $\beta \notin A \circledast \alpha$, then there is some $A' \subseteq A$ such that $A' \cup \{\alpha\} \nvdash \perp$ and $A' \cup \{\alpha, \beta\} \vdash \perp$.

In addition, in order to characterize this operation we need a new postulate that is a weaker version of pre-expansion. The original idea of the this postulate is that the consecutive act of expansion by α and revision by α should bring the same outcome as that of only performing the revision. In our context, due to the partial acceptance nature of selective revision, we cannot always guarantee that the successive performance of expansion by α and selective revision by α has the same outcome as that of only performing the selective revision. Nevertheless, if α is in the outcome of a selective revision by α, we can ensure the mentioned property:

(Weak pre-expansion) If $\alpha \in A \circledast \alpha$ then $(A \cup \{\alpha\}) \circledast \alpha = A \circledast \alpha$

4.2 Constructing the Operation

The construction is similar to the one for bases in the previous section. The only difference is on some of the properties for the transformation function, also to eliminate negation of sentences:

If $\alpha \nvdash \bot$, then $f(\alpha) \nvdash \bot$ (Consistency preservation)

If $A \cup \{\alpha\} \nvdash \bot$, then $f(\alpha) = \alpha$ (Weak maximality)

Observation 1 is still valid here. Its proof just needs to be adapted by replacing $\nvdash \neg\alpha$ by $\alpha \nvdash \bot$ and $A \nvdash \neg\alpha$ by $A \cup \{\alpha\} \nvdash \bot$.

4.3 Representation Results

The following representation theorems have been obtained for four classes of negation free selective base revision:

Theorem 11. *Let $\mathcal{L}$ be a finite language, A be a belief base in $\mathcal{L}$ and $\circledast$ be an operator on A. Then the following conditions are equivalent:*

(a) $\circledast$ satisfies weak inclusion, consistency, conditional uniformity, proxy success, stability, uniform success, weak relevance (respectively, weak core-retainment) and weak pre-expansion.

(b) There exists a negation free external partial meet (respectively, kernel) revision operator $*$ for A and a transformation function f that satisfies lower boundary, idempotence, implication, uniform identity and such that $A \circledast \alpha = A * f(\alpha)$, for all α.

Theorem 12. *Let $\mathcal{L}$ be a finite language, A be a belief base in $\mathcal{L}$ and $\circledast$ be an operator on A. Then the following conditions are equivalent:*

(a) $\circledast$ satisfies weak inclusion, consistency, conditional uniformity, weak proxy success, stability, uniform success, weak relevance (respectively, weak core-retainment) and weak pre-expansion.

(b) There exists a negation free external partial meet (respectively, kernel) revision operator $*$ for A and a transformation function f that satisfies lower boundary, consistency preservation, idempotence, uniform identity and such that $A \circledast \alpha = A * f(\alpha)$, for all α.

5 Conclusion and Future Work

In this paper, we adapted the selective revision operators that were defined in (Fermé and Hansson 1999) to the belief base context. We proposed new properties for the transformation functions, several new postulates to characterize selective base revision operators and presented axiomatic characterizations for several classes of such operators.

We also addressed the issue of revision in languages which are not closed under negation, as it is the case of DLs. Instead of defining revisions based on contraction through the Levi identity, we adapted negation free revision to the context of selective revision.

The postulates used on those axiomatic characterisations are either the same or weaker versions of the base revisions postulates on which the selective revisions are based. Thus the selective base revisions operators presented in those theorems are weaker variants of the base revisions that the former are based on, and thus are a broader class of operators. In the particular case where $f(\alpha) = \alpha$ holds for all α, both types of operators coincide.

Future work includes defining negation free selective revision for belief sets and generalizing the results to deal with multiple change, where the input is a set of formulas.

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