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Syntax Splitting for Iterated Contractions

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Abstract

Parikh developed the notion of syntax splitting to describe belief sets with independent parts. He also formulated a postulate demanding that belief revisions respect syntax splittings in belief sets. The concept of syntax splitting was later transferred to epistemic states with total preorders and ranking functions by Kern-Isberner and Brewka along with corresponding postulates for belief revisions. Besides revision, contraction is also a central operation in the field of general belief change. In this paper, we analyse belief contractions with respect to syntax splitting. Based on the work on syntax splitting for revision, we develop syntax splitting postulates for contractions on ranking functions, on epistemic states with total preorder, and on belief sets. Finally, we evaluate different contractions from the literature, namely moderate contraction, natural contraction, lexicographic contraction, and c-contractions with respect to the newly developed contraction postulates.

1 Introduction

Any intelligent agent must be able to represent her beliefs and to reason with it, and in any evolving environment these beliefs must evolve accordingly. Adding new beliefs to existing beliefs while maintaining consistency is called *belief revision*. A belief revision might require the removal of some of the agent's existing beliefs, that contradicts the new information. Removing beliefs (e.g. because some information added earlier is from an unreliable source) is called belief contraction.

Many different postulates have been proposed for belief change. The best known postulates are probably the AGM postulates for the revision and contraction of belief sets (Alchourrón, Gärdenfors, and Makinson 1985). Though this approach is widely accepted, it is also criticised. One reason is that the AGM framework does not capture iterative changes properly, i.e., applying several changes after each other to a belief set. To address this, additional revision postulates were proposed (Darwiche and Pearl 1997). Along with this, Darwiche and Pearl developed a framework based on epistemic states with total preorders. Later, corresponding contraction postulates were proposed in (Konieczny and Pino Pérez 2017). Another drawback of the AGM framework is that the revisions in general do not respect the internal structure of belief sets. Parikh noted that a belief set might contain independent beliefs about different parts of the signature (Parikh 1999).

Example 1. *Consider an agent having the following beliefs about a certain object: (1) If it is a car, it can move. (2) It is blue. While the belief about the object being a car and its ability to move is connected, the colour of the object is independent of the objects type and ability to move. If the agent learns that the object is in fact yellow, it would be unreasonable to change the belief that cars can move.*

Parikh formalized this by the notion of syntax splitting and formulated a postulate for belief revision demanding that independent parts should be revised independently (Parikh 1999). This postulate was characterised in the framework of total preorders in (Peppas et al. 2015). In (Aravanis, Peppas, and Williams 2017), Parikh's postulate was characterised in the context of epistemic entrenchment. The concept of syntax splitting was transferred to epistemic states with total preorders and ranking functions along with corresponding postulates for iterated belief revisions (Kern-Isberner and Brewka 2017).

In the last years, belief contraction gained more attention as one of the central belief change operations, e.g. (Nayak et al. 2006; Ramachandran, Nayak, and Orgun 2012; Caridroit, Konieczny, and Marquis 2017; Konieczny and Pino Pérez 2017; Sauerwald, Kern-Isberner, and Beierle 2020). But as far as we are aware, postulates for contractions in the presence of a syntax splitting are missing.

In this paper, we develop syntax splitting postulates for contractions. These postulates are based on the work on syntax splitting for revision and deal with contractions that respect syntax splittings in the agent's beliefs. We consider contractions on ranking functions, on total preorders, and on belief sets, which are all well-known and established representations of epistemic states.We study and establish the precise connection among the new contraction postulates. Among other things, our analysis reveals a subtle but important difference in the concepts of belief change used in (Parikh 1999) and (Kern-Isberner and Brewka 2017). To show the applicability of the new postulates, we evaluate different types of contractions from the literature, namely moderate contraction, natural contraction, lexicographic contraction, (Ramachandran, Nayak, and Orgun 2012) and c-contractions (Kern-Isberner et al. 2017) with respect to the newly developed contraction postulates. In summary, the main contributions of this paper are:

- Propose new syntax splitting postulates for contractions on ranking functions, epistemic states with total preorders, and belief sets.
- Establish the precise relationships among these postulates.
- Analyse natural contraction, moderate contraction and lexicographic contraction on total preorders with respect to syntax splitting.
- Analyse c-contractions on ranking functions with respect to syntax splitting.

The rest of the paper is structured as follows. In Sec. 2, we outline previous work on belief representation and belief change as far as needed here. In Sec. 3, we recall the definition of syntax splitting and corresponding revision postulates for different beliefs representation frameworks. In Sec. 4, we develop new syntax splitting postulates for contractions of ranking functions, epistemic states with TPOs, and belief sets. The new postulates are applied to some contraction operators from literature in Section 5. In Section 6, we conclude and point out future work.

2 Background: Belief Representation and Belief Change

Propositional Logic Let Σ be a (propositional) finite signature. The set of all propositional formulae over Σ is denoted by $\mathrm{Form}(\Sigma)$. We will use $\bar{A}$ as shorthand for $\neg A$ and AB as shorthand for $A \wedge B$ with $A, B \in \mathrm{Form}(\Sigma)$. The set of all interpretations, also called worlds, of Σ will be denoted as $\mathrm{Int}(\Sigma)$ or Ω. An interpretation $\omega \in \mathrm{Int}(\Sigma)$ is a *model* for $A \in \mathrm{Form}(\Sigma)$, denoted as $\omega \models A$, if A holds in ω. The set of models for a formula is $\mathrm{Mod}_\Sigma(A) = \{\omega \in \mathrm{Int}(\Sigma) \mid \omega \models A\}$. A formula with at least one model is called *consistent*, otherwise *inconsistent*. For $A, B \in \mathrm{Form}(\Sigma)$ we say A entails B if $\mathrm{Mod}_\Sigma(A) \subseteq \mathrm{Mod}_\Sigma(B)$. The concepts of models, consistency and entailment are analogously used for sets of formulae. For $\mathcal{M} \subseteq \mathrm{Form}(\Sigma)$, the *deductive closure* of $\mathcal{M}$ is $\mathrm{Cn}_\Sigma(\mathcal{M}) = \{A \in \mathrm{Form}(\Sigma) \mid \mathcal{M} \models A\}$. If $M = \mathrm{Cn}_\Sigma(M)$ then M is called *deductively closed*. Note that the outcome of the Cn operator depends on the logical language used. As we only use propositional logic in this paper, the outcome of the Cn operator depends solely on the considered signature here. The deductive closure of any set of formulae is deductively closed and $\mathcal{M} \subseteq \mathrm{Cn}_\Sigma(\mathcal{M})$. The *theory* of a set $I \subseteq \mathrm{Int}(\Sigma)$ is $\mathrm{Th}(I) = \{A \in \mathrm{Form}(\Sigma) \mid \omega \models A \text{ for all } \omega \in I\}$. Every theory is deductively closed, $\mathrm{Mod}_\Sigma(\mathrm{Th}(I)) = I$ for all $I \subseteq \mathrm{Int}(\Sigma)$, and $\mathrm{Th}(\mathrm{Mod}_\Sigma(\mathcal{M})) = \mathrm{Cn}_\Sigma(\mathcal{M})$ for all $\mathcal{M} \subseteq \mathrm{Form}(\Sigma)$.

Belief Revision We assume that an agent has an *epistemic state* or *belief state* Ψ that is based on some propositional signature Σ and contains all information that is relevant for the reasoning of the agent. Furthermore, the agent has an inference relation $\approx$ connecting belief states over Σ and formulae in $\mathrm{Form}(\Sigma)$. The relation $\Psi \mathrel{|\!\!\approx} A$ holds iff the agent with belief state Ψ believes that the formula A is true. We consider the belief state in a very abstract sense here, but more concrete concepts for belief states are introduced later in this paper.

A *belief change* is the change of an agent's belief state from Ψ to Ψ° given a finite consistent set $\mathcal{A}$ of formulae as input. For the belief changes considered in this paper, Ψ and Ψ° are based on the same signature. Belief changes can be categorized by their outcome: We call a change of Ψ with $\mathcal{A} = \{A_1, \ldots, A_n\}$ to Ψ° a *revision* if $\Psi^\circ \mathrel{|\!\!\approx} A_i$ for every $A_i \in \mathcal{A}$, and a *contraction* if $\Psi^\circ \not\mathrel{|\!\!\approx} A_i$ for every $A_i \in A$. If $\mathcal{A}$ contains exactly one element, we call the change a *single change* and may write A_1 instead of $\mathcal{A} = \{A_1\}$.

Belief changes can be realized by a *belief change operator*, i.e. a function $\circ$, that maps an agent's belief state Ψ before the change and a set $\mathcal{A}$ of formulae to the belief state $\Psi^\circ = \Psi \circ \mathcal{A}$ after the change of Ψ with $\mathcal{A}$. In this paper, if Ψ is based on the signature Σ, then we allow only $\mathcal{A} \subseteq \mathrm{Form}(\Sigma)$ and require that Ψ° is also based on Σ. For the case of belief sets this means that for a subsignature $\Sigma_1 \subseteq \Sigma$, a belief set $K \subseteq \mathrm{Form}(\Sigma_1)$, and a set of formulae $\mathcal{A} \subseteq \mathrm{Form}(\Sigma_1)$ it holds that $K \circ \mathcal{A} \subseteq \mathrm{Form}(\Sigma_1)$, i.e. a belief change may not introduce new variables to the belief set. Change operators that are only defined for changes with one formula (i.e. single changes) are called single belief change operators. While finding good revision and contraction operators is a complex task, we will not make further assumptions about the considered belief changes here.

In the following, we will specify three approaches to belief representation that will be used in this paper.

Belief Sets One way to analyse the belief state of an agent is to look at the set of propositions the agent considers to be true (Hansson 1999). We call this set of formulae the *belief set* $K \subseteq \mathrm{Form}(\Sigma)$ of an agent. We assume that an agent believes all implications of her beliefs, i.e., K is deductively closed. If an agent's epistemic state is modelled by a belief set K, then $K \mathrel{|\!\!\approx} A$ iff $A \in K$ by definition.

A belief set $\mathrm{Bel}(\Psi)$ can be assigned to an epistemic state Ψ by the following: $A \in \mathrm{Bel}(\Psi)$ iff $\Psi \mathrel{|\!\!\approx} A$ for $A \in \mathrm{Form}(\Sigma)$.

Total Preorders and Iterated Revision In (Darwiche and Pearl 1997) another framework for belief revision is introduced that was developed to deal with iterated belief revisions. This framework takes into account how (un-)likely certain scenarios are. The belief states are called *epistemic states*. Every epistemic state Ψ is assigned a total preorder (TPO) $\preccurlyeq_\Psi$ on Ω. If $\omega_1 \preccurlyeq_\Psi \omega_2$, then the agent with state Ψ considers ω_1 to be as least as plausible as ω_2 for $\omega_1, \omega_2 \in \Omega$. The minimal worlds are considered most plausible. The set of minima of a set S with respect to an order $\leq$ on S is $\min(S, \leq) = \{a \in S \mid a \leq b \text{ for all } b \in S\}$. The belief set associated with Ψ is therefore $\mathrm{Bel}(\Psi) = \mathrm{Th}(\min(\Omega, \preccurlyeq_\Psi))$. The total preorder is lifted to formulae: $A \preccurlyeq_\Psi B$ iff for an $\omega_1 \in \min(\mathrm{Mod}_\Sigma(A), \preccurlyeq_\Psi)$ and $\omega_2 \in \min(\mathrm{Mod}_\Sigma(B), \preccurlyeq_\Psi)$ it holds that $\omega_1 \preccurlyeq_\Psi \omega_2$.

Ranking Functions Another way to model epistemic states is the use of ranking functions. A *ranking function* is a function $\kappa : \Omega \to \mathbb{N}_0$ with $\kappa^{-1}(0) \neq \emptyset$ (Spohn 1988).

The rank of $\omega \in \Omega$ is $\kappa(\omega)$. Ranking functions are introduced in a more general form in (Spohn 1988) by mapping to ordinal numbers, but our definition is sufficient for the purpose of this paper. The lower the rank of a world, the more plausible it is. The most plausible worlds are those with rank 0 and the belief set of a ranking function is $\mathrm{Bel}(\Psi) = \mathrm{Th}(\min(\Omega, \preccurlyeq_\kappa)) = \mathrm{Th}(\kappa^{-1}(0))$. The rank of a formula A is $\kappa(A) = \min_{\omega \in \mathrm{Mod}(A)} \kappa(\omega)$. Every ranking function induces a purely qualitative epistemic state represented by a total preorder $\preccurlyeq_\kappa$ defined by $\omega_1 \preccurlyeq_\kappa \omega_2$ iff $\kappa(\omega_1) \leq \kappa(\omega_2)$.

3 Syntax Splitting and Revisions

In (Parikh 1999) it was noticed that the AGM postulates do not take into account that a belief set might be based on completely independent propositions. For example, the revision operator *trivial update* defined as

$$K *_{\mathrm{TU}} A = \begin{cases} K & \text{if } A \in K \\ \mathrm{Cn}_\Sigma(A) & \text{otherwise} \end{cases}$$

for a belief set K over Σ and $A \in \mathrm{Form}(\Sigma)$ fulfils all AGM revision postulates but yields unintuitive results like $\mathrm{Cn}_\Sigma(a, b) *_{\mathrm{TU}} \bar{a} = \mathrm{Cn}_\Sigma(\bar{a})$. Information in the knowledge base that is not affected by the new information is unnecessarily removed by the revision.

Parikh defined the term *syntax splitting* to describe belief sets with independent information and formulated a postulate that describes the revision of belief sets with syntax splitting (Parikh 1999). The concept of syntax splitting and Parikh's postulate were later transferred to epistemic states with total preorders and to ranking functions in (Kern-Isberner and Brewka 2017). We will now recall the definition of syntax splitting and corresponding postulates for each of the three frameworks: belief sets, TPOs, and ranking functions.

3.1 Syntax Splitting on Belief Sets

A syntax splitting for a belief set is a partitioning of the knowledge base such that the independent parts of information can be each expressed using only one partition of the syntax splitting. In the following, the symbol $\dot\cup$ indicates a union of disjunctive sets. It is used in this paper to highlight partitions.

Definition 1 (syntax splitting for belief sets (Parikh 1999))**.** *Let Σ be a signature and K a belief set over Σ. For $A \in \mathrm{Form}(\Sigma)$ the set $\Sigma(A) \subseteq \Sigma$ denotes the smallest set of variables that is necessary to represent a formula that is equivalent to A.*

A partitioning $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a syntax splitting *for K, if there are formulae $A_1, \ldots, A_n \in \mathrm{Form}(\Sigma)$ with $K = \mathrm{Cn}_\Sigma(\{A_1, \ldots, A_n\})$ and $\Sigma(A_i) \subseteq \Sigma_i$ for $i = 1, \ldots, n$.*

With this definition Parikh proposed a postulate for revision operators. It is intended as addition to other postulates.

Postulate (P) (following (Parikh 1999))**.** *Let Σ be a signature and $*$ a revision operator on $\mathrm{Form}(\Sigma)$. For every partitioning $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ there is a revision operator $\diamond$ on $\mathrm{Form}(\Sigma_1)$, such that for every $A, C \in \mathrm{Form}(\Sigma_1), B \in \mathrm{Form}(\Sigma_2), K = \mathrm{Cn}_\Sigma(A, B)$ it holds that:*

$$K * C = \mathrm{Cn}_\Sigma(\mathrm{Cn}_{\Sigma_1}(A) \diamond C \cup \{B\})$$

It is sufficient to consider syntax splittings with two partitions in this postulate: If a belief set has a syntax splitting with more than two partitions, all partitions that do not share variables with the new formula C can be merged into a single partition of a syntax splitting with only two partitions.

Note that the formulation we present was altered slightly with respect to the version "strong P" given in (Peppas et al. 2015) to avoid misunderstandings. In (Peppas et al. 2015) it was claimed that there are two different ways to read the original Postulate (P): "weak (P)" and "strong (P)".

3.2 Syntax Splitting on Preorders of Worlds

Considering syntax splitting only on the belief sets of belief states can lead to problems.

Example 2. *Let $\Sigma = \{a, b\}$ with $a \neq b$. An agent strongly beliefs $a \to b$. Further she believes a. Let her epistemic state Ψ be associated with the TPO $\preccurlyeq_\Psi$ with $ab \prec_\Psi \bar{a}b, \bar{a}\bar{b} \prec_\Psi a\bar{b}$. Her belief set is therefore $K = \mathrm{Cn}_\Sigma(a, a \to b) = \mathrm{Cn}_\Sigma(a, b)$. Now the agent learns $\bar{a}$. The revision with $\mathrm{Bel}(\Psi * \bar{a}) = \mathrm{Cn}_\Sigma(\bar{a}b \vee \bar{a}\bar{b})$ that is implied by $\preccurlyeq_\Psi$ violates (P).*

Since belief sets cannot capture different degrees of belief, these are not taken into account in Parikh's syntax splitting. Kern-Isberner and Brewka developed a definition for syntax splitting on epistemic states with total preorders on worlds (Kern-Isberner and Brewka 2017). A concept necessary for their approach is that of the marginalisation of an epistemic state. A marginalisation reduces the signature of an epistemic state to a given subsignature. In the following definition, the worlds $\omega_1, \omega_2 \in \mathrm{Int}(\Sigma_1)$ are seen as formulae over Σ in the inequation $\omega_1 \preccurlyeq_\Psi \omega_2$.

Definition 2 (marginalisation of TPOs (Kern-Isberner and Brewka 2017))**.** *Let Σ be a signature and Ψ an epistemic state with TPO $\preccurlyeq_\Psi$ on $\Omega = \mathrm{Int}(\Sigma)$. Let $\Theta \subseteq \Sigma$. The marginalisation of $\preccurlyeq_\Psi$ to Θ is the unique TPO $\preccurlyeq_{\Psi|_\Theta}$ on $\Omega_\Theta = \mathrm{Int}(\Theta)$ defined by $\omega_1 \preccurlyeq_{\Psi|_\Theta} \omega_2$ iff $\omega_1 \preccurlyeq_\Psi \omega_2$ for worlds $\omega_1, \omega_2 \in \Omega_\Theta$.*

Definition 3 (syntax splitting for TPOs (Kern-Isberner and Brewka 2017))**.** *Let Σ be a signature and Ψ an epistemic state with TPO $\preccurlyeq_\Psi$ on $\Omega = \mathrm{Int}(\Sigma)$. Let $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ be a partitioning, ω^j be the variable assignment of the variables in Σ_j as in ω, and $\omega^{\neq i} := \bigwedge_{\substack{j=1,\ldots,n \\ i \neq j}} \omega^j$ for $\omega \in \Omega$ and $i = 1, \ldots, n$. The partitioning $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a* syntax splitting *for Ψ if, for $i = 1, \ldots, n$,*

$$\omega_1^{\neq i} = \omega_2^{\neq i} \quad \textit{implies} \quad \left(\omega_1 \preccurlyeq_\Psi \omega_2 \textit{ iff } \omega_1^i \preccurlyeq_{\Psi|_{\Sigma_i}} \omega_2^i\right).$$

This basically states that the order of two worlds that differ only in one partition of a syntax splitting does not depend on the actual variable assignment outside this partition.

Based on postulate (R1) and (R2) in (Peppas et al. 2015) Kern-Isberner and Brewka developed the following syntax splitting postulate for TPOs.

Postulate (MR) (Marginalised Revision (Kern-Isberner and Brewka 2017))**.** *Let $*$ be a revision operator on epistemic states with TPO. For every state Ψ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ it holds that:*

$$\preccurlyeq_{(\Psi * \mathcal{C})|_{\Sigma_i}} = \preccurlyeq_{(\Psi|_{\Sigma_i}) * C_i} \quad \textit{for } i = 1, \dots, n$$

(MR) describes that revision and marginalisation can be swapped if they affect the same partition of the syntax splitting. One implication of (P) is that the syntax splitting remains after a revision with a formula that affects only one partition. As this is not captured by (MR), another postulate was added.

Postulate (P^{it}) (iterated P (Kern-Isberner and Brewka 2017))**.** *Let $*$ be a revision operator on epistemic states with TPO. For every state Ψ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$, the partition $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a syntax splitting for $\Psi * \mathcal{C}$.*

Note that (MR) does not imply (P^{it}) or vice versa. A counterexample for this can be found in (Kern-Isberner and Brewka 2017).

3.3 Syntax Splitting on Ranking Functions

As a ranking function induces a total preorder on the worlds, the postulates (MR) and (P^{it}) could be applied to revisions on ranking functions. But the additional expressiveness of ranking functions allows to define a stricter notion of syntax splitting and syntax splitting postulates more specific to ranking functions.

Definition 4 (syntax splitting for ranking functions (Kern-Isberner and Brewka 2017))**.** *Let Σ be a signature and κ a ranking function over $\Omega = \mathrm{Int}(\Sigma)$. Let ω^j be the variable assignment of the variables in Σ_j as in ω.*

A partitioning $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a syntax splitting *for κ if there are ranking functions $\kappa_i : \Sigma_i \mapsto \mathbb{N}_0$ for $i = 1, \dots, n$ such that $\kappa(\omega) = \kappa_1(\omega^1) + \cdots + \kappa_n(\omega^n)$. This is denoted as $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$.*

Marginalisation can be defined for ranking functions as well. Again, the signature is reduced to a given subsignature Θ. The rank of a world ω_Θ over Θ is determined by the rank of the minimal world that matches ω_Θ on Θ.

Definition 5 (marginalisation on ranking functions (Kern-Isberner and Brewka 2017))**.** *Let Σ be a signature and κ be a ranking function over $\Omega = \mathrm{Int}(\Sigma)$. Let $\Theta \subseteq \Sigma$. The* marginalisation *of κ to Θ is the function $\kappa|_\Theta : \Omega_\Theta \mapsto \mathbb{N}_0$ with $\kappa|_\Theta(\omega) = \kappa(\omega)$ for $\omega \in \Omega_\Theta$.*

Note that ω is seen as a formula in the right hand side of the last equation in Definition 5 and that the marginalisation of a ranking function to a subsignature Θ is a ranking function over $\mathrm{Int}(\Theta)$. Marginalization of semantical structures has also been studied in (Beierle and Kern-Isberner 2012) where many different semantics for conditional logics, including total preorders, ranking functions, probablity distributions, possibility measures, conditional objects, and variants thereof, are formalized as *institutions* (Goguen and Burstall 1992). The marginalization of total preorders and of ranking functions in Definitions 2 and 5, respectively, are special cases of the general forgetful functor $Mod(\sigma)$ from Σ-models to Θ-models given in (Beierle and Kern-Isberner 2012) where $\Theta \subseteq \Sigma$ and σ is the inclusion from Θ to Σ.

The marginalisation of ranking functions is compatible with the marginalisation of total preorders in the following sense:

Proposition 1. *Let Σ be a signature, κ be a ranking function over $\Omega = \mathrm{Int}(\Sigma)$, and Ψ be an epistemic state such that $\preccurlyeq_\Psi = \preccurlyeq_\kappa$. Let $\Theta \subseteq \Sigma$. Then $\preccurlyeq_{\Psi|_\Theta} = \preccurlyeq_{\kappa|_\Theta}$.*

Also for a ranking function $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ with syntax spitting $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ it holds that $\kappa|_{\Sigma_i} = \kappa_i$ for $i = 1, \dots, n$. Using these definitions the postulates (MR) and (P^{it}) can be transferred to ranking functions.

Postulate (MR^{ocf}) (Marginalised Revision for OCFs (Kern-Isberner and Brewka 2017))**.** *Let $*$ be a revision operator on ranking functions. For every ranking function $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ it holds that:*

$$(\kappa * \mathcal{C})|_{\Sigma_i} = \kappa|_{\Sigma_i} * C_i = \kappa_i * C_i \quad \textit{for } i = 1, \dots, n.$$

Postulate (P^{ocf}) (P for OCFs (Kern-Isberner and Brewka 2017))**.** *Let $*$ be a revision operator on ranking functions. For every ranking function $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ it holds that:*

$$\kappa * \mathcal{C} = (\kappa_1 * C_1) \oplus \cdots \oplus (\kappa_n * C_n)$$

(P^{ocf}) implies (MR^{ocf}) (Kern-Isberner and Brewka 2017). Another implication of (P^{ocf}) is the following postulate (P^{it-ocf}) that is inspired by (P^{it}).

Postulate (P^{it-ocf}) (iterated P for OCFs)**.** *Let $*$ be a revision operator on ranking functions. For every ranking function $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ the partitioning $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a syntax splitting for $\kappa * \mathcal{C}$.*

It can be shown that (MR^{ocf}) and (P^{it-ocf}) together are equivalent to (P^{ocf}).

4 Syntax Splitting Postulates for Contractions

In this section, we will look at the contraction of beliefs with syntax splitting. Based on the syntax splitting postulates for revision presented in Section 3, we will develop syntax splitting postulates for contractions on ranking functions, on epistemic states with total preorders, and on belief sets. To do so, we will first transfer the postulates for the revision of ranking functions to the contraction of ranking functions. Then we will transfer these postulates to contractions on epistemic states with total preorders. Finally, we will develop postulates for the contraction of belief sets. Figure 1 shows an overview of the syntax splitting postulates for contractions developed in this section.

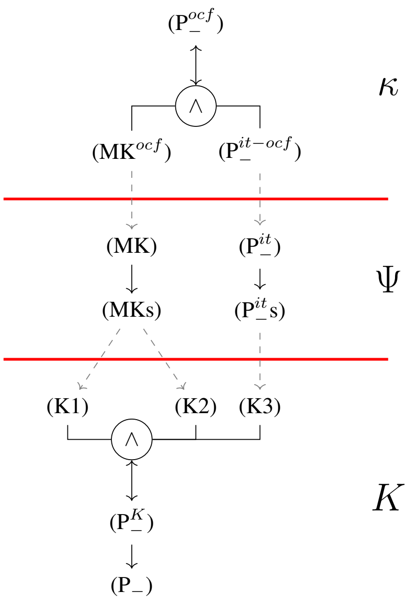


Figure 1: Overview of the developed syntax splitting postulates for contraction. The solid arrows indicate that one postulate implies another postulate (possibly with some assumptions). The dashed lines indicate that the conclusion of one postulate implies the conclusion of another postulate.

4.1 Contraction of Ranking Functions

The postulate (P^{ocf}) can be transferred to a similar postulate for contractions.

Postulate (P^{ocf}_{-}). *Let $-$ be a contraction operator on ranking functions. For every ranking function $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ it holds that:*

$$\kappa - \mathcal{C} = (\kappa_1 - C_1) \oplus \cdots \oplus (\kappa_n - C_n)$$

The postulates (P^{ocf}) and (P^{ocf}_{-}) are so similar that they could be generalized to a single postulate for belief changes on ranking functions. An implication of the postulate (P^{ocf}_{-}) is that the syntax splitting remains unchanged after the contraction. Another implication is that the marginalization of $\kappa - \mathcal{C}$ on a partition Σ_i is the same as $\kappa_i - C_i$. Both statements can be formulated as autonomous postulates.

Postulate $(\mathrm{P}^{it-ocf}_{-})$. *Let $-$ be a contraction operator on ranking functions. For every ranking function $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ the partition $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a syntax splitting for $\kappa - \mathcal{C}$.*

Postulate (MK^{ocf}). *Let $-$ be a contraction operator on ranking functions. For every ranking function $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ it holds that:*

$$(\kappa - \mathcal{C})|_{\Sigma_i} = \kappa_i - C_i = \kappa|_{\Sigma_i} - C_i \quad \text{for } i = 1, \dots, n$$

The following proposition shows that $(\mathrm{P}^{it-ocf}_{-})$ and (MK^{ocf}) together are equivalent to (P^{ocf}_{-}).

Proposition 2. *A contraction operator $-$ fulfils (P^{ocf}_{-}) iff it fulfils (MK^{ocf}) and $(\mathrm{P}^{it-ocf}_{-})$.*

Proof. The $\Rightarrow$ direction is obvious. For the $\Leftarrow$ direction let $-$ be a contraction operator fulfilling (MK^{ocf}) and $(\mathrm{P}^{it-ocf}_{-})$. Let $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ be a ranking function with syntax splitting $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ such that $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$. Using $(\mathrm{P}^{it-ocf}_{-})$ it follows that $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a syntax splitting for $\kappa - \mathcal{C}$, i.e.,

$$\kappa - \mathcal{C} = \kappa_1^- \oplus \cdots \oplus \kappa_n^-$$

for some $\kappa_1^-, \dots, \kappa_n^-$. From (MK^{ocf}) it follows that $\kappa_i^- = (\kappa - \mathcal{C})|_{\Sigma_i} = \kappa|_{\Sigma_i} - C_i = \kappa_i - C_i$ for $i = 1, \dots, n$. Therefore, $-$ fulfils (P^{ocf}_{-}). $\square$

4.2 Contraction of Epistemic States with TPOs

Consider now epistemic states with total preorders over worlds. The postulates $(\mathrm{P}^{it-ocf}_{-})$ and (MK^{ocf}) can be transferred from ranking functions to this case.

Postulate (P^{it}_{-}). *Let $-$ be a contraction strategy on epistemic states with total preorders. For every state Ψ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ with $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ the partition $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ is a syntax splitting for $\Psi - \mathcal{C}$.*

Postulate (MK). *Let $-$ be a contraction operator on epistemic states with total preorders. For every state Ψ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \dots, C_n\}$ with $C_i \in \mathrm{Form}(\Sigma_i)$ for $i = 1, \dots, n$ it holds that:*

$$\preccurlyeq_{(\Psi - \mathcal{C})|_{\Sigma_i}} = \preccurlyeq_{(\Psi|_{\Sigma_i}) - C_i} \quad \text{for } i = 1, \dots, n$$

Neither (MK) implies (P^{it}_{-}) nor vice versa.

As a ranking function κ can be seen as an epistemic state with total preorder $\preccurlyeq_\kappa$, the postulates (MK) and (P^{it}_{-}) can be applied to ranking functions as well. But it should be noted that (MK^{ocf}) does not imply (MK) even though the conclusion of (MK^{ocf}) implies (MK). A ranking function κ contains more information than the derived TPO $\preccurlyeq_\kappa$, and a syntax splitting for $\preccurlyeq_\kappa$ is not necessarily a syntax splitting for κ. Therefore, there are cases where (MK) is applicable, but (MK^{ocf}) is not. The same holds for $(\mathrm{P}^{it-ocf}_{-})$ and (P^{it}_{-}).

The postulates (P^{it}_{-}) and (MK) describe multiple contractions, but many contraction operators on total preorders are single contractions, i.e., they contract one formula at a time. For a contraction that does not distinguish between a formula C and the set $\{C, \perp\}$ and satisfies *trivial vacuity*, i.e. $\Psi - \perp = \Psi$, the following two postulates for single contractions follow from (P^{it}_{-}) and (MK).

Postulate $(\mathrm{P}^{it}_{-}\mathrm{s})$. *Let $-$ be a contraction operator on epistemic states with total preorders. For every state Ψ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ and $C \in \mathrm{Form}(\Sigma_1)$ the partitioning $\Sigma_1 \dot\cup \Sigma_2$ is a syntax splitting for $\Psi - C$.*

Postulate (MKs). *Let $-$ be a contraction operator on epistemic states with total preorders. For every state Ψ with syntax splitting $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ and $C \in \mathrm{Form}(\Sigma_1)$ we have:*

$$\preccurlyeq_{(\Psi - C)|_{\Sigma_1}} = \preccurlyeq_{(\Psi|_{\Sigma_1}) - C} \quad \text{(MKs1)}$$

$$\preccurlyeq_{(\Psi - C)|_{\Sigma_2}} = \preccurlyeq_{(\Psi|_{\Sigma_2})} \quad \text{(MKs2)}$$

The first part of the postulate (MKs) restricts the change of the partition that is affected by the contraction. The second part restricts the change on the part that is not affected by the contraction.

4.3 Syntax Splitting on Belief Sets

Now we want to transfer the syntax splitting postulates to contractions on belief sets. As most contraction operators do not support multiple contractions, we will start with (MKs) and (P^{it}_{-}s). Let us look at the conclusions of this postulates.

Proposition 3. *Let Ψ be an epistemic state over signature Σ, $C \in \mathrm{Form}(\Sigma_1)$ such that $\Sigma_1 \subseteq \Sigma$ and $-$ a contraction operator on epistemic states with TPOs.*

- *If $\Sigma_1 \dot\cup \Sigma_2$ with $\Sigma_2 = \Sigma \setminus \Sigma_1$ is a syntax splitting for $\Psi - C$, then $\Sigma_1 \dot\cup \Sigma_2$ is a syntax splitting for $\mathrm{Bel}(\Psi - C)$ as well (Kern-Isberner and Brewka 2017).*
- *If $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}} = \preccurlyeq_{(\Psi|_{\Sigma_1})-C}$, then $\mathrm{Bel}(\Psi - C) \cap \mathrm{Form}(\Sigma_1) = \mathrm{Bel}((\Psi|_{\Sigma_1}) - C)$.*
- *If $\preccurlyeq_{(\Psi-C)|_{\Sigma_2}} = \preccurlyeq_{(\Psi|_{\Sigma_2})}$, then $\mathrm{Bel}(\Psi - C) \cap \mathrm{Form}(\Sigma_2) = \mathrm{Bel}(\Psi) \cap \mathrm{Form}(\Sigma_2)$.*

Proof. The first statement is a special case of Proposition 3 in (Kern-Isberner and Brewka 2017).

Let Σ be a signature, Ψ, Φ epistemic states over Σ and $\Sigma_1 \subseteq \Sigma_2$. To prove the other two statements, we show that $\preccurlyeq_{\Psi|_{\Sigma_1}} = \preccurlyeq_{\Phi|_{\Sigma_1}}$ implies that $\mathrm{Bel}(\Psi) \cap \mathrm{Form}(\Sigma_1) = \mathrm{Bel}(\Phi) \cap \mathrm{Form}(\Sigma_1)$. Let $\Omega = \mathrm{Int}(\Sigma)$ and $\Omega_{\Sigma_1} = \mathrm{Int}(\Sigma_1)$. Let ω^1 be the variable assignment of the variables in Σ_1 as in ω for a world $\omega \in \Omega$.

$$\begin{aligned}
&\mathrm{Bel}(\Psi) \cap \mathrm{Form}(\Sigma_1)\\
&= (\mathrm{Th}(\min(\Omega, \preccurlyeq_\Psi))) \cap \mathrm{Form}(\Sigma_1)\\
&= \{A \in \mathrm{Form}(\Sigma_1) \mid \omega \models A \text{ for all } \omega \in \min(\Omega, \preccurlyeq_\Psi)\}\\
&\overset{\dagger}{=} \{A \in \mathrm{Form}(\Sigma_1) \mid \omega^1 \models A \text{ for all } \omega \in \min(\Omega, \preccurlyeq_\Psi)\}\\
&\overset{\ddagger}{=} \{A \in \mathrm{Form}(\Sigma_1) \mid \omega_1 \models A \text{ for all } \omega_1 \in \min(\Omega, \preccurlyeq_{\Psi|_{\Sigma_1}})\}\\
&= \{A \in \mathrm{Form}(\Sigma_1) \mid \omega_1 \models A \text{ for all } \omega_1 \in \min(\Omega, \preccurlyeq_{\Phi|_{\Sigma_1}})\}\\
&= \mathrm{Bel}(\Phi) \cap \mathrm{Form}(\Sigma_1)
\end{aligned}$$

Equation † holds, because A uses only variables from Σ_1. The assignment of variables from $\Sigma \setminus \Sigma_1$ has no influence on the evaluation of A. Equation ‡ holds, because $\{\omega^1 \mid \omega \in \min(\Omega, \preccurlyeq_\Psi)\} = \{\omega_1 \mid \omega_1 \in \min(\Omega, \preccurlyeq_{\Psi|_{\Sigma_1}})\}$. The world $\omega^1 \in \mathrm{Int}(\Sigma_1)$ is the restriction of $\omega \in \mathrm{Int}(\Sigma)$ to Σ_1, and ω_1 is a world directly selected from $\mathrm{Int}(\Sigma)$. □

Based on this we can formulate the following syntax splitting postulate for contractions on belief sets.

Postulate (K). *Let $-$ be a contraction operator on belief sets. For every belief set K with syntax splitting $\Sigma = \Sigma_1 \dot\cup \Sigma_2$, i.e. $K = \mathrm{Cn}_\Sigma(A, B)$ such that $A \in \mathrm{Form}(\Sigma_1), B \in \mathrm{Form}(\Sigma_2)$, and $C \in \mathrm{Form}(\Sigma_1)$ it holds that:*

(K1) $(K - C) \cap \mathrm{Form}(\Sigma_2) = K \cap \mathrm{Form}(\Sigma_2)$

(K2) $(K - C) \cap \mathrm{Form}(\Sigma_1) = (K \cap \mathrm{Form}(\Sigma_1)) - C$

(K3) *$\Sigma_1 \dot\cup \Sigma_2$ is a syntax splitting of $K - C$.*

The postulate (K) can be formulated as the following, more compact postulate.

Postulate (P^K_-). *Let $-$ be a contraction operator on belief sets. For every belief set K with syntax splitting $\Sigma = \Sigma_1 \dot\cup \Sigma_2$, i.e. $K = \mathrm{Cn}_\Sigma(A, B)$ such that $A \in \mathrm{Form}(\Sigma_1), B \in \mathrm{Form}(\Sigma_2)$, and $C \in \mathrm{Form}(\Sigma_1)$ it holds that:*

$$K - C = \mathrm{Cn}_\Sigma((\mathrm{Cn}_{\Sigma_1}(A) - C) \cup \{B\})$$

The following proposition shows that (K) and (P^K_-) are indeed equivalent.

Proposition 4. *Let $-$ be a contraction operator. Then $-$ fulfils (P^K_-) iff $-$ fulfils (K).*

Proof. Let $-$ be a contraction operator, $K = \mathrm{Cn}_\Sigma(A, B)$ be a belief set with syntax splitting $\Sigma = \Sigma_1 \dot\cup \Sigma_2$, and $A, C \in \mathrm{Form}(\Sigma_1), B \in \mathrm{Form}(\Sigma_2)$.

$\Rightarrow$: Let $-$ fulfil (P^K_-). This implies:

$$\begin{aligned}
&(K - C) \cap \mathrm{Form}(\Sigma_2)\\
&\overset{(\mathrm{P}^K_-)}{=} \mathrm{Cn}_\Sigma(\underbrace{\mathrm{Cn}_{\Sigma_1}(A) - C}_{\subseteq \mathrm{Form}(\Sigma_1)} \cup \{B\}) \cap \mathrm{Form}(\Sigma_2)\\
&= \mathrm{Cn}_\Sigma(B) = \mathrm{Cn}_\Sigma(A, B) \cap \mathrm{Form}(\Sigma_2)\\
&= K \cap \mathrm{Form}(\Sigma_2)
\end{aligned}$$

Therefore $-$ fulfils (K1).
Furthermore, we have:

$$\begin{aligned}
&(K - C) \cap \mathrm{Form}(\Sigma_1)\\
&\overset{(\mathrm{P}^K_-)}{=} \mathrm{Cn}_\Sigma(\mathrm{Cn}_{\Sigma_1}(A) - C \cup \{B\}) \cap \mathrm{Form}(\Sigma_1)\\
&= \mathrm{Cn}_\Sigma(\mathrm{Cn}_{\Sigma_1}(A) - C) \cap \mathrm{Form}(\Sigma_1)\\
&= \mathrm{Cn}_\Sigma((K \cap \mathrm{Form}(\Sigma_1)) - C) \cap \mathrm{Form}(\Sigma_1)\\
&\overset{\substack{(K\cap\mathrm{Form}(\Sigma_1))-C\\ \subseteq\mathrm{Form}(\Sigma_1)}}{=} (K \cap \mathrm{Form}(\Sigma_1)) - C
\end{aligned}$$

Therefore $-$ fulfils (K2).

Because $\Sigma_1 \dot\cup \Sigma_2$ is a syntax splitting for $K - C = (\mathrm{Cn}_\Sigma(\mathrm{Cn}_{\Sigma_1}(A) - C \cup \{B\}))$ the contraction fulfils (K3).

$\Leftarrow$ (inspired by the proof of Theorem 1 in (Peppas et al. 2015)): Let $-$ fulfil (K). From (K2) it follows

$$\begin{aligned}
&(K - C) \cap \mathrm{Form}(\Sigma_1)\\
&= ((K \cap \mathrm{Form}(\Sigma_1)) - C) \cap \mathrm{Form}(\Sigma_1)\\
&= \mathrm{Cn}_{\Sigma_1}(A) - C
\end{aligned}$$

and from (K1) it follows:

$$(K - C) \cap \mathrm{Form}(\Sigma_2) = K \cap \mathrm{Form}(\Sigma_2)$$

$$= \mathrm{Cn}_\Sigma(A, B) \cap \mathrm{Form}(\Sigma_2) = \mathrm{Cn}_\Sigma(B)$$

Because $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ is a syntax splitting for $K - C$ (see (K3)) we have

$$\begin{aligned} & K - C \\ & = \mathrm{Cn}_\Sigma((K - C \cap \mathrm{Form}(\Sigma_1)) \cup (K - C \cap \mathrm{Form}(\Sigma_2))) \\ & = \mathrm{Cn}_\Sigma((\mathrm{Cn}_{\Sigma_1}(A) - C) \cup (\mathrm{Cn}_\Sigma(B))) \\ & = \mathrm{Cn}_\Sigma((\mathrm{Cn}_{\Sigma_1}(A) - C) \cup \{B\}) \end{aligned}$$

Therefore $-$ fulfils (P^K_-). □

Another way to get a syntax splitting postulate for contractions of belief sets is to directly transfer (P) to contractions. Doing this, we obtain the following postulate.

Postulate (P_-). *Let Σ be a signature and $-$ be a contraction operator on $\mathrm{Form}(\Sigma)$. For every partitioning $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ there is a contraction operator $\div$ on $\mathrm{Form}(\Sigma_1)$ such that for all $A, C \in \mathrm{Form}(\Sigma_1), B \in \mathrm{Form}(\Sigma_2), K = \mathrm{Cn}_\Sigma(A, B)$ it holds that:*

$$K - C = \mathrm{Cn}_\Sigma(\mathrm{Cn}_{\Sigma_1}(A) \div C \cup \{B\})$$

While (P^K_-) and (P_-) look similar, they are not identical. This is because (P^K_-) is a postulate for contraction strategies, while (P_-) is a postulate for contraction operators with a fixed signature (cf. Section 3.2). In (P_-) a new contraction operator $\div$ for subsignatures is introduced. In (P^K_-) the same contraction operator is used for belief sets over Σ and over subsignatures $\Sigma_1 \subseteq \Sigma$. The postulates are nevertheless connected: If a contraction operator fulfils (P^K_-), then this operator fulfils (P_-).

5 Application to Known Contractions

We now apply the new syntax splitting postulates to contractions proposed in the literature. We examine the moderate, the natural, and the lexicographic contraction (Ramachandran, Nayak, and Orgun 2012) on epistemic states with total preorders with the postulates (MKs) and (P^{it}s). Afterwards, we examine c-changes on ranking functions (Kern-Isberner et al. 2017) with (MK^{ocf}) and (P^{it-ocf}_-).

5.1 Moderate Contraction

After a moderate contraction, all models of the contracted formula that are not maximally plausible, are ordered after the models of this formula.

Definition 6 (moderate contraction (following (Ramachandran, Nayak, and Orgun 2012))). *Let Ψ be an epistemic state with TPO $\preccurlyeq_\Psi$ over Σ. A* moderate contraction *with $C \in \mathrm{Form}(\Sigma)$ yields the state $\Psi -_M C$ with ordering $\preccurlyeq_{\Psi -_M C}$ such that:*

- *if $\omega_1, \omega_2 \models C$ or $\omega_1, \omega_2 \models \overline{C}$, then $\omega_1 \preccurlyeq_{\Psi -_M C} \omega_2$ iff $\omega_1 \preccurlyeq_\Psi \omega_2$*
- *if $\omega_1 \models \overline{C}$ and $\omega_2 \models C, \omega_2 \notin \min(\mathrm{Mod}(C), \preccurlyeq_\Psi)$, then $\omega_1 \prec_{\Psi -_M C} \omega_2$*
- *if $\omega_1 \in \min(\mathrm{Mod}(C), \preccurlyeq_\Psi) \cup \min(\mathrm{Mod}(C), \preccurlyeq_\Psi)$, then $\omega_1 \preccurlyeq_{\Psi -_M C} \omega_2$*

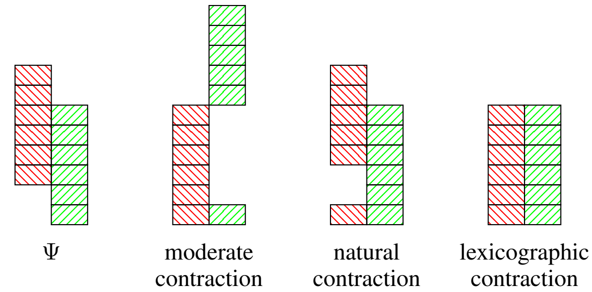


Figure 2: Graphic representation of different contraction operators on epistemic states with total preorders. An epistemic state Ψ is contracted by a formula α. The models of α are represented by this area, the counter models by this area. The lower an area is, the lower and thus the more plausible are the represented models in the epistemic state's TPO.

First a general observation: If a contraction $-$ fulfils the postulates (CK1) and (CK2) from (Konieczny and Pino Pérez 2017, Theorem 4, Properties 4 and 5) which are

(CK1) $\omega_1 \preccurlyeq_\Psi \omega_2$ iff $\omega_1 \preccurlyeq_{\Psi - C} \omega_2$ for $\omega_1, \omega_2 \in \mathrm{Mod}(C)$

(CK2) $\omega_1 \preccurlyeq_\Psi \omega_2$ iff $\omega_1 \preccurlyeq_{\Psi - C} \omega_2$ for $\omega_1, \omega_2 \in \mathrm{Mod}(\overline{C})$.

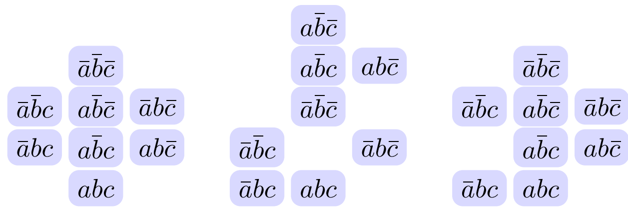
for an epistemic state Ψ over a signature Σ and $C \in \mathrm{Form}(\Sigma)$, then $-$ fulfils the (MKs2) part of (MKs). (CK1) and (CK2) prohibit unnecessary changes to the TPO, and (MKs2) prohibits changes to the part of the belief state not affected by the contraction. Therefore, these postulates are connected. However, (CK1) and (CK2) are not enough to imply the whole (MKs). The moderate, the natural, and the lexicographic contraction considered here fulfil (CK1) and (CK2).

Proposition 5. *A contraction operator on epistemic states with TPOs that fulfils (CK1) and (CK2) fulfils (MKs2).*

Proof. Let Ψ be an epistemic state with syntax splitting $\Sigma = \Sigma_1 \dot\cup \Sigma_2$ and $C \in \mathrm{Form}(\Sigma_1)$. First we show $\preccurlyeq_{(\Psi - C)|_{\Sigma_2}} \subseteq \preccurlyeq_{\Psi|_{\Sigma_2}}$. To do so, consider any $\omega_1, \omega_2 \in \Sigma_2$ such that $\omega_1 \preccurlyeq_{(\Psi - C)|_{\Sigma_2}} \omega_2$. Let ω_1^l be one of the smallest worlds with respect to $\preccurlyeq_{\Psi - C}$ such that ω_1^l and ω_1 coincide on Σ_2. Let ω_2^l be the world that coincides with ω_1^l on Σ_1 and with ω_1 on Σ_2. Then $\omega_1^l \preccurlyeq_{\Psi - C} \omega_2$. Because ω_1^l and ω_2^l coincide on Σ_1, either both or none of them fulfils C. Therefore $\omega_1^l \preccurlyeq_\Psi \omega_2^l$.

Let ω_1^m be one of the smallest worlds with respect to $\preccurlyeq_\Psi$ such that ω_1^m and ω_1 coincide on Σ_2. Let ω_2^m be the world that coincides with ω_1^m on Σ_1 and with ω_1 on Σ_2. The syntax splitting of Ψ and $\omega_1^l \preccurlyeq_\Psi \omega_2^l$ imply $\omega_1^m \preccurlyeq_\Psi \omega_2^m$. Because ω_1^m was chosen minimal, $\omega_1 \preccurlyeq_{\Psi|_{\Sigma_2}} \omega_2$.

Now we will show that $\preccurlyeq_{\Psi|_{\Sigma_2}} \subseteq \preccurlyeq_{(\Psi - C)|_{\Sigma_2}}$. To do so, consider $\omega_1, \omega_2 \in \Sigma_2$ such that $\omega_1 \preccurlyeq_{\Psi|_{\Sigma_2}} \omega_2$. Let ω_2^l be one of the smallest worlds with respect to $\preccurlyeq_{\Psi - C}$ that coincides with ω_2 on Σ_2 and let ω_1^l be the world that coincides with ω_2^l on Σ_1 and with ω_1 on Σ_2. Because of the syntax splitting of Ψ we have $\omega_1^l \preccurlyeq_\Psi \omega_2^l$. Because either both or none of ω_1^l and ω_2^l fulfil C it holds that $\omega_1^l \preccurlyeq_{\Psi - C} \omega_2^l$. Because ω_2^l was chosen as small as possible, $\omega_1 \preccurlyeq_{(\Psi - C)|_{\Sigma_2}} \omega_2$. □



(a) Ψ TPO before contraction (b) $\Psi -_M a$ after moderate contraction (c) $\Psi -_N a$ after natural contraction

Figure 3: The syntax splitting $\{a\}\dot{\cup}\{b\}\dot{\cup}\{c\}$ of Ψ is not preserved by moderate contraction (3b) nor by natural contraction (3c).

Applying (MKs) and (P^{it}_{-}s) yields:

Proposition 6. *Moderate contraction fulfils (MKs) but not (P^{it}_{-}s).*

Proof. To prove that moderate contraction fulfils (MKs), consider a state Ψ with syntax splitting $\Sigma_1 \dot{\cup} \Sigma_2$ and $C \in \mathrm{Form}(\Sigma_1)$. For showing $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}} \subseteq \preccurlyeq_{(\Psi|_{\Sigma_1})-C}$, we consider any $\omega_1, \omega_2 \in \Sigma_1$ with $\omega_1 \preccurlyeq_{(\Psi-C)|_{\Sigma_1}} \omega_2$ and show $\omega_1 \preccurlyeq_{(\Psi|_{\Sigma_1})-C} \omega_2$ by distinguishing the following five cases:

Case 1 $\omega_1 \models \overline{C}, \omega_2 \models \overline{C}$

Case 2 $\omega_1 \models C, \omega_2 \models C$

Case 3 $\omega_1 \models C, \omega_2 \models \overline{C}$

Case 4a $\omega_1 \models \overline{C}, \omega_2 \models C$ and $\omega_1 \approx_{(\Psi-C)|_{\Sigma_1}} \omega_2$

Case 4b $\omega_1 \models \overline{C}, \omega_2 \models C$ and $\omega_1 \prec_{(\Psi-C)|_{\Sigma_1}} \omega_2$

We now show $\preccurlyeq_{(\Psi|_{\Sigma_1})-C} \subseteq \preccurlyeq_{(\Psi-C)|_{\Sigma_1}}$. To do so, consider any $\omega_1, \omega_2 \in \Sigma_1$ with $\omega_1 \preccurlyeq_{(\Psi-C)|_{\Sigma_1}} \omega_2$ and we can show $\omega_1 \preccurlyeq_{(\Psi|_{\Sigma_1})-C} \omega_2$ by a similar case discrimination.

The moderate contraction fulfils (MKs2) because of Proposition 5.

The following counter example shows that moderate contraction does not fulfil (P^{it}_{-}s). Let $\Sigma = \{a, b, c\}$. The partitioning $\{a\} \dot{\cup} \{b\} \dot{\cup} \{c\}$ is a syntax splitting for the TPO depicted in Figure 3a. Contracting this TPO with a using moderate contraction yields the TPO depicted in Figure 3b. The partitioning $\{a\} \dot{\cup} \{b\} \dot{\cup} \{c\}$ is not a syntax splitting for the latter. ☐

5.2 Natural Contraction

Natural contraction is a contraction operator inspired by natural revision.

Definition 7 (natural contraction (following (Ramachandran, Nayak, and Orgun 2012)))**.** *Let Ψ be an epistemic state with TPO $\preccurlyeq_\Psi$ over Σ. A* natural contraction *with $C \in \mathrm{Form}(\Sigma)$ yields the state $\Psi -_N C$ with the TPO $\preccurlyeq_{\Psi-_N C}$ such that:*

- *If $\omega_1 \in \min(\mathrm{Mod}(C), \preccurlyeq_\Psi) \cup \min(\mathrm{Mod}(C), \preccurlyeq_\Psi)$, then $\omega_1 \preccurlyeq_{\Psi-_N C} \omega_2$*
- *If $\omega_1, \omega_2 \notin \min(\mathrm{Mod}(C), \preccurlyeq_\Psi) \cup \min(\mathrm{Mod}(C), \preccurlyeq_\Psi)$, then $\omega_1 \preccurlyeq_{\Psi-_N C} \omega_2$ iff $\omega_1 \preccurlyeq_\Psi \omega_2$*

The natural contraction tries to change the total preorder as little as possible.

Proposition 7. *Natural contraction fulfils (MKs) but not (P^{it}_{-}s).*

Proof. To prove that natural contraction fulfils (MKs1), consider a state Ψ with syntax splitting $\Sigma_1 \dot{\cup} \Sigma_2$ and $C \in \mathrm{Form}(\Sigma_1)$. For showing $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}} \subseteq \preccurlyeq_{(\Psi|_{\Sigma_1})-C}$, we consider any $\omega_1, \omega_2 \in \Sigma_1$ with $\omega_1 \preccurlyeq_{(\Psi-C)|_{\Sigma_1}} \omega_2$ and show $\omega_1 \preccurlyeq_{(\Psi|_{\Sigma_1})-C} \omega_2$ by distinguishing the following cases:

Case 1 ω_1 is minimal with respect to $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}}$

Case 2 ω_1 is not minimal with respect to $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}}$

Now we show $\preccurlyeq_{(\Psi|_{\Sigma_1})-C} \subseteq \preccurlyeq_{(\Psi-C)|_{\Sigma_1}}$. To do so, we consider any $\omega_1, \omega_2 \in \Sigma_1$ with $\omega_1 \preccurlyeq_{(\Psi-C)|_{\Sigma_1}} \omega_2$ and show $\omega_1 \preccurlyeq_{(\Psi|_{\Sigma_1})-C} \omega_2$ by a similar case discrimination:

Case 1a ω_1 is minimal with respect to $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}}$ and ω_1 is minimal with respect to $\preccurlyeq_{\Psi|_{\Sigma_1}}$

Case 1b ω_1 is minimal with respect to $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}}$ and ω_1 is minimal in $\mathrm{Mod}_{\Sigma_1}(C)$ with respect to $\preccurlyeq_{\Psi|_{\Sigma_1}}$

Case 2 ω_1 is not minimal with respect to $\preccurlyeq_{(\Psi-C)|_{\Sigma_1}}$

Natural contraction fulfils (MKs2) due to Proposition 5. The following counter example shows that natural contraction does not fulfil (P^{it}_{-}s). Let $\Sigma = \{a, b, c\}$. The partitioning $\{a\} \dot{\cup} \{b\} \dot{\cup} \{c\}$ is a syntax splitting for the TPO depicted in Figure 3a. A natural contraction of this TPO with a yields the TPO depicted in Figure 3c. The partitioning $\{a\} \dot{\cup} \{b\} \dot{\cup} \{c\}$ is not a syntax splitting for the latter. ☐

5.3 Lexicographic Contraction

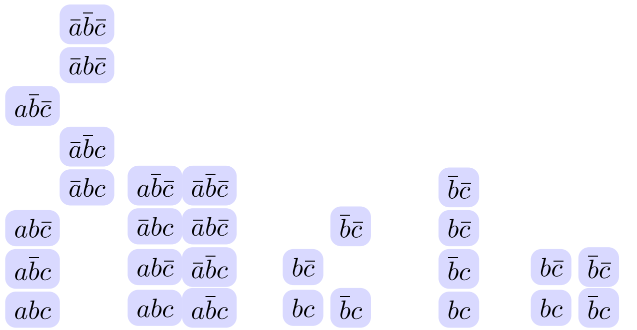
Lexicographic contraction places models and counter models of the contracted formula "parallel" to each other.

Definition 8 (complete chain (following (Ramachandran, Nayak, and Orgun 2012)))**.** *Let M be a set with a total preorder $\preccurlyeq$. An ordered set $K = \{k_1, \ldots, k_n\} \subseteq M$ is a* complete chain *in M with respect to $\preccurlyeq$ if either $K = \emptyset$ or $k_1 \in \min(M, \preccurlyeq)$ and $K \setminus \{k_1\}$ is a complete chain in $M \setminus \min(M, \preccurlyeq)$. A complete chain $K = \{k_1, \ldots, k_n\} \neq \emptyset$* ends *in k_n.*

Definition 9 (lexicographic contraction (following (Ramachandran, Nayak, and Orgun 2012)))**.** *Let Ψ be an epistemic state with TPO $\preccurlyeq_\Psi$ over Σ. A* lexicographic contraction *with $C \in \mathrm{Form}(\Sigma)$ yields a state $\Psi -_L C$ with TPO $\preccurlyeq_{\Psi-_L C}$ such that:*

- *If $\omega_1, \omega_2 \models C$ or $\omega_1, \omega_2 \models \overline{C}$, then $\omega_1 \preccurlyeq_{\Psi-_L C} \omega_2$ iff $\omega_1 \preccurlyeq_\Psi \omega_2$*
- *If $\omega_1 \models X$ and $\omega_2 \models \overline{X}$ with $X = C$ or $X = \overline{C}$, then $\omega_1 \preccurlyeq_{\Psi-_L C} \omega_2$ if a complete chain in $\mathrm{Mod}(X)$ with respect to $\preccurlyeq_\Psi$ that ends in ω_1 is at least as short as a complete chain in $\mathrm{Mod}(\overline{X})$ with respect to $\preccurlyeq_\Psi$ that ends in ω_2.*

Proposition 8. *The lexicographic contraction fulfils (MKs2) but not (MKs1) or (P^{it}_{-}).*



(a) Ψ (b) $\Psi -_L b$ (c) $(\Psi -_L b)|_{\Sigma_2}$ (d) $\Psi|_{\Sigma_2}$ (e) $\Psi|_{\Sigma_2} -_L b$

Figure 4: The syntax splitting $\{a\}\dot\cup\{b,c\}$ is lost in this contraction (see Figure 4b). Furthermore $\preccurlyeq_{(\Psi-b)|_{\Sigma_2}} \neq \preccurlyeq_{\Psi|_{\Sigma_2}-b}$.

Proof. The lexicographic contraction fulfils (MKs2) because of Proposition 5.

That the lexicographic contraction does not fulfil (MKs1) and (P^{it}_{-}) is demonstrated by the following example. Let $\Sigma = \{a, b, c\}$. The partitioning $\Sigma_1 \dot\cup \Sigma_2 = \{a\} \dot\cup \{b, c\}$ is a syntax splitting for the TPO $\preccurlyeq_\Psi$ illustrated in Figure 4a. Contracting b yields the TPO $\preccurlyeq_{\Psi-b}$ depicted in Figure 4b. The partition $\{a\} \dot\cup \{b, c\}$ is not a syntax splitting for $\preccurlyeq_{\Psi-b}$. Furthermore $\preccurlyeq_{(\Psi-b)|_{\Sigma_2}} \neq \preccurlyeq_{\Psi|_{\Sigma_2}-b}$. □

5.4 C-Contraction

C-changes are a special kind of change operations on ranking functions (Kern-Isberner et al. 2017; Beierle et al. 2019) that are based on the principle of conditional preservation (Kern-Isberner 2001; Kern-Isberner 2004). While c-changes are defined to handle changes with conditionals, we only consider changes with formulae here.

Definition 10 (c-change (Kern-Isberner et al. 2017))**.** *Let κ be a ranking function and $\mathcal{C} = \{C_1, \ldots, C_n\}$ be a set of propositional formulae. A change from κ with $\mathcal{C}$ to κ° is a* c-change *if there are rational numbers $\gamma_1^+, \gamma_1^-, \ldots, \gamma_n^+, \gamma_n^-, \kappa_0$ such that*

$$\kappa^\circ(\omega) = \kappa_0 + \kappa(\omega) + \sum_{\substack{i=1 \\ \omega\models C_i}}^{n} \gamma_i^+ + \sum_{\substack{i=1 \\ \omega\models \overline{C_i}}}^{n} \gamma_i^-$$

and κ° is a ranking function.

In practise, the value κ_0 is used to adjust the ranks such that the minimal worlds have rank 0. If $\kappa^\circ(\overline{C_i}) - \kappa^\circ(C_i) > 0$ for every $i = 1, \ldots, n$, then the change is called a *c-revision*. If $\kappa^\circ(\overline{C_i}) - \kappa^\circ(C_i) \leq 0$ for every $i = 1, \ldots, n$, then the change is called a *c-contraction*. A special case of the c-contraction is the *c-ignoration* with $\kappa^\circ(\overline{C_i}) - \kappa^\circ(C_i) = 0$ for every $i = 1, \ldots, n$.

Proposition 9. *C-contractions fulfil (P^{it-ocf}_{-}) in the sense that every contraction operator that uses only c-contractions, fulfils (P^{it-ocf}_{-}).*

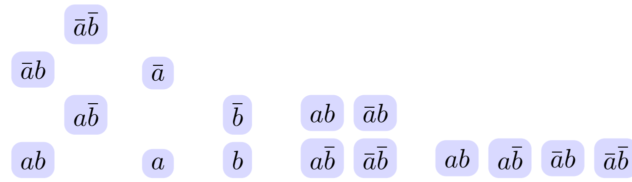
Proof. Let $-$ be a contraction operator that uses only c-changes. Let $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ be a ranking function with syntax splitting $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \ldots, C_n\}$ be a set of propositional formulae such that $C_i \subseteq \mathrm{Form}(C_i)$ for $i = 1, \ldots, n$. Then $\kappa^- = \kappa - C$ has the form:

$$\begin{aligned}\kappa^-(\omega) &= \kappa_0 + \kappa(\omega) + \\ &\begin{cases}\gamma_1^+ & \text{if } \omega \models C_1 \\ \gamma_1^- & \text{if } \omega \not\models C_1\end{cases} + \cdots + \begin{cases}\gamma_n^+ & \text{if } \omega \models C_n \\ \gamma_n^- & \text{if } \omega \not\models C_n\end{cases} \\ &= \kappa_1^-(\omega_1) \oplus \cdots \oplus \kappa_n^-(\omega_n)\end{aligned}$$

where $\omega = \omega_1\omega_2 \ldots \omega_n$ with $\omega_i \in \mathrm{Int}(\Sigma_i)$ for $i = 1, \ldots, n$ can be any world. The κ_0^i are chosen such that

$$\kappa_i^-(\omega_i) = \kappa_0^i + \kappa_i(\omega_i) + \begin{cases}\gamma_i^+ & \text{if } \omega \models C_i \\ \gamma_i^- & \text{if } \omega \not\models C_i\end{cases}$$

is a ranking function for $i = 1, \ldots, n$. □



(a) κ (b) κ_1 (c) κ_2 (d) $\kappa - \{a, b\}$ (e) $(\kappa - a) \oplus (\kappa - b)$

Figure 5: This contraction operator uses only c-contractions and does not fulfil (MK^{ocf}).

Arbitrary c-contractions do not fulfil (P^{ocf}_{-}), as the choice of the impacts does not necessarily respect syntax splittings. Consider the following counter example:

Example 3. *Let $\kappa = \kappa_1 \oplus \kappa_2$ be the ranking function shown in Figure 5a with the syntax splitting $\Sigma = \{a\} \dot\cup \{b\}$. Consider a contraction operator that uses the impact ${}^a\gamma^- = -2$ for the contraction $\kappa_1 - a$, the impact ${}^b\gamma^- = -1$ for the contraction $\kappa_2 - b$, and the impacts $\gamma_1^- = -2, \gamma_2^- = -2$ for the contraction $\kappa - \{a, b\}$. The impacts γ^+ are zero for each contraction. This contraction operator does not fulfil (MK^{ocf}) as $(\kappa - \{a, b\})(ab) = 1$ but $((\kappa - a) \oplus (\kappa - b))(ab) = 0$.*

For c-changes we can show a property similar to (P^{ocf}_{-}).

Proposition 10. *Let $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ be a ranking function with syntax splitting $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \ldots, C_n\}$ a set of formulae such that $C_i \subseteq \mathrm{Form}(C_i)$ for $i = 1, \ldots, n$. Let $\kappa^- = \kappa - \mathcal{C}$ be a c-contraction with impacts $\gamma_1^-, \ldots, \gamma_n^-$ and $\gamma_1^+, \ldots, \gamma_n^+$. Then it holds that*

$$\kappa - \mathcal{C} = (\kappa_1 -_1 C_1) \oplus \cdots \oplus (\kappa_n -_n C_n)$$

where $\kappa_i -_i C_i$ is a c-contraction with impacts γ_i^+, γ_i^-.

Proof. Let κ and $\mathcal{C}$ be as in the proposition. Let $\kappa^- = \kappa - \mathcal{C} = \kappa_1^-(\omega_1) \oplus \cdots \oplus \kappa_n^-(\omega_n)$ with ranking functions

$$\kappa_i^-(\omega_i) = \kappa_0^i + \kappa_i(\omega_i) + \begin{cases}\gamma_i^+ & \text{falls } \omega \models C_i \\ \gamma_i^- & \text{falls } \omega \not\models C_i\end{cases}$$

for $i = 1, \ldots, n$ as in the proof of Proposition 9. Every $\kappa_i^- = \kappa_i -_i C_i$ is a c-change with impacts γ_i^+, γ_i^-. It is left to show that this is a contraction, i.e. $\kappa_i^-(\overline{C_i}) = 0$. Assumption: There is an i such that $\kappa_i^-(\overline{C_i}) \neq 0$. Therefore, $\kappa^-(\overline{C_i}) \neq 0$. This is a contradiction, as $\kappa - \mathcal{C}$ is a c-contraction. □

Additionally, contraction operators that are based on c-ignorations fulfil (MK^{ocf}) and therefore (P$_-^{ocf}$).

Proposition 11. *A contraction operator that uses only c-ignorations fulfils (MK^{ocf}).*

Proof. Let $\kappa = \kappa_1 \oplus \cdots \oplus \kappa_n$ be a ranking function with syntax splitting $\Sigma_1 \dot\cup \cdots \dot\cup \Sigma_n$ and $\mathcal{C} = \{C_1, \ldots, C_n\}$ such that $C_i \subseteq \mathrm{Form}(C_i)$ for $i = 1, \ldots, n$. Let $\kappa^- = \kappa - \mathcal{C} = \kappa_1^-(\omega_1) \oplus \cdots \oplus \kappa_n^-(\omega_n)$ such that

$$\kappa_i^-(\omega_i) = \kappa_0^i + \kappa_i(\omega_i) + \begin{cases} \gamma_i^+ & \text{if } \omega \models C_i \\ \gamma_i^- & \text{if } \omega \not\models C_i \end{cases}$$

are ranking functions for $i = 1, \ldots, n$ as in the proof of Proposition 9.

Assumption: There is an i such that $\kappa_i - C_i \neq \kappa_i^-$. Because c-ignorations with a single formula are unique, κ_i^- cannot result from a c-ignoration of κ_i with C_i. κ_i^- is the result of a c-change of κ_i with C_i. Therefore, either $\kappa_i^-(C_i) \neq 0$ or $\kappa_i^-(\overline{C_i}) \neq 0$. This implies either $\kappa^-(C_i) \neq 0$ or $\kappa^-(\overline{C_i}) \neq 0$, contradicting the fact that $\kappa - \mathcal{C}$ is a c-ignoration. □

In summary, none of the examined contraction operators for total preorders fulfils (P$_-^{it}$), while every c-change fulfils (P$_-^{it-ocf}$). The Postulates (MK^{ocf}) and (MKs) are fulfilled by some, but not all considered contraction operators.

6 Conclusions and Further Work

In this paper, we introduced new syntax splitting postulates for belief contractions. Each of the postulates (P$_-^{it-ocf}$), (P$_-^{it}$), and (K3) describes, for different frameworks, that a contraction preserves a syntax splitting if the contracted information uses only variables from one partition of it. The postulates (MK^{ocf}), (MK), and (K1) together with (K2) describe that a contraction of beliefs with a syntax splitting only affects the parts that share variables with the removed information. For ranking functions, (P$_-^{it-ocf}$) and (MK^{ocf}) together are equivalent to (P$_-^{ocf}$). In a similar way, (K1), (K2), and (K3) together are equivalent to (P$_-^{K}$). An overview over the new postulates is shown in Figure 1.

The postulates (MKs) and (P$_-^{it}$s) were applied to the natural, the moderate, and the lexicographic contraction. The natural and the moderate contraction fulfil (MK), the lexicographic contraction does not fulfil it. None of the three contraction operators fulfils (P$_-^{it}$). C-changes were analysed with respect to the postulates (MK^{ocf}) and (P$_-^{it-ocf}$). While all c-changes fulfil (P$_-^{it-ocf}$), only some of the contraction operators based on c-changes fulfil (MK^{ocf}).

In our current work, we are developing characterisations for c-change operators that satisfy not only (P$_-^{it-ocf}$) but (P$_-^{ocf}$). For this, we will extend the selection strategies for c-representations introduced in (Kern-Isberner, Beierle, and Brewka 2020) to the more general c-change framework. Further work also includes analysing more contraction operators with respect to the new postulates and a deeper analysis of the relation between syntax splitting and c-changes. This might not only provide insights on the operators but also on syntax splitting in general and how it should be handled. Furthermore, it will be interesting to generalize the postulates so that they can be applied also in other settings.

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